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Determination of Relative Signs of Neutron and Proton Transition Matrix Elements: Strong Cancellation Observed for the $^{34}S(0^+ \to 2_2^+)$ Transition

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Hadron scattering is shown to be sensitive to the relative sign of neutron and proton transition matrix elements. The relative signs for the 2_2^+ states are determined to be positive for ^{26}Mg, ^{30}Si, and ^{42}Ca, and negative for ^{34}S on the basis of proton differential cross-section measurements at 650 and 800 MeV. Suppression of the one-step amplitude in the ^{34}S 2_2^+ state causes the interference of one-step and multistep reactions to be experimentally apparent.

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The sensitivity of inelastic hadron scattering (h,h') to both neutrons and protons offers the possibility of increasing our understanding of the neutron structure of nuclei by determining the relative signs of neutron and proton transition multipole matrix elements, M_n and M_p. In this Letter we present, for the first time, an example of hadronic measurements of these relative signs under conditions where the magnitudes (but not the signs) are previously known by electromagnetic (EM) methods.[1] This enables us to compare the experimental (h,h') results with predictions which sensitively depend on the relative sign of M_n and M_p.

A purely EM technique for obtaining the magnitudes of M_n and M_p using $B(E\lambda)$ values from mirror transition rates has been previously developed.[1] For a given analog transition one obtains $M_p(T_z)$ from $B(E\lambda, T_z; J_i \to J_f) = |M_p(T_z)|^2/(2J_i+1)$. To obtain $M_n(T_z)$ one uses the equivalent isospin representation for the matrix elements,

$$M_{n,p}(T_z) = [M_0(T_z) \pm M_1(T_z)]/2, \tag{1}$$

where $M_0(T_z)$ and $M_1(T_z)$ are the isoscalar and isovector transition multipole matrix elements. From charge independence M_0 is independent of

T_z while $M_1(T_z=0)=0$. From charge symmetry $M_1(T_z=-1)=-M_1(T_z=+1)$ so that $M_n(T_z)=M_p(-T_z)$; e.g., $M_p(^{42}\mathrm{Ti})=M_n(^{42}\mathrm{Ca})$. Note that for a pure isoscalar (isovector) transition $M_n=\pm M_p$.

In general, isoscalar nuclear interactions are attractive, and therefore the lowest states of nuclei have the lowest isospin and tend to have large values of M_0. The isovector interaction is repulsive, and the M_1 values tend to be small for transitions between low-lying states. Because of this $M_n(T_z)$ and $M_p(T_z)$ are generally expected to have the same sign for transitions between low-lying states. This general expectation is supported by shell and collective-model calculations[2,3] and experimental data.[4,5]

From an experimental point of view it is of interest both to verify that $|M_1|<|M_0|$ in general and also to identify those rare cases where the opposite is true. From $B(E\lambda)$ one measures the magnitudes of $M_{n,p}(T_z)$ but not their relative sign. Thus there are two possible solutions to Eq. (1): $|M_1|<|M_0|$ and $|M_1|>|M_0|$. For $T=\frac{1}{2}$ doublets there is no separate EM measurement which can be used to determine the relative signs of M_n and M_p. For $T\geq 1$ isobaric systems one can make three or more measurements and resolve this ambiguity by measurements of the lifetime of the analogous transition in the $T_z=0$ nucleus. Lifetime measurements in the $T_z=0$, odd-odd nuclei are generally more difficult since there are more γ branches; nevertheless there have been four cases for the lowest $2^+\to 0^+$ transition in $T=1$ nuclei for which all three transitions have been measured ($A=26$, 30, 34, and 42).[6] The results show that in all cases for the lowest 2^+ transition $M_1/M_0 \ll 1$, i.e., the transitions are predominantly isoscalar in character as one expects and therefore M_n and M_p have the same sign. For the second $2^+\to 0^+$ transition all three lifetime measurements have been performed only for $A=26$ and 30.[6-8] The relative sign is positive for ^{30}Si but because of large experimental errors is undetermined for ^{26}Mg.

