Thermal Conductivity and Charge Relaxation in Strong-Coupling Superconductors

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The temperature-dependent thermal conductivity and the charge relaxation time of a strong-coupling superconductor are determined from solutions to the Eliashberg equations and a kinetic equation for the distribution function. The results yield a pronounced maximum at low temperatures in the thermal conductivity of pure Pb and a gradual disappearance of the maximum with increasing amounts of impurity scattering.

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Shortly after the advent of the BCS theory of superconductivity, considerable effort, starting with the work of Bardeen, Rickayzen, and Tewordt,[1] went into explaining the observed temperature dependence of the electronic contribution to the thermal conductivity $\kappa_s(T)$ and the marked difference in the observed behavior of weak- and strong-coupling superconductors immediately below the transition temperature T_c. In the case of very pure Pb the slope of the thermal conductivity ratio $\kappa_s(T)/\kappa_n(T)$, as a function of T/T_c, was found experimentally to be approximately 9,[2] compared to a slope of about 1.6 (Ref. 3) for a typical weak-coupling superconductor like Sn, $\kappa_n(T)$ being the normal-state conductivity at the temperature T. The explanation of this dramatic difference was given by Ambegaokar and Tewordt,[4] who derived a kinetic equation valid for strong-coupling superconductors, and by Ambegaokar and Woo,[5] who calculated the thermal conductivity in the relaxation-time approximation and obtained a slope of about 11 at T_c.

The renewed interest in nonequilibrium superconductivity in recent years has mainly focused on those nonequilibrium properties that are peculiar to the superconducting state, such as charge imbalance and gap relaxation. There exist, however, a number of reasons for reconsidering the thermal conductivity: (1) The kinetic equations describing thermal transport in superconductors with strong pair breaking have been derived only in the limiting cases where electrons are scattered solely by phonons[4] or solely by nonmagnetic and magnetic impurities.[6] (2) The kinetic equation for pure strong-coupling superconductors[4] has never been solved. All the numerical calculations based on this equation, of which we are aware, have employed the relaxation-time approximation, which does not allow one to assess the validity of the approximation. (3) The kinetic equations that describe electronic thermal conductivity and charge imbalance possess identical electron-phonon collision operators in the absence of band anisotropy. The equations differ only in the driving terms, which have opposite parity like in the case of electrical and thermal conductivity of normal metals. Just as these normal-state transport coefficients are related by the temperature-dependent Lorenz number, we can relate the thermal-transport and charge-relaxation times by a temperature-dependent function, which is universal for weak-coupling superconductors, but may differ for strong-coupling materials.

A kinetic equation for the case of charge imbalance in strong-coupling superconductors has been derived recently by Beyer Nielsen *et al.*,[7] who generalized the equation of Schmid and Schön[8] to allow for strong coupling. For the case of thermal conductivity we have derived a similar kinetic equation which includes electron-phonon scattering as well as collision terms arising from scattering against ordinary and magnetic impurities. The details of the derivation will be published elsewhere.[9] The collision terms are not independent, since the generalized densities of states that appear in them are affected by a combination of all pair-breaking processes, irrespective of whether they originate in electron-phonon collisions, spin-flip scattering, or superflow. The kinetic equation which we have derived contains an electron-phonon collision operator, which differs in one respect from that of Ambegaokar and Tewordt,[4] who neglected certain small terms, which they interpreted as originating in a motion of the condensate. Such terms arose naturally within the Kubo formalism employed by these authors but are absent in our approach. Our kinetic equation differs from theirs by terms in the scattering-in part of the collision operator exhibited below. When solving the kinetic equation we have explicitly verified that these terms play no prac-

tical role on account of their smallness.

