Density Threshold for Magnetohydrodynamic Activity in Alcator C

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Experimental measurements of magnetic fluctuations arising from resistive magnetohydrodynamic tearing modes reveal unexpected behavior in Alcator C. After the plasma current has risen to its steady-state level, no significant magnetohydrodynamic activity is observed unless the electron density is above a threshold value, $\bar{n}_c$. For a given toroidal field, $\bar{n}_c$ is remarkably repeatable. We find $\bar{n}_c$ to be proportional to B^2 and independent of I_p.

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Resistive magnetohydrodynamic (MHD) theory predicts a class of instabilities which cause helical filamentation of the equilibrium plasma current resulting in the formation of magnetic island structures in the flux contours.[1-3] These tearing modes have been the subject of intense theoretical and experimental study in recent years, as a result of their implications in energy confinement,[4] current diffusion,[5] and major disruptions.[6] From elementary resistive theory, the instability criterion is found to be

$$\Delta' > 0, \tag{1}$$

where Δ' is defined as the relative jump in the radial derivative of $\tilde{B}_r$ across the tearing layer at the resonant surface. If $\vec{\tilde{B}}$ is defined in terms of a perturbed flux function,

$$\vec{\tilde{B}} = \nabla\{\tilde{\psi}(r)\exp[i(m\theta - nz/R]\}\times\hat{z}, \tag{2}$$

then

$$\Delta' \equiv \frac{(\partial\tilde{\psi}/\partial r)|_{r_s+\epsilon} - (\partial\tilde{\psi}/\partial r)|_{r_s-\epsilon}}{\tilde{\psi}(r_s)}, \tag{3}$$

where ϵ is the narrow tearing layer width. In steady state, Δ' is found by solving the differential equation

$$\frac{1}{r}\frac{\partial}{\partial_r}\left(r\frac{\partial\tilde{\psi}}{\partial r}\right) - \frac{m^2}{r^2}\tilde{\psi} = \frac{-(m/r)(\mu_0 R/B_z)\partial J_z/\partial r}{n - m/q(r)}\tilde{\psi}, \tag{4}$$

subject to proper boundary conditions. Equation (4) is for a cylindrical model of the plasma and is derived by operating with $\hat{z}\cdot\nabla\times$ on the equilibrium pressure balance equation $0 = \vec{J}\times\vec{B} - \nabla p$, substituting in Eq. (2), and linearizing. Note that the stability criterion for tearing modes is shown to depend *solely* on the equilibrium current profile, $J_z(r)$, the toroidal magnetic field, and geometric constants. *No* explicit dependence on electron density is predicted and strong dependence on the total plasma current, I_p, is expected.

On Alcator C, a novel instrument known as the m-spectrum analyzer[7] is used to study fluctuations of the poloidal magnetic field at the vacuum wall due to magnetic islands. The device simultaneously yields the amplitudes and rotation frequencies of the $m = 1$, 2, 3, 4, and 5 modes in real time for the entire duration of the discharge. The time response to changes in mode amplitude is on the order of ~100 μsec. For a typical Alcator plasma shot, the initial current-rise phase lasts 150 msec and is sometimes accompanied by complicated MHD activity. However, that issue will not be addressed here; instead this paper will concentrate on the portion of the discharge where the current has reached its quasi-steady-state plateau. This period may last up to 300 msec and electron-cyclotron emission measurements indicate that the temperature profile is always fitted by a Gaussian to within experimental error, provided the soft x-ray emission is sawtoothing.[8] In Alcator C, where $Z_{\rm eff}(r) \simeq$ const,[9] one can assume $J(r) \sim T^{3/2}(r)$ and use this experimental profile in Eq. (4). Solving for the $m = 2, n = 1$ perturbed flux functions, one finds that $\Delta' > 0$ for the range of limiter q values achieved so far. Therefore we expect to see $m = 2$ magnetic islands throughout the steady-state portion of the discharge. Given that the limit on the m-spectrum analyzer's sensitivity is $\sim\frac{1}{2}$ G, the theory also predicts that the expected signal-to-noise ratio is >10.