Inelastic hadron scattering offers a direct and experimentally convenient method to determine the relative signs of M_n and M_p. Assuming that the reaction mechanism is one step and can be described by the distorted-wave Born or impulse approximation (DWA) one can write[5]

$$\sigma_{h,h'}(\theta)\simeq\sigma_{\mathrm{DWA}}'(\theta)|M_p+(b_n^h/b_p^h)M_n|^2, \qquad (2)$$

where $\sigma_{\mathrm{DWA}}'(\theta)$ contains the usual reaction dynamics; the second factor contains the nuclear-structure matrix elements, and b_n^h/b_p^h is the ratio of hadron-neutron to hadron-proton interaction strengths.[5] In general these b values are complex. For high-energy protons the volume integrals of the interactions[9] give ratios which are almost real. The empirical justification for the use of Eq. (2) with real values of b_n^h/b_p^h has been given previously[5]; a small imaginary part will not appreciably affect our conclusions.

It is clear from Eq. (2) that the relative sign of M_n and M_p will make a large difference in the magnitude of the predicted cross section. This has been utilized for a long time to ascertain that transitions to the lowest 2^+ and 3^- states are predominantly isoscalar by comparing (h,h') reactions to EM transition rates.[4,5] This sensitivity has also been recently exploited in (π,π') and charge-exchange reactions.[10,11]

For the second 2^+ state (2_2^+) the relative sign of M_n and M_p is not as certain, and shell-model calculations[3] predict a negative sign for the $0^+\to 2_2^+$ transition in ^{30}Si and ^{34}S. We form the ratio $R=d\sigma(0^+\to 2_2^+)/d\sigma(0^+\to 2_1^+)$ which has the advantage of being independent of DWA calculations. To reduce nuclear structure uncertainties we have chosen nuclei for which the magnitudes of M_n and M_p for the $0^+\to 2_2^+$ transitions are known from EM transition rates (^{26}Mg, ^{30}Si, ^{34}S, and ^{42}Ca). Using experimental[1,6-8] matrix elements for $M_{n,p}$ we can obtain the ratio

$$R_\pm=\left[\frac{M_p(2_2^+)\pm(b_n^h/b_p^h)M_n(2_2^+)}{M_p(2_1^+)+(b_n^h/b_p^h)M_n(2_1^+)}\right]^2, \qquad (3)$$

where the $\pm$ sign refers to the relative signs of $M_{n,p}$ for the $0^+\to 2_2^+$ transition. For the $0^+\to 2_1^+$ transition $M_{n,p}(2_1^+)$ have the same sign.

Inelastic proton scattering to the first and second 2^+ states of ^{30}Si, ^{34}S, and ^{42}Ca was performed with the high resolution spectrometer at the Clinton P. Anderson Meson Physics Facility. Proton scattering on ^{26}Mg was performed previously.[12] Details of the experimental apparatus have been described elsewhere.[13,14] 650-MeV polarized protons were used on ^{30}Si, ^{34}S, and ^{42}Ca, and 800-MeV polarized protons were used on ^{26}Mg and ^{30}Si. Enriched targets were used. The energy resolution was approximately 100 keV full width at half maximum.

The observed relative cross sections to the first and second 2^+ states are presented in Fig. 1. To compare cross sections more easily the nuclear size has been factored out by plotting the data relative to qR_D where R_D is the diffraction radius determined by fitting the positions of the first minimum in the elastic cross sections,

