Scalar Mesons in the Unitarized Quark Model

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(Received 1 February 1982)

It is shown that in spite of their unconventional experimental properties the scalar mesons ϵ, δ, κ, and S^* can be understood as the unitarized remnants of conventional $q\bar{q}$ states. The S^* and δ have large components of $q\bar{q}q\bar{q}$ in the form of virtual two-meson states (mainly $K\bar{K}$).

PACS numbers: 14.40.-r, 12.35.Ht

The understanding of the scalar mesons has been controversial for long because of their quite unconventional experimental properties:

(1) The $\kappa(1400)$ is about 400 MeV heavier than the $\delta(980)$ in contrast to typically 100 MeV (cf. $K^*-\rho$). This difficulty persists in the currently popular four-quark model, although the P-matrix framework by Jaffe[1] can describe the data.
(2) The width ratio $\Gamma(\kappa)/\Gamma(\delta)$ is experimentally >6, while flavor symmetry predicts a much smaller ratio:

$$\Gamma(\kappa)/\Gamma(\delta) = \left|\frac{g_{\kappa K\pi}}{g_{\delta\pi\eta}}\right|^2 \frac{k_K}{k_\eta}\frac{m_\delta{}^2}{m_\kappa{}^2} = 0.71\sin^{-2}(\theta_P - 35.3°) < 2. \quad (1)$$

See Flatté[2] and Morgan.[3] (3) The $S^*(990)$ is nearly degenerate in mass with the $\delta(980)$ although in a $q\bar{q}$ model one expects the $s\bar{s}$ state (S^*) to be 200 MeV heavier than the $u\bar{d}$ state (δ). (4) The S^* has a small coupling to $\pi\pi$ as one expects from an ideally mixed $s\bar{s}$. On the other hand conventional Gell-Mann–Okubo mass formulas predict a large deviation from ideal mixing. (5) The mass of the ϵ has been controversial for long. Should the slow passing of the phase shift through 90° at 700–900 MeV be interpreted as a broad resonance $\epsilon(700)$, or is the phase shift rather described by a very broad $\epsilon(1400)$ with a narrow S^* superimposed?

In this paper I resolve these questions within a rather conventional model, the unitarized quark model (UQM),[4,5] which we have previously applied successfully to the 1^{--}, 0^{-+}, 1^{++}, and 1^{+-} multiplets, explaining signs and magnitudes of mass splittings, deviations from ideal mixings, as well as the Q_1Q_2 mixing. In the model I assume a simple unmixed input spectrum for the bare mesons. The physical meson masses are shifted down through hadronic shifts, and mix with each other at the same time as they acquire finite widths to open channels.

For more details on the model I refer readers to our previous work.[4,5] Below I list the most important ingredients relevant to this application:

(1) Complete groups of flavor-related thresholds are taken into account. Thus, e.g., for the flavor-less mesons (ϵ, δ^0, S^*, ϵ_c) the pseudoscalar-pseudoscalar (PP) thresholds are:

$$\pi^0\pi^0,\ \pi^+\pi^-,\ K^+K^-,\ K^0\bar{K}^0,\ \eta\eta,\ \eta\eta',\ \eta'\eta',\ D^+D^-,\ D^0\bar{D}^0,\ F^+F^-,\ \eta_c\eta_c. \quad (2)$$

(2) The coupling constant for each threshold is given by the "quark counting rule"[6] $g_{iab} = \gamma^{SPP}\mathrm{Tr}(\Delta_i\Delta_a\Delta_b)_+$, where γ^{SPP} is an overall coupling constant and where Δ_i are 4×4 matrices given by the quark content of the meson. Thus Δ for η and η' depend on the mixing angle θ_P, which is fixed to $-11°$.

(3) The partial width and mass shift, which are related to a given threshold, are given by the real and imaginary parts of the function $\Pi(s)$. Thus

$$\mathrm{Im}\Pi_{ij}(s) = -\sum_{a,b} g_{iab}g_{jab}(k_a/\sqrt{s})F^2(k_a)\theta(s-s_{ab}) \quad (3a)$$

$$\mathrm{Re}\Pi_{ij}(s) = \pi^{-1}\int_{s_{ab}}^{\infty}[\mathrm{Im}\Pi_{ij}(s')/(s'-s)]ds', \quad (3b)$$

where $F(k)$ is the hadronic form factor which provides the cutoff and $s_{ab}=(m_a+m_b)^2$.

(4) The hadronic form factor is assumed to be flavor independent and of the form $F(k) = \exp[-(k/k_{\mathrm{cutoff}})^2]$.