In this Letter we present a general kinetic equation for the thermal conductivity of strong-coupling superconductors when phonons as well as nonmagnetic and magnetic impurities are present. We also give results of numerically solving the equation for a model electron-phonon coupling suitable for Pb. Near T_c we determine a slope of about 7.2 for κ_s/κ_n, in good agreement with experiment,[2] but somewhat different from the approximate result of Ambegaokar and Woo.[5] At lower temperatures we find that the thermal conductivity of pure Pb begins to rise again, reaching a maximum which is higher than the value of the thermal conductivity at T_c (Fig. 1). When impurities are added this maximum is shifted towards higher temperatures and eventually disappears (Fig. 2). In the presence of two different scattering mechanisms, such as impurities and phonons, the electronic part of the thermal resistivity is not simply the sum of the resistivities calculated for each scattering mechanism considered separately. The difference, commonly referred to as the deviation from Matthiessen's rule (DMR), may be sizable. In the present work we find the DMR to be up to 30% of the total resistivity. Finally, we have performed the first strong-coupling calculation of charge imbalance for the case of tunnel injection and compared with the weak-coupling result (Fig. 3).

In the following we denote the distribution function by f and we introduce the usual deviation function ψ, which for thermal conductivity is defined by $f=f_0+f_0(1-f_0)\hat{p}\cdot\nabla T\psi$, where f_0 is the Fermi function and $\hat{p}$ a unit vector in momentum space, while ∇T is the temperature gradient. When band anisotropy is neglected the kinetic equation for thermal conductivity becomes

$$X(E)=-\left(2N_1\,\mathrm{Im}ZE-2R_2\,\mathrm{Im}\varphi+\frac{N_1^2-R_2^2}{\tau_{\mathrm{imp}}}+\frac{N_1^2+R_2^2}{\tau_s}\right)\psi(E)$$
$$+\int_{-\infty}^{\infty}dE'\,B(E',E)(N_1^2-R_2^2)(N_1'^2-R_2'^2)\left(1+\frac{N_2N_2'}{N_1N_1'}\right)\psi(E'), \tag{1}$$

where we have used the fact that at low temperatures the momentum of the phonons may be neglected compared with the Fermi momentum. The quantity $B(E,E')$ is given in terms of the usual electron-phonon coupling function α^2F as

$$B(E,E')=\pi\alpha^2F(|E'-E|)\frac{\cosh E'/2T}{\cosh(E/2T)\sinh(|E'-E|/2T)} \tag{2}$$

and τ_{imp} and τ_s are the scattering times for nonmagnetic and magnetic impurities, respectively. The functions Z and φ are the complex renormalization and gap functions which we obtain from numerical solutions of the Eliashberg equation, and N_i and R_i $(i=1,2)$ are generalized densities of states which include smearing effects due to pair breaking.[7,8] The function $X(E)$ is the driving term, which in the case of thermal conductivity is $X(E)=(N_1^2-R_2^2)v_FE/T^2$, where v_F is the (unrenormalized) Fermi velocity. For charge relaxation the electron-phonon part of the collision term is identical, but the nonmagnetic-impurity-scattering term is absent and the magnetic one replaced by

$$(1/\tau_s)[N_1^2+R_2^2-(N_1^2-R_2^2)^2+(N_1N_2+R_1R_2)^2].$$

The driving term for charge relaxation due to tunnel injection is proportional to $f_0(E-eV)-f_0(E+eV)$, where V is the injection voltage. It follows that thermal conductivity and charge relaxation are described by distribution functions that are odd and even in E, respectively. In the absence of impurity scattering a comparison with the kinetic equation derived by Ambegaokar and Tewordt[4] shows that it is identical to Eq. (1) except for the N_2N_2'/N_1N' term. The thermal conductivity κ is

$$\kappa=\frac{N(0)v_F}{3}\int_0^{\infty}dE(N_1^2-R_2^2)\frac{E\psi}{\cosh^2(E/2T)}, \tag{3}$$

and the charge imbalance time τ_Q* is given by a similar integral which apart from the absence of the factor of E under the integral differs only by a constant factor from (3). Here $N(0)$ is the (unrenormalized) normal density of states for a single spin at the Fermi surface.