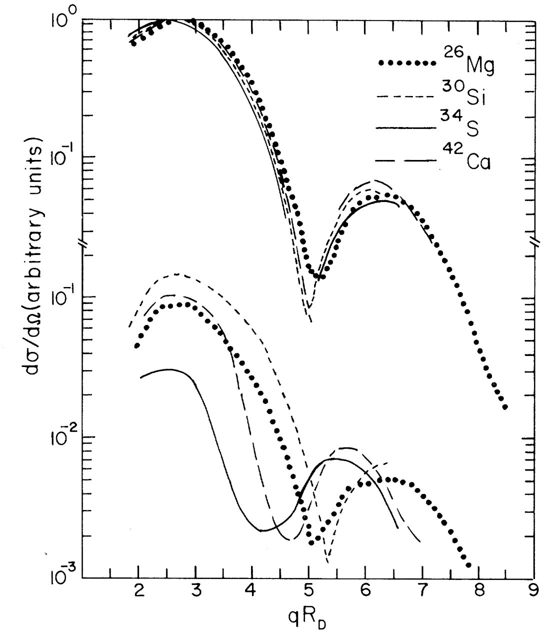

FIG. 1. Proton scattering relative differential cross sections to the 2_1^+ and 2_2^+ states of ^{26}Mg, ^{30}Si, ^{34}S, and ^{42}Ca. The proton energy is 800 MeV for ^{26}Mg and 650 MeV for ^{30}Si, ^{34}S, and ^{42}Ca. The maximum of each 2_1^+ distribution in the upper set of curves has been normalized to 1 and the 2_2^+ distribution in the lower set of curves is shown with the proper strength relative to the 2_1^+. q is momentum transfer and R_D is the diffraction radius. The values of R_D used for ^{26}Mg, ^{30}Si, ^{34}S, and ^{42}Ca are 3.37, 3.45, 3.62, and 3.98 fm, respectively. Smooth lines were drawn through the data.

which are at approximately 9° c.m., by the first zero of $J_1(qR_D)$.[4] It can be seen that the 2_1^+ angular distributions for the four nuclei form an almost universal curve. By contrast there are large variations in the shape of the four 2_2^+ angular distributions. The relative 2_2^+ cross sections at the first maximum for ^{26}Mg, ^{30}Si, and ^{42}Ca are clustered around 0.1, and for ^{34}S is approximately 0.03. This suggests that the relative signs of M_n and M_p are positive for ^{26}Mg, ^{30}Si, and ^{42}Ca, and negative for ^{34}S.

The observed ratios of $d\sigma(0^+ \to 2_2^+)/d\sigma(0^+ \to 2_1^+)$ at the first maximum along with the $R_\pm$'s of the EM method and the shell-model predictions are presented in Table I. The parameter b_n^h/b_p^h was calculated for 650- and 800-MeV protons by using nucleon-nucleon t matrix parametrizations.[9] For low-energy protons and α particles b_n^h/b_p^h is taken from Ref. 5. Table I shows that the experimental ratios, excluding ^{34}S, are systematically larger than R_+ with 800- and 650-MeV protons showing the smallest deviation. This qualitatively agrees with the belief that at lower energies multistep amplitudes, which are not included in Eq. (3), become important and increase the magnitude of the $0^+ \to 2_2^+$ transition. Comparing ratios for 800- and 650-MeV protons, the relative sign of ^{26}Mg is positive, resolving the ambiguity in the EM measurements and confirming the shell-model prediction. The relative sign of ^{30}Si is positive, confirming the EM prediction and contradicting the shell-model result. The relative sign of ^{42}Ca is also positive. Experimental ratios from α particles and low-energy protons on ^{26}Mg and ^{42}Ca are larger than R_+ and support the inference of positive relative sign for both cases.

In ^{34}S the experimental ratio is less than R_+,

TABLE I. Predicted and measured ratios of 2_2^+ to 2_1^+ cross sections. The experimental ratios are evaluated at the peak of the first maximum and the numbers in parentheses are errors. R_{sm} uses matrix elements from the shell-model calculation of Ref. 3. $R_\pm$ use the following M_n's and M_p's in units of $e\cdot$fm^2 obtained from EM transition rates (Refs. 1, 6–8): ^{26}Mg $0^+ \to 2_1^+$ $M_n = 18.76(0.91)$ and $M_p = 17.73(0.40)$; ^{26}Mg $0^+ \to 2_2^+$ 6.00(0.79) and 3.02(0.17); ^{30}Si $0^+ \to 2_1^+$ 20.5(1.9) and 14.30(0.40); ^{30}Si $0^+ \to 2_2^+$ 3.65(0.49) and 6.47(0.30); ^{34}S $0^+ \to 2_1^+$ 21.1(0.32) and 13.97(0.29); ^{34}S $0^+ \to 2_2^+$ 2.75(0.52) and 4.88(0.18); ^{42}Ca $0^+ \to 2_1^+$ 27.9(2.6) and 20.41(0.34); ^{42}Ca $0^+ \to 2_2^+$ 5.0(1.2) and 8.6(1.3). Unreferenced experimental ratios are from this paper.