I have also done calculations with $F=1$, and traded the cutoff into a subtraction constant, whereby Eq. (3b) gives the Chew-Mandelstam function (see, e.g., Basdevant, Froggatt, and Petersen,[7] and Achasov, Devianin, and Shestakov[8]). The relation between the present model and that of the P matrix has been discussed by Maciel and Paton.[9]

The parameters of the model are as follows:

(1) The overall coupling constant γ^{SPP}, which is determined mainly by the $K\pi$ S-wave phase shift. We find a best value $\gamma^{SPP}=0.96$ GeV.

(2) The cutoff parameter k_{cutoff}, which was in

our previous applications found to be larger than 0.6 GeV/c with best value near 0.7 GeV/c. For the 0^{++} mesons cutoffs >0.6 GeV/c give reasonable fits. In the fits shown $k_{\rm cutoff}=0.7$ GeV/c, which corresponds to a hadron radius 0.7 fm.

(3) The bare $u\bar{u}$ meson mass m_0, which for the 0.7-GeV/c cutoff is found to be 1.34 GeV or very near the value found for the axial mesons[5] (1.32 GeV). This suggests that the mass splittings between the 0^{++}, 1^{++}, and 1^{+-} mesons are to a large extent due to the hadronic mass shifts and the $\vec{L}\cdot\vec{S}$ and tensor forces are small.

(4) The quark mass difference $m_d - m_u$ is fixed at 4 MeV and $m_c - m_u$ is fixed by the $\epsilon_c = \chi(3414)$ mass. For $m_s - m_u$, remarkably enough the fit gives a value 80 MeV, and $m_c - m_u = 1.042$ GeV near the values found in our previous applications.

(5) Other possible parameters are the couplings to other groups of thresholds, in particular γ^{SVV}, which, however, only marginally affects the fit. This is understandable since these thresholds are above 1.5 GeV. In the fits shown $\gamma^{SVV}=0$.

The predicted resonance parameters are shown in Table I. Since the scalar mesons are very broad (κ,ϵ) or lie close to the $K\bar{K}$ threshold (δ, S^*), it is not sufficient to understand quoted masses and widths, but we must look at the partial waves themselves. If the latter are dominated by the resonances in question the T-matrix elements normalized to "Argand" units can be written:

$$T_{ab\text{-}cd} = \sum_{i,j} g_{abi}[m_{0,ij}{}^2 + \Pi_{ij}(s)]^{-1} g_{cdj} \times [(k_a k_c)^{1/2}/s] F(k_a) F(k_c), \qquad (4)$$

where the sum runs over the resonances. These partial-wave amplitudes satisfy by construction the coupled-channel unitarity condition $T^\dagger - T = 2iTT^\dagger$, which follows from Eq. (3a). The K matrix associated with the T matrix can be obtained simply by putting $\mathrm{Im}\Pi_{ij}(s)=0$ in Eq. (4).

TABLE I. Predictions of the model.

Meson	Mass (MeV)	$\Gamma = \mathrm{Im}M^2/m$ (MeV)	Peak width (MeV)
δ	970	500	100
κ	1350	430	
S^*	985	400	50
ϵ	1237	1400	
$D_s = c\bar{u}$	2327	140	
$F_s = c\bar{s}$	2361	90	
$\epsilon_c = c\bar{c}$	3414	1	

In Fig. 1(a) I show the S-wave $K\pi$ phase shift and the associated Argand diagram. The curves are the predictions of the model and the data are from Kelly *et al.*,[6] Estabrooks,[10] and Aston *et al.*[11] From these data two parameters, γ^{SPP} and $m_0(\kappa) = m_0 + m_s - m_u$, are determined. The prediction deviates from a Breit-Wigner resonance in two important respects: One has a flatter energy dependence of the phase shift below 1300 MeV and a rapid increase in the phase shift from 1350 MeV to the $K\eta'$ threshold at 1452 MeV. The reason for this can be seen most clearly from Fig. 1(b), where $m^2 + \mathrm{Re}\Pi(s)$ is shown. The $\kappa(1350)$ falls between the two most important thresholds $K\pi$ and $K\eta'$. Therefore below the resonance $\mathrm{Re}\Pi$ increases, making the apparent width larger. On the other hand, from 1350 to 1425 MeV there is

