## Cluster Approach to Intranuclear Cascade for Relativistic Heavy-Ion Collisions

T. Kodama, S. B. Duarte, and K. C. Chung

Centro Brasileiro de Pesquisas Fisicas, Rio de Janeiro, Brazil

and

R. A. M. S. Nazareth

Instituto de Fisica, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil (Received 21 June 1982)

<sup>A</sup> new approach to the intranuclear cascade model for relativistic heavy-ion reactions is presented. The effect of nucleon concentration on the collision process is explicitly included. It is found that the contributions from the nonbinary processes are far from being negligible. Such processes are shown to broaden the angular distribution of inclusive proton spectra for  $20Ne + 238U$  head-on collisions.

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In the last few years, a number of theoretical and experimental studies on relativistic heavyion collisions (RHIC) have been done. The ultimate aim of such investigations is to learn about the properties of nuclear matter at extreme conditions of density and temperature. Up to the present moment, our understanding of this subject is still far from being clear. Most of the available experimental data are inclusive, averaged over all impact parameters. Such data are shown to be unable, in general, to give detailed information about the collision dynamics. Thus they hardly detect any differences among the current models.

However, a new type of information is available from recent multiplicity-selected experiments. ' Stöcker et  $al^2$  discussed the results of the highmultiplicity-selected Ne(393 MeV/u)+ U reaction (head on) with respect to the predictions of several theoretical models. They claimed that the model based on nuclear fluid dynamics has some advantage in reproducing the side-peaked angular distribution over the conventional intranuclear cascade calculation.

Nevertheless, the Monte Carlo approach is useful to simulate the microscopic process, because it furnishes a direct way of calculating microscopic quantities without the introduction of any phenomenological parameter. It still remains to discuss further to what extent the intranuclear cascade model is adequate in simulating the RHIC.

The basic point of the conventional cascade calculation is the assumption of sequential binary collisions.<sup>3</sup> However, this is not a trivial assumption. Rigorously speaking, such treatment is justified only in the limit of dilute-gas approximation, where correlations, during the nucleon-

nucleon collisions, are negligible. In RHIC, even when the incident energy is high enough so that the nucleon de Broglie wavelength is smaller than the mean nucleon interdistance at the normal nuclear density, we hardly expect that the system behaves as a dilute gas during the whole process. In fact for central collisions of heavy systems, a local density increase due to compression may cause correlations in nucleon-nucleon collisions. Such correlations manifest the nonbinary character of RHIC. These nonbinary correlations are fundamental for the possible appearance of novel collective phenomena, such as hydrodynamic shock waves, pion condensation, and density isomeric states of nuclear matter. Thus it is essential to study the nonbinary correlations in the context of the intranuclear cascade model.

In this work, we propose a new approach to the intranuclear cascade process which permits us, in a simple way, to incorporate correlation effects due to nucleon concentration, although it is clear that any intranuclear cascade simulation only gives a necessarily restricted description of actual processes. The procedure is the following.

At first, we simulate the projectile and target nuclei by randomly generated coordinates and momenta for  $A_P+A_T$  nucleons, where  $A_P$  and  $A_T$ are the mass number of projectile and target nucleus, respectively. The surface diffuseness of the density distribution is taken into account and the degenerate Fermi-gas model is used. Then the incident nucleus with impact parameter b is boosted by a Lorentz transformation, according to a laboratory incident energy  $E_{in}$ . The time evolution of the system is followed step by step with a time increment  $\Delta t$  appropriately chosen (see later discussion).

At each time  $t$ , we define clusters of nucleons

according to the following prescription: (a) List, for each nucleon, say the *i*th one, all nucleons  $j$ whose closest proximity  $r_{i j}$  to this *i*th nucleon occurs during the time interval  $\Delta t$ , under the condition

$$
\pi r_{i j}^{2} \leqslant \sigma^{\text{tot}}(E_{i j}),
$$

where  $\sigma^{\text{tot}}$  is the total nucleon-nucleon cross section, and  $E_{ij}$  is the relative energy. The above procedure gives an estimate of nucleon concentration around each nucleon. According to this concentration we classify all nucleons into groups. (b) Then, for each group, select the pair  $(i, j)$ which has the smallest relative de Broglie wavelength  $\lambda_{\min}$  and associate to this pair all other nucleons in the group whose distance from  $i$  or  $j$ is smaller than  $\lambda_{\min}$ . Such a subgroup is called a cluster. This procedure is continued until all nucleons are grouped into clusters. In this way  $m$  nucleons in a cluster are supposed to collide, during the time interval  $\Delta t$ , in a correlated manner, which we refer to as an  $m$ -body collision.