Nucleus	Probe	b_n/b_p	R_+	R_-	R_{sm}	Expt. ratio	Ref.
^{26}Mg	(p,p') 0.8 GeV	0.81	0.057(0.010)	0.003(0.002)	0.188	0.085(0.002)	12
^{26}Mg	(p,p') 20 MeV	3	0.081(0.019)	0.041(0.013)	0.346	0.26(0.02)	15,16,17
^{26}Mg	(α,α') 120 MeV	1	0.061(0.011)	0.007(0.004)	0.211	0.14(0.02)	18
^{30}Si	(p,p') 0.8 GeV	0.81	0.093(0.014)	0.013(0.004)	0.015	0.128(0.006)	
^{30}Si	(p,p') 0.65 GeV	0.81	0.093(0.014)	0.013(0.004)	0.015	0.137(0.007)	
^{34}S	(p,p') 0.65 GeV	0.81	0.052(0.011)	0.007(0.003)	0.0006	0.032(0.003)	
^{42}Ca	(p,p') 0.65 GeV	0.81	0.087(0.024)	0.011(0.008)		0.098(0.006)	
^{42}Ca	(p,p') 50 MeV	3	0.052(0.018)	0.005(0.005)		0.14(0.04)	19
^{42}Ca	(α,α') 31 MeV	1	0.079(0.022)	0.006(0.006)		0.14(0.01)	20

and is approximately 4 times greater than R_{-}. If one assumes a minus relative sign for ^{34}S and ascribes the extra strength in the $0^+ \to 2_2^+$ transition to multistep processes not included in Eq. (3), the multistep amplitude would be approximately equal to the one-step amplitude (assuming that they are coherent) to account for the factor of 4 difference over R_{-}. Multistep amplitudes of approximately the same magnitude and sign as that in ^{34}S also explain the enhancements found over R_{+} for 800- and 650-MeV protons on ^{26}Mg, ^{30}Si, and ^{42}Ca. The shape of the ^{34}S 2_2^+ angular distribution when compared with the other 2_2^+ and 2_1^+ states shows the strong signature of a multistep reaction,[21] as described below. Relative to the minima of the 2_1^+ states at $qR_D \simeq 5.1$, the minima of the 2_2^+ states vary, with that of ^{34}S occurring at the smallest value of $qR_D \simeq 4.2$. For the 2_1^+ states the ratio of the second to first maximum varies between 0.05 and 0.07; for the 2_2^+ states in ^{26}Mg, ^{30}Si, and ^{42}Ca this ratio is between 0.06 and 0.09, and for ^{34}S is 0.23. These numbers indicate the large difference between the slope of the ^{34}S 2_2^+ angular distribution and all of the others. The conclusion is that the relative sign of M_n and M_p in the ^{34}Si 2_2^+ is negative, thereby suppressing the one-step direct reaction and making multistep effects more apparent.

In summary, the ^{34}S $0^+ \to 2_2^+$ transition appears to be a valuable case in which to study the interference of one-step and multistep amplitudes. Data-to-data relations derived by Amado, McNeil, and Sparrow[21] have qualitatively reproduced the ^{34}S 2_2^+ shape when admixtures of one step and two step were used[22] and the work on this analysis is continuing. Knowledge of M_n and M_p from the mirror method can constrain calculations by reducing the uncertainty of the one-step strength and we plan to do a coupled-channels calculation with this constraint. Because branching ratios from higher excited states to the ground state are small and matrix elements difficult to measure, this method of determining relative signs is generally limited to 2_1^+ and 2_2^+ states.

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