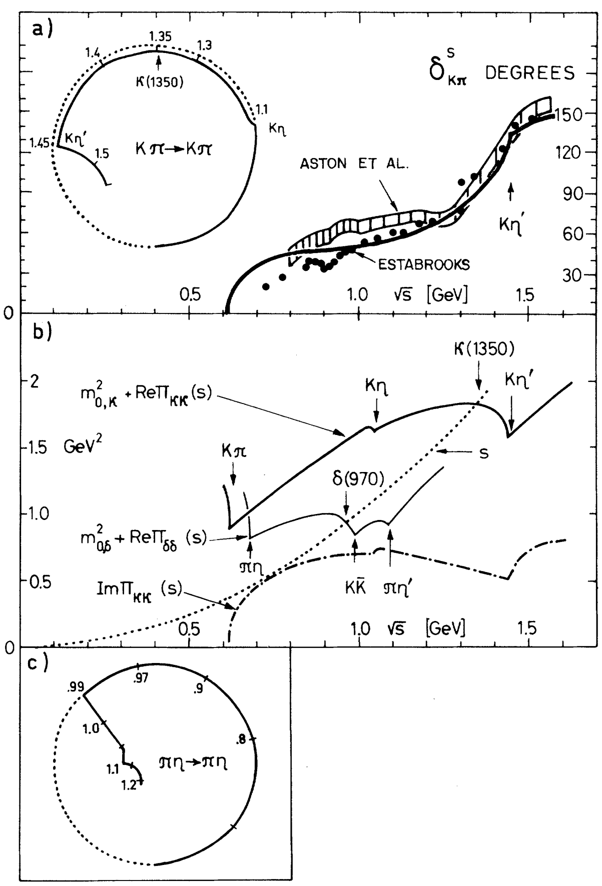

FIG. 1. (a) the $K\pi$ S-wave phase shift and the associated Argand diagram with data from Refs. 10 and 11. (b) The $\sqrt{s}$ dependence of the renormalized mass for the κ and the δ. The intersections of the dotted curve (s) with the solid curves determine the resonance masses. The dot-dashed curve is $\mathrm{Im}\Pi(s)$ for the κ. (c) The predicted Argand diagram for the $\pi\eta$-$\pi\eta$ S wave.

a square-root-like drop in ReΠ coming from the $K\eta'$ giving a rapid motion in the Argand diagram. Previously this rapid motion was used as an argument for a heavier κ(1450 MeV), with a rather narrow width, superimposed upon a background. We need no background and the analysis suggests strongly that the resonance is at 1350 MeV, where the phase shift passes 90°, in agreement with early simpler analyses.

In Fig. 1(b) the function $m_0^2 + \mathrm{Re}\Pi(s)$ for the δ is also shown. One observes that the hadronic shift is much larger for the δ than for the κ, simply because the three important thresholds $\pi\eta$, $K\bar{K}$, and $\pi\eta'$ are all near 1 GeV and all contribute to the mass shift of the δ. Thus the dominant part of the κ-δ mass difference is due to a larger hadronic shift for the δ compared to that for the κ. This answers the first question in the introduction. Therefore we can fit simultaneously the δ and the κ mass with a small $m_s - m_u$ value (< 100 MeV) as we found for other multiplets.

Another important observation is that Re$\Pi(s)$ drops strongly before the $K\bar{K}$ threshold. Therefore the apparent width of the δ is reduced by a factor 2–3. Together with the absorption above the $K\bar{K}$ threshold the observed width of the δ peak should be much smaller than expected from a definition $\Gamma = \mathrm{Im}[\Pi(m^2)]/m$; cf. the Argand diagram in Fig. 1(c). Essentially the same mechanism for a narrow δ in spite of a large $\pi\eta'$ coupling was pointed out by Flatté[2] using a simpler analytic form, and was also discussed more recently by Achasov, Devianin, and Shestakov.[8] Together with the nearly opposite situation for the κ width, this answers the second question in the introduction.

With two parameters fixed by the $K\pi$ phase shift, with $m_s - m_u$ at 80 MeV, and a fixed cutoff, no other free parameters remain for the ϵ and the S^* resonances.

When we look at the $\pi\pi$ phase shift δ_0^0 [Fig. 2(a)] the model predicts, remarkably enough, the well-known rapid phase shift increase near 980 MeV due to the S^*.

Qualitatively it is easy to understand this result because the $K\bar{K}$ Clebsch-Gordan coefficient is twice as large for an $s\bar{s}$ state as for the δ and, in addition, the nearby $\eta\eta$ channel also contributes to the S^* mass shift. This makes the S^* mass shift the largest of the four resonances and answers the third question in the introduction: The quark mass difference $2(m_s - m_u)$ in $m(S^*) - m(\delta)$ is compensated by a larger mass shift for the S^* than the δ.