We now present the solution for thermal conductivity and charge relaxation in the strong-coupling superconductor Pb, with nonmagnetic impurities included. To do this we use the model for the effective electron-phonon coupling α^2F that is discussed by Scalapino,[10] but we include

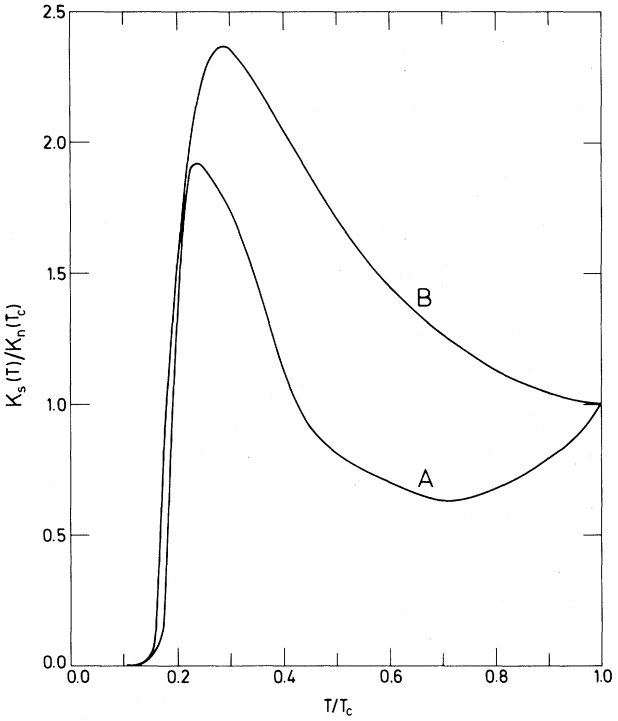

FIG. 1. The temperature dependence of the electronic thermal conductivity κ_s for pure superconductors, normalized to the value at T_c (instead of the value at T). Curve A is the result for Pb and curve B the result for a weak-coupling superconductor.

a Debye tail for frequencies below the transverse peak, $\alpha^2F(E) = 2.75\times10^{-3}E^2$ (E is measured in millielectronvolts). The Debye tail is only important at low temperatures, below $T \lesssim 0.4T_c$, where it causes the turnover and subsequent exponential decrease of $\kappa_s(T)$.

Figure 1 shows our calculated κ_s for a pure weak-coupling superconductor within the Debye model and for pure Pb, normalized to its value at T_c. Near T_c the curves are very different, the weak-coupling result increasing with decreasing temperature while the curve for Pb decreases because of the rapid increase in the gap and the strong energy dependence of the electron-phonon scattering rate. The weak-coupling result has a maximum at $T \simeq 0.3T_c$ and then falls off very rapidly because of the exponential freezing out of the number of quasiparticles. The behavior for Pb is very different, since κ_s shows a minimum at $T \simeq 0.7\ T_c$ and then increases very rapidly, as a result of the decrease in the number of phonons. Below the maximum at $T = 0.25T_c$ the thermal conductivity eventually drops to zero. Figure 2 shows the influence of impurities, which is conveniently discussed in terms of the parameter $c = \tau_{in}/\mathrm{Re}Z(0)\tau_{imp}$, τ_{in} being the inelastic relaxation time at T_c and at the Fermi energy. For our model electron-phonon scattering $\mathrm{Re}Z(0)/\tau_{in}T_c = 0.15$. It is seen that curves B and C follow the $c = 0$ curve down to $T \simeq 0.6T_c$. The temperature dependence below $0.6T_c$ depends critically on the value of c, but the general behavior

