Comment on the Applicability of Lagrangian Methods to the London Equations

Edwards[1] claims to derive the London equations from a classical action in the limit of arbitrarily long collision times. His conclusions are fallacious for the following reasons.

Edward's Eulerian action I_2 [his Eq. (6)] is indeed equivalent to the full particle-field action I_1 [his Eq. (1)] for noncolliding particles of constant mass-to-charge ratio. Note, however, that the definition [his Eq. (5)] of four-current,

$$j^\beta(x^\nu)=\sum_i q_i \int u_i{}^\beta(\tau_i)\delta^4(x^\nu - z_i{}^\nu(\tau_i))d\tau_i,$$

where $z_i{}^\nu(\tau_i)$ is the world line of the ith particle, is a highly singular object: It vanishes off world lines and is δ^3-like on them. Taking the variation of I_2 with respect to $j_\beta(x^\nu)$, we get

$$0=\delta I_2=\int d^4x\,\{A^\beta \delta j_\beta + (mc/q)\delta[j^\beta j_\beta]^{1/2}\}$$
$$=\int d^4x\left[A^\beta+\frac{mc}{q}\frac{j^\beta}{(j^\beta j_\beta)^{1/2}}\right]\delta j_\beta, \qquad (1)$$

provided $j_\beta \neq 0$! Edwards sets the brackets equal to zero and defines the fluid four-velocity $u^\beta \equiv j^\beta/\rho_0$ to get his Eq. (7). But, even if δj_β is smooth over all space-time, $j_\beta/(j_\beta j^\beta)^{1/2}$ is highly singular: It is 0/0 off world lines and $\delta^3/(\delta^3\delta^3)^{1/2}$-like on them. To set $[\cdots]=0$, to get the usual Euler-Lagrange equations, all the terms must exist and be continuous.[2] This criterion fails when applied to (1).

To use the Lagrangian method and avoid such singularities, j_β must be smoothed. In fact, it is straightforward to show that if j_β is replaced by $\langle j_\beta \rangle$ in Edwards's I_2, where $\langle\cdots\rangle$ represents spatial averaging on a scale much smaller than the mean free path, then one obtains his Eq. (7) by defining $\langle j_\beta \rangle = \rho_0 u_\beta$. But then the Lagrangian I_2 used with j_β replaced by $\langle j_\beta \rangle$ does *not* reduce to his original Lagrangian I_1. Moreover, the current densities in the London equations are smoothed and not the δ-function variety which Edwards has constrained his solutions to be. These difficulties do not arise when dealing with I_1, for then all objects are well defined. In particular, no variation of $u_{i\beta}$ is made off the ith world line, so that $u_{i\beta}$ never vanishes.

Finally, even if (7), as written, were justifiable, it could not describe a superconductor because it is a local relationship between currents and fields. As is well known,[3] the current response of an electron gas to a field is nonlocal: j_β at a point depends on a spatial average of extent $\sim l$ and a time average of duration $\sim l/v_F$ (l is the mean free path, v_F the Fermi velocity). Thus even if one could use Edwards's[1] (7) at all in superconductors, it would be restricted to the case of static uniform fields, precluding its application to penetration phenomena.

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[1]W. F. Edwards, Phys. Rev. Lett. 47, 1863 (1981).

[2]G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw-Hill, New York, 1968), 2nd ed., pp. 344–349.

[3]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975), p. 5.