Edwards Responds: Treatments of collections of particles as continuous systems are common and the literature covering action-integral approaches using Eulerian variables where variations are taken with respect to the velocity fields is extensive.[1] In fact, without suggesting that the derivation is classical or that there might be classical applications, Geurst recently used an action very similar to I_2 in my recent paper[2] to derive the Ginzburg-Landau equations as well as the London equations.[3]

The key question arising from my paper is whether or not the Eulerian action I_2 properly describes certain classical physical systems. Each of the preceding Comments[4] attempts to negate such a possibility primarily through arguments concerning the relationship of I_2 with the particle action I_1. However, as I shall argue, rather than disproving the concept they actually aid our understanding of the precise nature of the physical systems to which I_2 applies.

On page 1864 of my paper I referred to I_2 as the "Eulerian equivalent" of I_1. By this I did not mean a strict mathematical equivalence as some apparently thought for, as I showed in some detail following Eq. (8), the set of possible solutions to the Lorentz force relation which follows from both actions is restricted by the London equations which come only from I_2, and hence there is clearly an important difference between them.

deVegvar questions my use of I_2 to represent a system of particles. Segall, Foldy, and Brown argue that my methods for going from I_1 to I_2 and to the London equations are inherently incorrect and Henyey shows in what way I_2 differs from I_1.

deVegvar's Comment.—In view of singularities that arise, deVegvar questions the use of the Euler-Lagrange equations with I_2 when j_σ represents a sum over particles. The proof of convergence based on the Schwartz distribution theory can be outlined as follows:

For fluids consisting of an ensemble of particles we approximate the world line of each particle, i, by a tube extending a small radius ξ_i from the world line and we approximate j_σ by a smooth function J_σ which is nonzero within the tube, falls to zero quickly and smoothly immediately outside the tube, and remains zero beyond radius ζ_i where $\zeta_i > \xi_i$. Within the tube, where the integrand of I_2 is differentiable with respect to J_σ, the Euler-Lagrange equations must be satisfied, and consequently $A_\sigma + (m/q)U_\sigma = 0$ where $U_\sigma = J_\sigma / (J_\alpha J^\alpha)^{1/2}$. Now we shrink each tube, letting ζ_i become small. As we do the tubes approach the world lines, J_σ approaches j_σ, and U_σ approaches u_σ, the velocity of the particles. We are not concerned with the conditions at the edges and outside the tubes, i.e., where $(J_\alpha J^\alpha)^{1/2}$ is not differentiable with respect to J_α, because we are interested in the behavior of the particles, not in the space between them.

Concerning the nonlocal nature of the response of an electron gas, the London theory is a local description and is valid to first order whether one considers its derivation to be classical or quantum mechanical. The nonlocal theory of Pippard, later derived from BCS theory, is a refinement which is yet to be investigated within the present classical context.

Segall, Foldy, and Brown's Comment.—Segall, Foldy, and Brown attempt to demonstrate that the results from I_2 are invalid unless one introduces constraints because when $q = 0$ the variation of I_2 with respect to u_σ results in $u_\sigma = 0$. The problem here is that for a charged fluid, I_2 inherently contains the conservation of charge (and consequently of mass as well) as seen from the fact that $\partial_\sigma j^\sigma = 0$ results from Maxwell's equations which themselves come from I_2. However, with $q = 0$ the conservation of mass is lost and must be reinserted as a constraint; consequently one no longer gets $u_\sigma = 0$ but $u_\sigma + \partial_\sigma \eta = 0$ where η is an undetermined multiplier.

However, as Segall, Foldy, and Brown point out, even that result is unacceptable for a neutral fluid because the curl of the velocity is zero. This apparent dilemma has been extensively discussed in the literature[1] and was the original motivation for introducing the Lin constraint[5] which removes the contradiction and allows one to derive the equations for an uncharged, perfect fluid and, when applied to I_2 for a charged fluid, leads directly to the Lorentz force equations without the London equation restrictions.

But does the neglect of the Lin constraint necessarily lead to absurdities and, if not, under what circumstances can it be neglected?