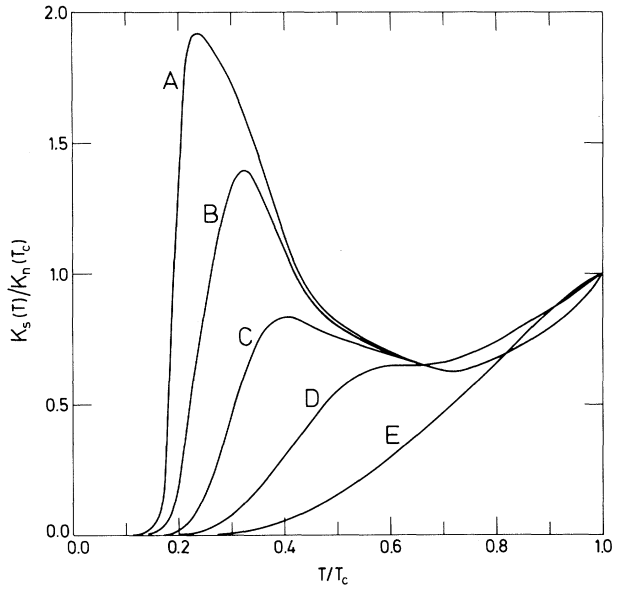

FIG. 2. The temperature dependence of the electronic thermal conductivity κ_s for Pb, normalized to the value at T_c, for various impurity concentrations in terms of $c = \tau_{in}/\mathrm{Re}Z(0)\tau_{imp}$. A, $c = 0$; B, $c = 0.0073$; C, $c = 0.073$; D, $c = 0.73$; E, $c = 73$.

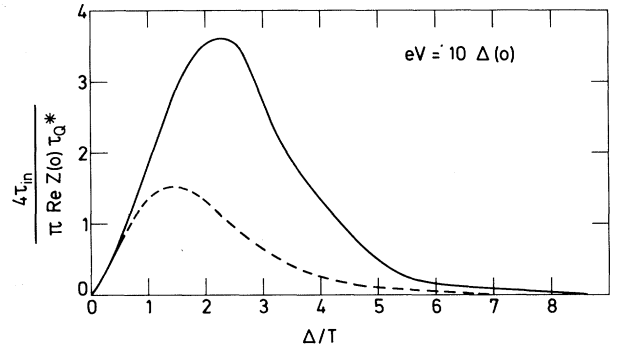

FIG. 3. Plot of the quantity $4\tau_{in}/\pi\ \mathrm{Re}Z(0)\tau_{Q^*}$ as a function of the energy gap Δ in units of the temperature T. The dashed curve shows the result for a weak-coupling superconductor, while the solid curve shows the result for Pb. In the strong-coupling case Δ is the value of the real part of the gap function φ/Z at the gap edge. The parameter τ_{in} is the normal-state inelastic relaxation time at T_c and at the Fermi energy.

is that if $c \lesssim 1$, κ_s has a maximum. If $c \gtrsim 1$ then κ_s always decreases with decreasing temperature.

Recently Mezahov-Deglin[11] measured κ_s in pure Pb and observed a maximum near $T \simeq 0.3T_c$ attributed to lattice conduction. Our calculations show that the maximum in the heat conductivity may be explained purely in terms of the electronic contribution, but we cannot at the moment conclude which mechanism is the more important, as the impurity time τ_{imp} cannot be estimated from the available experimental information.[11]

Turning now to the case of charge imbalance we exhibit in Fig. 3 the charge relaxation rate $\tau_Q{}^{*-1}$ for Pb, for a constant injection voltage V equal to ten times the $T=0$ gap energy $\Delta(0)$. The dashed curve is the result for a weak-coupling superconductor in the Debye model while the solid curve is the result for Pb. In both cases the rate has been normalized to the $\tau_{in}{}^{-1}$ appropriate to the two different α^2F. It is seen that the broadening of quasiparticle states leads to a larger charge relaxation rate than in weak-coupling superconductors. A comparison with experiment shows that the calculated value of $\tau_Q{}^*$ at $T=0.6T_c$ is 30% larger than that measured by Clarke and Paterson.[12] The calculation shows a fast rise of $\tau_Q{}^*$ at lower temperatures, much faster than that observed in the experiment. This may be due to our neglect of gap anisotropy, which excludes charge relaxation due to elastic scattering. Further experiments on Pb to test this would clearly be desirable.

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