Explanation of Flow Dissipation in ^{3}He-*B*

We propose an explanation for the large dissipation observed by Eisenstein and Packard[1] in U-tube oscillations of superfluid ^{3}He-*B*. It turns out that the qualitative change in behavior (compared to superfluid ^{4}He) is simply due to the increased viscosities, which are four orders of magnitude larger in superfluid ^{3}He than in ^{4}He.[2] This means that a dissipation mechanism, not previously discussed (and negligible in ^{4}He), becomes important. The observation does not therefore bring into doubt our basic understanding of the superfluid phases.

The experiments[1] show that the flow of liquid ^{3}He-*B* between two reservoirs connected by a capillary and driven by the height difference between the levels is overdamped, and not oscillatory as found in the classic experiment in superfluid ^{4}He.[3] Furthermore, the authors find no dissipation mechanisms that can lead to a quality factor less than 100. Here we consider the dissipation implied by the conversion of superflow to normal flow that occurs near the liquid surfaces so that the excitations of the normal fluid can follow the surface. This extra dissipation may be treated within the macroscopic two-fluid equations, and is governed by the mean of the "second viscosity" ζ_3 and shear viscosity η.

The equations we use are the standard two-fluid equations.[4] In the layer near the surface we may neglect inertial effects so that the important terms are

$$\rho^{-1}\partial_x P - \rho_s \zeta_3 \partial_x{}^2(v_s - v_n) = 0\,, \tag{1}$$

$$\rho^{-1}\partial_x P + (\eta_{\text{eff}}/\rho d^2)v_n = 0\,. \tag{2}$$

The first equation is the linearized static equation for the superfluid velocity $\boldsymbol{v}_s$, with P the pressure, ρ_s the superfluid mass density. The second equation is the linear equation for the mass current. Although the profile of v_n (and hence v_s) over the reservoir is not uniform, we neglect this effect and consider only some average velocity over the cross section. The second term in Eq. (2) then represents the viscous force due to the sidewalls (separation d), with η_{eff} an effective shear viscosity. A uniform pressure across the section gives $\eta_{\text{eff}} = 12\eta$. We also neglect the second viscosity coefficients ζ_1 and ζ_2 entering the normal fluid equation, since these are probably small,[5] and following the conclusions of Ref. 1 we neglect temperature effects, which may however be easily included. Finally we make the approximation of incompressible flow in the reservoirs,

$$\rho_s v_s + \rho_n v_n = \text{const}\,, \tag{3}$$

and assume the macroscopically plausible boundary condition that $\boldsymbol{v}_n$ is equal to the velocity of the surface.

Equations (1)–(3) lead to a normal-fluid velocity decaying to zero away from the liquid surfaces over a healing length $\lambda = (\rho^2\zeta_3/\eta_{\text{eff}})^{1/2}d$ implying a pressure drop over this depth [given by Eq. (2)]

$$\delta P = \rho(\zeta_3\eta_{\text{eff}})^{1/2}\dot{x}/d\,, \tag{4}$$

with x the change in the height of the surface and $\dot{x}$ its velocity. The equation of motion for the U-tube oscillations are obtained in the usual manner, but with δP subtracted from the driving pressure head $\rho g x$. We find Eq. (1) of Ref. 1 with damping coefficient $L = (\rho_s/\rho)(a/A)(dl)^{-1}(\zeta_3\eta_{\text{eff}})^{1/2}$ and characteristic frequency ω given by $\omega^2 = 2(\rho_s/\rho)(a/A)(g/l)$. Here a/A is the ratio of cross-sectional areas of capillary and reservoirs, and l is the capillary length.

Using the theoretical value[5] of $\rho\zeta_3 \sim 9.2$ cm^2 sec^{-1}, and $\eta\rho^{-1} \sim 0.36$ cm^2 sec^{-1} at 0.7 mK from Ref. 6, we find rather good agreement with experiment. It is easiest to compare the quantity $2L/\omega^2 = (\zeta_3\eta_{\text{eff}})^{1/2}/gd$, from which most geometric factors cancel. For this quantity, and $d = 0.02$ cm (Ref. 1), we find the value 0.3 sec, compared with the 0.5 sec estimated from the experimental data. The scaling of L with $\omega_0{}^2$ is also in rough agreement with experiment. Considering the crudeness of the calculation we consider this agreement satisfactory.

H. Brand
M. C. Cross
Bell Laboratories
Murray Hill, New Jersey 07974

Received 21 September 1982
PACS numbers: 67.50.Fi

[1] J. P. Eisenstein and R. E. Packard, Phys. Rev. Lett. 49, 564 (1982).
[2] S. Putterman, *Superfluid Hydrodynamics* (North-Holland, Amsterdam), p. 139.
[3] J. F. Allen and A. D. Misener, Proc. Roy. Soc. London, Ser. A 172, 467 (1939).
[4] I. M. Khalatnikov, *Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965), p. 66.
[5] P. Wolfle and D. Einzel, J. Low Temp. Phys. 32, 39 (1978).
[6] G. Eska, K. Neumaier, W. Schoepe, K. Uhlig, W. Wiedemann, and P. Wolfle, Phys. Rev. Lett. 44, 1337 (1980).