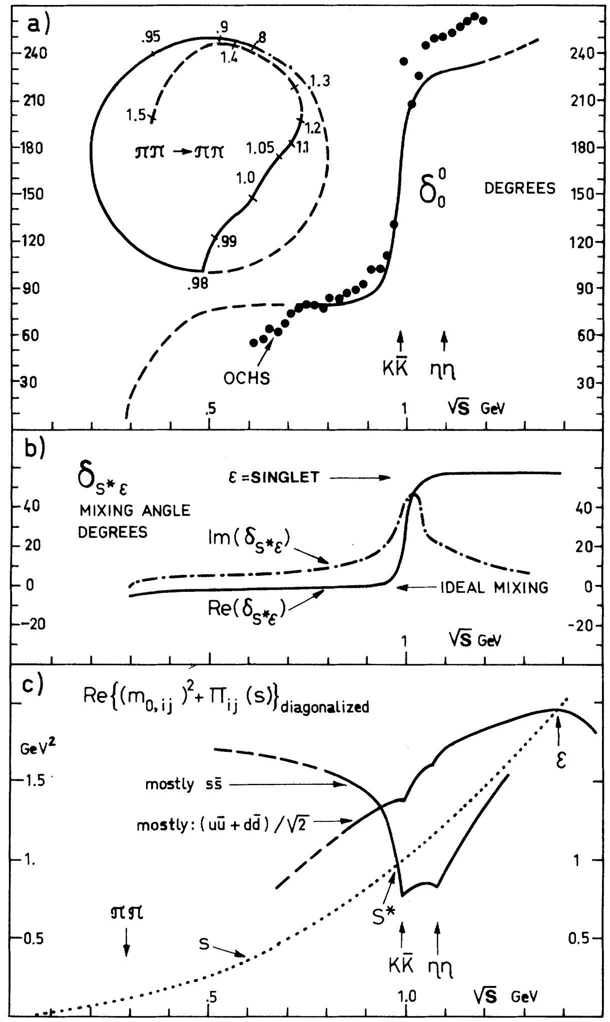

FIG. 2. (a) The $\pi\pi$ S-wave phase shift and the associated Argand diagram. The data points are from Ref. 12. (b) The complex mixing angle between the S^* and ϵ. At the S^* mass we have nearly ideal mixing, while at the ϵ mass the ϵ is nearly an SU(3) singlet. (c) The real parts of the eigenvalues of the mass matrix for the S^* and ϵ. The masses are determined from the intersections with the curve s.

As to the fourth question it is clear that because of the strong energy dependence of the mass matrix, simple Gell-Mann–Okubo mass formulas break down. In fact, since the mass matrix $m_{0,ij}^2 + \Pi_{ij}(s)$ is diagonalized by a complex orthogonal matrix, the mixing angles become both energy dependent and complex. In Fig. 2(b) the S^*-ϵ mixing angle is shown. Below the $K\bar{K}$ threshold we have nearly ideally mixed states, but above the $K\bar{K}$ threshold the diagonal elements of $m_{0,ij}^2 + \Pi_{ij}(s)$ become nearly equal for the S^*-ϵ submatrix and therefore there is a rapid change in the mixing angle from near ideally mixed to a near singlet ϵ.

In Fig. 2(c) the real parts of the eigenvalues of the mass matrix are shown for the S^*-ϵ system. As can be seen the mass of the S^* is predicted at 980 MeV and the ϵ mass is at 1390 MeV. The mass is defined as the energy where the real part of an eigenvalue of the inverse propagator passes zero.

As to the fifth question our analysis agrees with the interpretation of Morgan[3] from 1974 that the $\pi\pi$ phase shift is described by a very broad $\epsilon(1400)$ superimposed upon a narrower S^*. The $\pi\pi$ phase shift passes 90° near 900 MeV because of the combined effect of the ϵ and the S^* and the motion is slow because of the strongly energy-dependent contribution to $\mathrm{Re}\Pi(s)$ from the $\pi\pi$ threshold.

A posteriori, we see two reasons why previous analyses have not succeeded in describing the 0^{++} mesons as $q\bar{q}$ states: (1) Previously no one has taken into account all PP thresholds. (2) The P-matrix analyses, as well as many K-matrix analyses, have assumed a too simple form for $\Pi(s)$. For example, the "complex phase space" obtained by naively continuing $-k_a/\sqrt{s}$ of Eq. (3a) to give a contribution to $\mathrm{Re}\Pi(s)$ like $[(4m^2-s)/s]^{1/2}\theta(4m^2-s)$ is a very bad approximation of the Chew-Mandelstam function obtained from Eq. (3b).

Some analyses, e.g., Refs. 7 and 8, use correct analytic forms but do not study the whole flavor multiplet nor include all thresholds.

As a final remark, I believe it would be a fruitful approach to calculate the bare-mass spectrum, the overall coupling constants, and vertex functions from QCD (or another fundamental theory), and then fold in the unitarity effects before comparing with data.

I thank Matts Roos for many interesting discussions, Carleton DeTar for useful comments, and Kauko Heikkilä for help with the computing.

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