This predicted behavior is not observed experimentally. Figure 1(a) shows a typical hydrogen discharge at 55 kG. The lower traces show the amplitude of $\dot{B}_\theta$ $(=\omega\tilde{B}_\theta)$ for the $m = 2$ and $m = 3$ modes. Note the absence of any measurable MHD activity during the current plateau until t

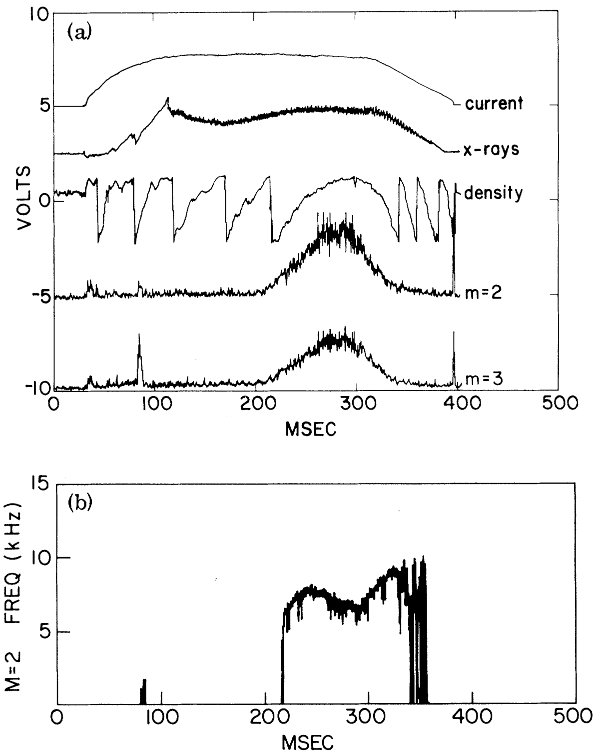

FIG. 1. (a) Plasma discharge illustrating the threshold effect. From top to bottom the traces are plasma current (125 kA/V), soft x-ray emission (arbitrary units), line-average density (0.57×10^{14} cm^{-3}/fringe), $m = 2$ and $m = 3$ amplitudes (24 G kHz/V). The apparent noise on the soft x-ray signal is due to the regular sawtoothing oscillation. (b) Measured frequency of the $m = 2$ mode for the discharge shown in (a).

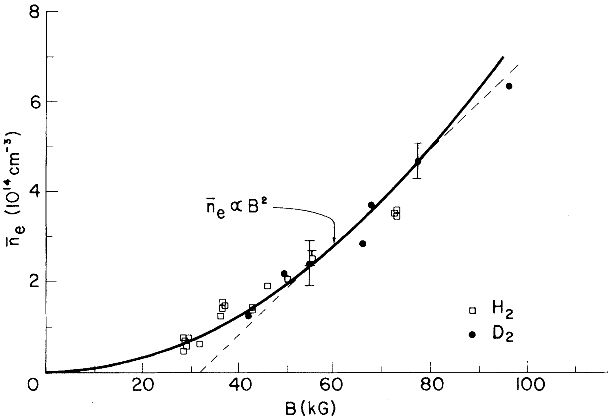

FIG. 2. Density at which MHD activity starts vs toroidal field. Both linear and quadratic least-squares fits are shown. The straight line does not accurately reflect the trend of the data at low field.

$= 205$ msec. When the line-average density reaches $2.4\times 10^{14}/\mathrm{cm}^3$ (~4.2 fringes), $m = 2$ and $m = 3$ modes are detected above the noise level and their amplitudes then roughly follow the density as it continues to rise above the threshold and then fall. The toroidal mode numbers are not measured but are assumed to be $n = 1$ and $n = 2$, respectively. The *maximum* $m = 2$ amplitude corresponds to a perturbation in $\tilde{B}_\theta/B_\theta$ of 0.3%. Only these two mode numbers show appreciable amplitude. This phenomenon is quite repeatable; virtually every H_2 shot which is sawtoothing will have measurable $m = 2$ and 3 magnetic fluctuations during the steady state *only* if the line-average density is above $\bar{n}_c \simeq 2.4\times 10^{14}/\mathrm{cm}^3$, with a remarkably small standard deviation in $\bar{n}_c$ of only $\pm 10\%$. The same behavior is also seen in deuterium discharges, although the standard deviation of $\bar{n}_c$ is then $\pm 20\%$.

