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Thermoelastic Effect in Niobium at the Superconducting Transition

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An abrupt decrease in the acoustic loss Q^{-1} has been observed in flexural modes of polycrystalline Nb disk samples on cooling through the superconducting transition temperature T_c. High-resolution measurements on a 1.6-kHz mode of one sample showed a 30% decrease in Q^{-1} in a 20-mK temperature interval. It is shown that these results are clearly identifiable with the reduction of the thermoelastic contribution to the acoustic loss in the superconducting state.

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Interest in the acoustic-loss mechanisms in solids at low temperatures has been stimulated by the requirement for very high mechanical Q factors in gravitational radiation antennas.[1,2] One material known to have a high Q is niobium. As a superconductor, Nb may be expected to show a decrease in Q^{-1} below T_c because of the BCS temperature dependence of the electronic contribution to Q^{-1}, which is well known from ultrasonic studies.[3] Previously reported measurements for Nb disk samples[4] showed decreased losses below T_c which appeared consistent with this interpretation.

The high-resolution measurements reported here show that the decrease is much too abrupt to be explained by the BCS electronic contribution. We show that the results are in fact the first observation of the thermoelastic effect[5] at low temperatures. In this effect, which is the thermodynamic reciprocal of thermal expansion, heat flow tends to restore thermal equilibrium between regions compressed (heated) and dilated (cooled) by the acoustic wave. The magnitude of the thermoelastic temperature gradients is determined by the coefficient of linear thermal expansion α, while the losses depend also on the thermal diffusion time τ_{th}. The reduction in thermoelastic contribution to Q^{-1} is due to the abrupt changes in α (Ref. 6) and the specific heat c (Ref. 7) on cooling through T_c.

We present results obtained from Q^{-1} measurements on the four different disk samples described in Table I. The disks were machined from polycrystalline Nb of 99.9% purity. The flexural modes for which measurements were made have the form

$$w(r,\theta)=w_0\cos n\theta\left[J_n(k_{0n}r)+\beta_n I_n(k_{0n}r)\right] \tag{1}$$

TABLE I. Description of the disk samples.

Sample number	Diameter (mm)	Thickness (mm)	Metallurgical treatment	Mean crystal grain size (mm)
1	125	3	Machined from rolled Nb plate, annealed for 2 h at 980 °C in 10^{-5} Torr vacuum	0.1
2	125	3	Machined from rolled Nb plate, annealed for 1 h at 980 °C in 10^{-5} Torr vacuum	0.2
3	122	3	Machined from rolled Nb plate, annealed for 1.5 h at 1150 °C in 10^{-5} Torr vacuum	0.5
4	126	5.7	Machined from electron-beam–melted Nb ingot and etched to remove machining damage	50

in polar coordinates. Here w is the transverse displacement and J_n and I_n are the Bessel functions of the first kind, of order n. The subscript 0 in the eigenvalue k_{0n} indicates the absence of nodal circles. The disk edge is free, the suspension point being at the center.[4]

Figure 1 shows the results of high-resolution measurements of Q^{-1} for the $n=3$ mode of sample 1 on cooling through T_c. The temperature scale is established by calibrating the mode frequency against a Ge resistance thermometer and then using the frequency as the thermometric parameter. This avoids the necessity of attaching a thermometer to the high-Q samples, which are in a vacuum of 10^{-4} Torr or less. The temperature scale has an accuracy of 40 mK in the range of Fig. 1 and a resolution of 3 mK above and 10 mK below T_c. The discontinuity in the temperature coefficients of the elastic constants of Nb at T_c (Ref. 8) results in a positive temperature coefficient of the frequency below T_c and for our samples its magnitude is about one-third that of the (negative) normal-state coefficient. The frequency maximum at T_c and the reduced temperature coefficient below T_c make it difficult to resolve the transition width by frequency measurement alone. Thus in Fig. 1, the temperature scale below 9.24 K has been determined by the extrapolation in time of the 30 mK/h cooling rate above 9.24 K, taking into account the effect of the rising specific heat on the cooling rate. We use the approximation that the specific heat is rising linearly in time from the normal to the superconducting value during the interval in which the frequency is stationary within the 10^{-4} Hz resolution. From this time interval we place an upper limit of 20 mK on the transition width.