Henyey's derivation.—Beginning with I_1, Henyey carefully transforms to the Eulerian form and shows that I_2 is strictly equivalent to I_1 only when I_2 is supplemented by the Lin constraint. This derivation is significant because in it the Lin constraint arises naturally and need not be imposed in an *ad hoc* manner as is so often done. Thus Henyey's derivation clarifies the difference between I_2 (unconstrained) and I_1. Using I_2 without the Lin constraint, as I did, is the same as setting $\lambda_\sigma = 0$ in Henyey's equations of motion. The result is the London equations, the solutions of which are a subset of the solutions to the Lorentz force relation. We therefore ask, are there physical systems that require the neglect of the Lin constraint?

When to neglect the Lin constraint.—The Lin constraint refers to the ability to follow the path of a fluid element from one position to another. For most fluids this is possible, hence the constraint applies. However, because of the Heisenberg uncertainty principle, one is unable to exactly follow the trajectory of a quantum fluid as discussed in a recent paper by Putterman.[6] Hence, at least for such fluids, the Lin constraint cannot be imposed. Putterman agrees that I_2 in my paper is classical and that the steps of the derivation from it, including the neglect of the Lin constraint, are permitted. However, he maintains that neglecting the Lin constraint is a subtle but crucial quantum mechanical assumption in an otherwise classical derivation.

Let me now sketch the reasons why collisionless, classical fluids also require one to neglect the Lin constraint and, consequently, are also governed by the London equations. Because space will only permit an outline, a full discussion will be submitted for publication elsewhere.

Consider first why one *can* follow elements in most classical fluids. It is because of local, binary, short-range, surface forces between the fluid element and its near neighbors. If, to observe an element of warm fluid whose particles are collisional, a photon were scattered from it the momentum imparted to it would be quickly transferred to its near neighbors and thence to the system as a whole. Consequently, the path of the element would be essentially undisturbed and hence the Lin constraint would apply and the London equations would not.

On the other hand, the momentum imparted by a photon to an element of a cold, classical plasma in collective, collisionless motion would change its path, removing it from the collective state, and subsequent collisions would not restore it but would randomize the motion and heat the system. The path could not be followed without destroying the collective state, and hence the Lin constraint would not apply and the London equations would.

Similarly for a charged fluid whose particles are in collective motion but whose collision cross sections are small, the momentum transferred by a photon to an element would remove it from the collective system and hence its path could not be followed. This argument can be extended to hot, collisionless plasmas. The extensive work on collective motion by Pines and Bohm[7] should be pertinent in guiding the development of quantitative criteria for systems to which the Lin constraint would not apply.

The possibility that there exist classical systems which require the London equations, as I have suggested, justifies considerable effort to experimentally establish or disconfirm the idea, because, if such systems do exist, e.g., thermonuclear plasmas, attempts to explain or control their behavior without using the London equations would be as contrived and ultimately unsuccessful as were early efforts to explain superconductivity without the London theory. In addition, the classical explanation of effects in superconductors and superfluids would contribute to our understanding of the foundations of quantum theory.

Positive experimental evidence is accumulating including flux ropes in the Venus ionosphere,[2] Saturn's ring structure,[8] and filamentation in the plasma focus.[9] It is hoped that a direct laboratory test will soon be made.

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[1]See, for example, R. L. Seliger, and G. B. Whitham, Proc. Roy. Soc. Ser. A 305, 1 (1968); H. W. Jackson, Phys. Rev. B 18, 6082 (1978).

[2]W. F. Edwards, Phys. Rev. Lett. 47, 1863 (1981).

[3]J. A. Geurst, Physica (Utrecht) 101B, 82 (1980). I regret that I was unaware of Geurst's papers when my Letter was published.

[4]Paul G. N. deVegvar, Phys. Rev. Lett. 49, 418 (1982); B. Segall, L. L. Foldy, and R. W. Brown, Phys. Rev. Lett. 49, 417 (1982); Frank S. Henyey, Phys. Rev. Lett. 49, 416 (1982).

[5]C. C. Lin, in *Liquid Helium*, edited by G. Careri (Academic, New York, 1963).

[6]S. Putterman, Phys. Lett. 89A, 146 (1982).

[7]D. Pines and D. Bohm, Phys. Rev. 85, 338 (1952).

[8]W. F. Edwards, B. F. Edwards, and R. C. Thompson, to be published.

[9]G. Vahala and L. Vahala, to be published.