Once clusters are formed, nucleon collisions are simulated in each cluster. For  $m=2$  (binary collision), we adopt the same procedure as  $\frac{1}{2}$  corrision), we adopt the same procedure as proton-proton, and neutron-proton collisions. Cross-section data are taken from the particle data compilation.

For *m*-body collisions  $(m \ge 3)$ , in principle, quantum correlations must be taken into account. Unfortunately we have neither theoretical nor experimental information for treating such correlations. Therefore, we tentatively simulate these  $m$ -body collisions by regarding them as an iso-



FIG. 1. Relative frequency of  $m$ -body collisions in the  $^{12}C + ^{12}C$  reaction (E<sub>in</sub>/A = 0.8 GeV) at  $\theta_{1ab} = 15^{\circ}$ . Note that the maximum contribution of binary collisions occurs in the quasielastic region (Ref. 6).

tropic decay of a compound state of  $m$  nucleons in the cluster, namely, the final  $m$  momenta are generated randomly in the c.m. system of the cluster, according to the invariant phase space. ' It should be noted that, by doing this, we are practically washing out the quantum correlations in  $m$ -body collisions, since such a momentum distribution is equivalent to that obtained by taking an ensemble average of many binary collisions among  $m$  particles. However, we hope that this particular choice will be harmless for estimating the amount of nonbinary processes in RHIC.

The intranuclear cascade described above is continued until all collisions cease. The ultimate momentum distribution is used to calculate the differential cross section of emitted particles.

In our approach, the time interval  $\Delta t$  has a crucial physical meaning. It should be identified with the time scale of nucleon-nucleon collisions  $\tau_{nn} = (1-2 \text{ fm})/c$ . Since the conventional classical binary cascade assumes  $\tau_{nn} = 0$ , our model falls back to the binary collision case in this limit. On



FIG. 2. Proton spectra for the  ${}^{12}C+{}^{12}C$  (E<sub>in</sub>/A = 0.8) GeV) reaction. Experimental data (dots) are compared with calculated spectra (solid line). The bars denote statistical uncertainties (total event number, 7000). The data are not corrected for the emission of light composite nuclei.



FIG. 4. Angular distribution for protons emitted from central collisions of  $^{20}$ Ne +  $^{238}$ U (E<sub>in</sub>/A = 393 MeV). Dashed curve: our calculation; solid curve: experi mental. Numbers indicated are the final proton energy in megaelectronvolts.

the other hand, for very large  $\Delta t$  and large effective nucleon-nuclon interaction range, our model tends to a fireball-type one.

In this Letter, we apply our model to the reactions  ${}^{12}C + {}^{12}C$  (E<sub>in</sub>/A = 800 MeV) and  ${}^{20}Ne + {}^{238}U$  $(E_{in}/A = 393$  MeV), head-on collision). We restricted ourselves here only to the nucleonic degree of freedom. We encountered in both cases an unexpectedly large frequency of nonbinary collisions. Even for a small system such as  $^{12}C$  $+$ <sup>12</sup>C, the contributions from nonbinary processes reach up to  $60\%$ . In Fig. 1, we show the percentage of each  $m$ -body contribution to the invariant differential cross section of the reaction  $^{12}C+^{12}C\rightarrow p+X$  (E<sub>in</sub>/A = 800 MeV) at  $\theta_{1ab}$  = 15<sup>°</sup> as a function of the final energy of emitted protons. The peak of the binary curve around the energy of elastic scattering indicates the existence of a relatively large contribution from knockout protons at this angle. It is worthwhile to note that this is consistent with the shoulder- arm structure of the proton spectra. $6$  Figure 2 shows the calculated proton spectra. The agreement with the experimental data is very good, as expected for such impact-parameter-averaged data.

In the case of the  $^{20}$ Ne+ $^{238}$ U reaction (E<sub>in</sub>/A) = 393 MeV, head on), we present the mean frequency of  $m$ -body collisions in Fig. 3. It is seen that the nonbinary collisions are far from being negligible. The rapid increase and slow decrease of these curves, which. attain their maxima at the same time, suggest a mechanism of a somewhat abrupt compression followed by an adiabatic-type expansion. As a matter of fact, it is found that the calculated relative frequencies behave as a Poisson distribution,  $P(m) = \lambda^{m-1} e^{-\lambda} / (m-1)!$ , where  $\lambda = \lambda(t)$  is proportional to the average den-



FIG. 3. Mean frequencies of  $m$ -body collisions as a function of time for the <sup>20</sup>Ne + <sup>238</sup>U reaction (E<sub>in</sub> /**A** = 393 MeV, head on). The whole process has a relatively long duration, around five times the time  $T_0$  necessary for Ne to pass through U without interaction.

sity at time  $t$ . This indicates that the nucleon concentration effect is correctly reflected on the formation of clusters.