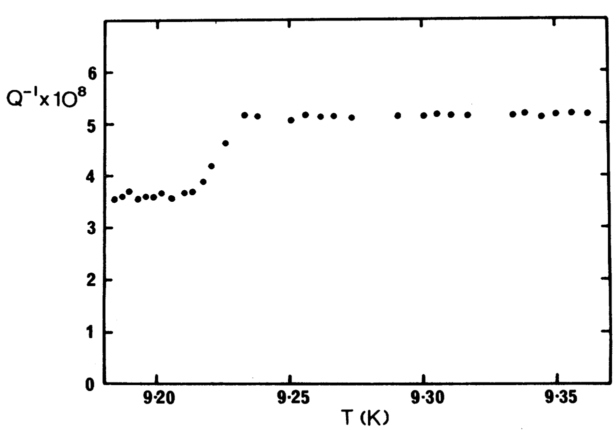

FIG. 1. Data showing the abrupt change in the acoustic loss Q^{-1} across T_c ($n=3$ mode, sample 1).

Figure 1 shows that Q^{-1} changes by 30% in a 20-mK interval at the transition. This interval is comparable to the transition widths obtained by specific-heat measurements[6] and suggests that the transition width in sample 1 is indeed about 20 mK. The data are reproducible and there is no observable hysteresis on thermally cycling through T_c. As confirmation of these results we present in Table II additional data for Q^{-1} measured at about 100 mK on either side of T_c for various modes of the four samples.

We note here that the difference in the values of Q^{-1} for the $n=2$ mode between samples 1 and 2 is due to suspension losses in sample 1 arising from a larger-diameter suspension shaft. The increased importance of the suspension losses for the $n=2$ mode can be understood from the fact that the radial eigenfunction in (1) behaves as r^n for small r. The $n=2$ mode therefore contains a larger fraction of its energy within a given small radius than do the higher modes. The magnitude of the decrease in Q^{-1} at the transition is not affected by this difference, however.

The thermoelastic loss due to transverse thermal currents in a thin plate undergoing flexural

TABLE II. Measured values of Q^{-1} near T_c for $n = 2$, 3, and 5 modes of the samples. Q_n^{-1} and Q_s^{-1} are the respective values in the normal and superconducting states.

Sample number	$n=2$ Frequency (Hz)	$n=2$ Q_n^{-1}	$n=2$ Q_s^{-1}	$n=3$ Frequency (Hz)	$n=3$ Q_n^{-1}	$n=3$ Q_s^{-1}	$n=5$ Frequency (Hz)	$n=5$ Q_n^{-1}	$n=5$ Q_s^{-1}
1	709	10.3×10^{-8}	9.11×10^{-8}	1604	5.13×10^{-8}	3.58×10^{-8}	4251	4.63×10^{-8}	3.57×10^{-8}
2	705	5.42×10^{-8}	4.22×10^{-8}	1703	4.37×10^{-8}	2.97×10^{-8}	4538	4.44×10^{-8}	3.45×10^{-8}
3	765	6.46×10^{-8}	4.93×10^{-8}	1755	0.58×10^{-8}	4.13×10^{-8}			
4	1243	6.49×10^{-8}	6.25×10^{-8}	3100	5.88×10^{-8}	5.52×10^{-8}			

vibrations is given by

$$Q_{\rm th}^{-1} = A_n Q_p^{-1}, \tag{2}$$

where A_n is a numerical factor characteristic of the particular vibration mode and Q_p^{-1} is the thermoelastic loss for plane bending, given to good approximation by[9,10]

$$Q_p^{-1} = \left(\frac{1+\sigma}{1-\sigma}\right)\frac{E\alpha^2 T}{c}\left(\frac{\omega\tau_{\rm th}}{1+\omega^2\tau_{\rm th}^2}\right). \tag{3}$$

Here

$$\tau_{\rm th} = h^2 c/\pi^2 K, \tag{4}$$

and σ, E, T, h, K, and ω are Poisson's ratio, Young's modulus, temperature, plate thickness, thermal conductivity, and angular frequency of vibration, respectively. The value of A_n is given by

$$A_n = \iint(\rho_1^{-1}+\rho_2^{-1})^2 dS\left[\iint(\rho_1^{-1}+\rho_2^{-1})^2 dS - 2(1-\sigma)\iint(\rho_1\rho_2)^{-1}dS\right]^{-1}, \tag{5}$$

where ρ_1 and ρ_2 are the principal radii of curvature and the integration is over the plate surface. For small vibration amplitudes we have[11]

$$\rho_1^{-1}+\rho_2^{-1} = \nabla^2 w \tag{6}$$

while the intrinsic curvature is given by

$$(\rho_1\rho_2)^{-1} = \frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) - \left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial\theta}\right)\right]^2. \tag{7}$$