The Alcator *C* machine is also commonly run at a toroidal field of 78 kG. Here the observed MHD behavior is qualitatively the same as that at 55 kG, except that the threshold density, $\bar{n}_c$, is at $4.8\times 10^{14}/\mathrm{cm}^3$. Again this threshold value is quite repeatable. This suggests the possibility of a magnetic field dependence of the density threshold for MHD activity. The Alcator *C* tokamak is an ideal machine for doing toroidal field scans and the resulting data are shown in Fig. 2. The magnetic field was varied by more than a factor of 3, from 30 to nearly 100 kG. The vertical bars show the standard deviation of $\bar{n}_c$ at 55 and 78 kG, where there are too many data points to plot individually. The only selection criteria for these data are as follows: (a) the shot exhibited sawtoothing soft x-ray emission (the norm for high-temperature tokamaks), and (b) the threshold density was reached during the current plateau. The solid curve is a best-fit quadratic:

$$\bar{n}_c(10^{14}/\mathrm{cm}^3) = 7.9\times 10^{-4} B^2(\mathrm{kG}^2). \qquad (5)$$

A straight-line fit cannot be absolutely ruled out; however, a least-squares fit would intercept the horizontal axis at 32 kG rather than the origin. This would mean that at low fields, $m = 2$ and 3 activity should be seen at any density—a prediction which is clearly not reflected in the trend of the data near 30 kG. Over the entire range of these data, the value of $\bar{n}/B^2$ at the start of MHD activity has a standard deviation of only 14%. The plasma β, which is proportional to this constant times the temperature, varies from 0.5% to 0.9% on axis and the poloidal β ranges between 0.25 and 0.50. It must be emphasized that the MHD threshold does not represent a density limit;

the line-average density often surpasses $\bar{n}_c$, as demonstrated in Fig. 1(a).

As stated previously, MHD activity is expected to be a function of plasma current. In order to quantify this dependence the limiter q was varied from 2.5 to 4.5 at 55 kG and the plasma current was varied from 167 to 550 kA over the entire range of data represented in Fig. 2. The unexpected result is displayed in Fig. 3. No dependence of the threshold effect on plasma current is detected.

The resolution of this surprising paradox is not obvious, given the MHD tearing theory presented here. The simplest hypothesis for explaining the absence of magnetic islands below $\bar{n}_c$ is to assume that the modes are indeed present, but just not rotating. The magnetic pickup loops would not detect the instabilities in this case. As the density is raised, some unspecified nonideal physics might cause the stationary islands to slowly begin rotating and therefore be observed. But this hypothesis is not supported by the measured frequency data. Typically a large rotation frequency is observed as soon as measurable MHD activity is detected. [Figure 1(b) shows the frequency of the $m=2$ mode for the shot displayed in Fig. 1(a). The $m=2$ frequency varied between 7 and 9 kHz while the $m=3$ was measured at 15–17 kHz.] No tendency for the mode frequency to build up from a slow value is observed in most cases. Given the fact that $\dot{B}_\theta$ in Fig. 1(a) rises slowly from noise level to finite amplitude, but rotation frequency does not, one can conclude that $\tilde{B}_\theta$ must also grow slowly from the noise level. Therefore the measurements do not support the hypothesis that large, stationary magnetic islands exist below the threshold density.

Three possibilities remain for explaining the MHD density-threshold effect: (1) The instability is not a tearing mode, (2) there is a subtle, unmeasurable dependence of the current profile on the line-average density, and (3) other nonideal terms must be added to the resistive MHD model. The possibility that a pressure-driven instability is being observed, rather than a resistive tearing mode, is most unlikely because of the relatively low β values obtained with only Ohmic heating. Ballooning modes are not expected at the peak, on-axis β's of 0.5% to 0.9% typical in Alcator C.