In Fig. 4, angular distributions of protons emitted from the same reaction are compared with the high-multiplicity-selected data.<sup>1</sup> The calculated curves exhibit little bit broader angular distributions than those of the conventional binary cributions than those of the conventional binary<br>cascade model,<sup>2</sup> although the experimental sidepeaked angular distributions are still far from our results. It should be remembered that our phase-space  $Ansatz$  for the m-body momentum distribution, which is almost equivalent to a local. thermalization, does not favor sideward maxima at large angles. The systematic lowness of our results with respect to the experimental data is probably due to the maximum impact parameter chosen  $(b_{\text{max}} = 1.5 \text{ fm})$ .<sup>7</sup>

In summary, we have presented a new approach to intranuclear cascade calculation, which includes the degree of freedom of nonbinary processes. It was shown that the simple phasespace  $Ansatz$  does not give the experimental sidepeaked angular distributions. However, it is expected that a more elaborate  $m$ -body momentum distribution may enhance the nonbinary collision effects, leading to possible collective phenomena. Hence, it is crucial to investigate further the quantum  $m$ -body correlations in RHIC.

On the other hand, it should be remembered that our approach deals only with local correlation effects. Long-range correlations can also be analyzed in our approach in terms of interaction between clusters. We are studying this ques-

tion, as well. as the problem of the action of the mean field on low-energy outgoing nucleons.

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## Two-Photon X-Ray Emission from Inner-Shell Transitions

Yigal Bannett and Isaac Freund

Department of Physics, Bar-Ilan University, Ramat-Gan, Israel (Received 1 July 1982)

Two-photon emission of x-rays from inner-shell transitions has been observed for the first time. The continuous  $K^{-1}$ - $L^{-1}$  two-photon spectrum of Mo appears to be in excellent agreement with a detailed theoretical analysis. This is not the case for transitions from higher shells.

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The lifetimes of metastable hydrogenic and heliumlike systems have been studied extensively, especially since the pioneering work of Lipeles, Novick, and Tolk' and the classic studies of Marrus and Schmieder.<sup>2</sup> Such metastable states decay either by two-photon electric dipole or by one-photon magnetic dipole or magnetic quadruple transitions. Here, we report the first measurements of two-photon x-ray emission from oneelectron inner-shell transitions. In a many-electron atom,  $K$ -shell hole states are, of course, not metastable, and are filled almost immediate<br>  $($   $\leq$  10<sup>-15</sup> sec) by strong one-photon or Auger tran- $($  < 10<sup>-15</sup> sec) by strong one-photon or Auger transitions. With much smaller probability, a  $K$ shell vacancy may also be filled by a one-electron two-photon process, and the study of such transitions yields a new, rich, multiphoton inner-shell<br>spectroscopy.<sup>3,4</sup> tions yields a new, rich, multiphoton inner-shell spectroscopy.<sup>3,4</sup>

In examining metastable states in hydrogeni and heliumlike ionic beams, $^{\rm 1.2}$  the background due tate<br>1,2 <sub>1</sub> to strongly allowed one-photon transitions is easily eliminated, since the intensities of these decay almost immediately leaving a clean metastable system. In the study of two-photon innershell transitions in solids, however, such as in

the present study of metallic Mo, no simple isolation of the desired process is possible, and the major experimental problem is suppression of spurious effects due to the intense one-photon  $K\alpha$ and  $K\beta$  background. vurious effects due to the intense one-photon  $K\alpha$ <br>dd  $K\beta$  background.<br>As in previous studies,<sup>1,2</sup> we detect both emitte

photons in fast time coincidence. For a pair of identical detectors, the expected ratio of the twophoton to the one-photon count rate may be writ-' $\lim_{\lambda \to 0} N^{(2)}/N^{(1)} \sim 10^{-3} Z^2 (\Delta\Omega/4\pi)$ , where  $\Delta\Omega$  is the collection solid angle, and hydrogenic results' for the  $2p - 1s$  one-photon and the  $2s - 1s$  twophoton transitions have been employed. Although we use a moderately heavy element  $(Z = 42)$  and employ large-area, nearly  $100\%$  efficient Si(Li) energy-dispersive detectors for which  $\Delta\Omega > 1$  sr, nonetheless,  $N^{(2)}/N^{(1)}$  is still only ~2×10<sup>-6</sup>. Under such circumstances great care must be exercised if artifacts are to be avoided.

We have found that when the detectors are illuminated by the large one-photon  $K$ -line flux of Mo, they will, if permitted, talk to each other via a mechanism in which an electron freed in one detector crystal by the primary photoionization event of the detection process generates brems-