On cooling through T_c, c increases from 16.6 kJ $m^{-3}\,K^{-1}$ to 29.5 kJ $m^{-3}\,K^{-1}$ (Ref. 7) and α decreases from 3.7×10^{-8} K^{-1} to 2.7×10^{-8} K^{-1},[6] significantly reducing $Q_{\rm th}^{-1}$. Numerical solutions for A_n, k_{0n}, and β_n were obtained from (1), (5), (6), and (7). These are given in Table III together with calculated and observed values of the change in Q^{-1} at the transition.

To obtain values for K, measurements were made in thin rods cut from the same plate material as sample 1. Two rods were annealed to 975° and 1500°C, respectively, giving $K = 55$ W m^{-1} K^{-1} at T_c in the former case, and $K = 90$ W m^{-1} K^{-1} in the latter. We have used $K = 55$ W m^{-1} K^{-1} for samples 1 and 2, and an interpolated value of 70 W m^{-1} K^{-1} for sample 3. On the basis of its metallurgical state we estimate $K = 100$ W m^{-1} K^{-1} for sample 4. The changes in E and σ across T_c are negligible: We have used $E = 1.2\times10^{11}$ N m^{-2} and $\sigma = 0.375$.

The agreement of the results in Table III is very good for the $n = 2$ mode of samples 1 and 2. For the higher modes of these samples and for

TABLE III. Comparison of measured and calculated values of the loss difference in the normal and superconducting states for modes $n = 2$, 3, and 5. Unless otherwise stated the error in the measured values is $\pm 2\times10^{-9}$. The calculated values of A_n, $k_{0n}a$, and β_n are also given (a is the disk radius).

Sample number	$(Q_n^{-1}-Q_s^{-1})_{\rm meas}$ $n=2$	$n=3$	$n=5$	$(Q_n^{-1}-Q_s^{-1})_{\rm calc}$ $n=2$	$n=3$	$n=5$	Mode parameter	$n=2$	$n=3$	$n=5$
1	1.2×10^{-8}	$1.5\pm0.1\times10^{-8}$	1.0×10^{-8}	1.2×10^{-8}	1.3×10^{-8}	8×10^{-9}	$k_{0n}a$	2.26	3.45	5.69
2	1.2×10^{-8}	1.5×10^{-8}	1.0×10^{-8}	1.2×10^{-8}	1.3×10^{-8}	8×10^{-9}	β_n	0.235	0.051	0.026
3	1.5×10^{-8}	1.7×10^{-8}		1.2×10^{-8}	1.5×10^{-8}		A_n	0.154	0.251	0.369
4	2×10^{-9}	4×10^{-9}		6×10^{-9}	4×10^{-9}					

the $n=2$ and $n=3$ modes of sample 3 the observed decrease is slightly larger than predicted. With the exception of the data for the $n=2$ mode of sample 3, a value $K=60$ W m^{-1} K^{-1} for samples 1 and 2 and 80 W m^{-1} K^{-1} for sample 3 would fit all the data for these three samples. For sample 4 the agreement is good for the $n=3$ mode but the observed decrease is smaller than predicted for the $n=2$ mode. This sample, which is twice as thick as the others, consists of about twelve large crystals, one of which occupies about 50% of the sample. The resulting large-scale elastic anisotropy, together with the uncertainty in the value of K, could possibly explain this result.

Conclusions.—(a) The detailed numerical agreement of our observations with the theoretical thermoelastic loss strongly supports the identification of this as the mechanism for the change in acoustic loss across T_c. In a 20-mK interval the BCS contribution is less than 1% of the observed effect. (b) The results verify the reciprocal relationship between the thermal expansion and the thermoelastic effect, expressed in Eq. (3), in a temperature regime where the conduction electrons contribute significantly[6] to these thermodynamic properties. (c) Precise measurements of Q^{-1} near T_c in polycrystalline samples with different geometry (e.g., a bar vibrating longitudinally) will allow an accurate measurement of the magnitude of the contribution of intercrystalline thermal currents[10] to the thermoelastic loss. This latter contribution is a general feature of polycrystalline materials, unlike the effect discussed here, which is significant only for flexural modes of thin rods or plates. An investigation of the intercrystalline loss will be made in the 1500-kg Nb antenna at the University of Western Australia, which has a typical crystal grain size of 50 mm.

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