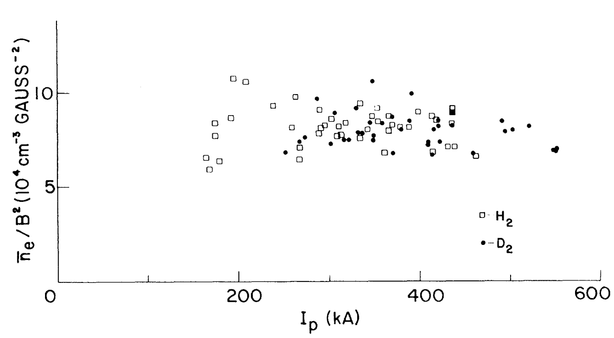

FIG. 3. $\bar{n}/B^2$ at start of MHD activity vs plasma current. No systematic dependence on I_p is observed.

The hypothesis that a subtle coupling between density and current profile is responsible for the n_e dependence and threshold phenomenon of tearing modes in Alcator C cannot be disproved. However, extensive measurements of both the temperature and $Z_{\rm eff}$ profiles show *no* variation with density[8,9] (during sawtoothing discharges). Therefore, we have absolutely no supporting evidence that the current profile is affected by density, at least within the resolution of the diagnostics. This does not preclude the possibility that the MHD threshold may be due to an unmeasurably small-scale flattening of the current profile near the resonant surface. However, this explanation is almost certainly implausible given the reproducibility of the effect, the small standard deviation in $\bar{n}_c$, and the clear, simple dependence on toroidal field.

The theoretical effects of nonideal physics appearing in the literature on tearing modes have been examined individually to determine their role in Alcator C plasmas. These include diamagnetic terms,[10] collisionality corrections,[11] finite β effects,[12] toroidicity,[13] etc. In every case it is found that either the nonideal terms modify the growth rate but do not change the stability criterion (finite gyroradius effects, for example), or they alter the stability criterion, but not in a way which agrees with the experimental measurements. Therefore none of the published modifications to resistive MHD theory can fully account for the observed behavior in Alcator C when examined on an individual basis. To study the combined effects of all of these nonideal terms, one would probably have to run large resistive MHD codes, and this work has not been carried out.

It is unclear whether this type of coupling between density and MHD is only a quirk of the Alcator C device or whether it occurs (or could occur) in other tokamaks as well. No other machine has reported such MHD behavior, although it is noted that the threshold density is relatively high for any given B field and this may explain the absence of such observations in other experiments. For example, during Ohmically heated discharges in the PLT tokamak, the density is in the range $(0.2-0.6)\times10^{14}/\mathrm{cm}^3$ and no $m=2$ activity is detected on sawtoothing discharges.[14] Perhaps a higher density is needed to trigger the tearing mode instability.

The threshold effect plays an important role for major disruptions in Alcator *C*—it divides these events into two distinct classes. For disruptions which occur at densities well above $\bar{n}_c$, $m=2$ and $m=3$ modes are seen to grow to large amplitudes prior to the disruption. This behavior certainly could agree with the present theories of disruptions; namely, the growth and overlap of large magnetic islands causing ergotic field lines and rapid loss of confinement.[15] However, on disruptions below $\bar{n}_c$, no measurable magnetic perturbations are detected prior to the current termination. A careful examination often shows density and soft x-ray anomalies during the millisecond prior to the disruption, but no oscillatory behavior is seen on the Mirnov loops. We presently have no explanation for the cause of disruptions below $\bar{n}_c$, since MHD activity apparently does not play a role here.

In conclusion, a density threshold for MHD activity in Alcator *C* has been observed. The threshold value is remarkably repeatable and increases with toroidal field, but is independent of plasma current. This behavior contradicts the predictions of elementary resistive MHD theory.

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