D-State Effects and J Dependence in (α,d) Reactions

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It is shown that inclusion of the D state of the alpha particle in the distorted-wave amplitude for (α,d) reactions causes a strong J dependence. The S-D-state interference is destructive for transitions with natural-parity J transfers and constructive for unnatural-parity J values, thus causing, for example, an odd-even staggering in the variation with J of the cross sections for exciting the states of a given multiplet. This provides a possible explanation for recent observations on the reaction ^{208}Pb$(\alpha,d)^{210}$Bi. A similar J dependence is predicted for $(^6$Li$,\alpha)$ reactions.

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One motivation of a recent study[1] of the reaction ^{208}Pb$(\alpha,d)^{210}$Bi was to see whether the theory of (α,d) reactions could be calibrated by considering a case where the wave functions of the target and residual nucleus were well known and simple. However, the observed[1,2] cross sections to the $(h_{9/2}g_{9/2})$ multiplet exhibited a strong odd-even dependence on J, the angular momentum transfer, which was not reproduced by a one-step distorted-wave Born-approximation (DWBA) calculation. The observations were explained[1,2] by allowing the coherent addition to the one-step transfer amplitude of the sequential $(\alpha,t;t,d)$ and $(\alpha,{}^3$He$;{}^3$He$,d)$ amplitudes. According to Daehnick *et al.*,[1,2] the interference between the one-step and two-step amplitudes was destructive for natural-parity values of J, constructive for unnatural-parity values. However, this has been questioned by Pinkston and Satchler[3] who showed on general grounds that the two amplitudes should have very similar structure and that, in particular, no relative phase emerged that was J dependent. Thus, a different source of the experimentally observed J dependence of the transfer cross sections has to be sought.

Recently, Santos *et al.*[4] presented evidence for the importance of the D state of the α particle in determining tensor analyzing powers in (d,α) reactions. The asymptotic D- to S-state ratio needed to explain the measurements was consistent with the predictions of the model of Jackson and Riska[5] for the α particle and corresponds to a value of the usual D-state parameter of $D_2 \approx -0.20$ fm^2. This is similar in magnitude to that observed for (d,t) and $(d,{}^3$He$)$ reactions, but in these reactions the S- and D-state contributions to the cross section are incoherent, in the absence of spin-dependent distorted waves, because they correspond to different spin transfers of $\frac{1}{2}$ and $\frac{3}{2}$. In contrast, the S- and D-state amplitudes in (α,d) reactions both correspond to spin-1 transfer and are coherent. Consequently, interference effects may be manifested in the differential cross sections. We now show that these effects are J dependent.

The one-step DWBA amplitude[6] for the reaction $A(\alpha,d)B$ depends upon the internal structure of the α particle through the potential overlap

$$U(\vec{\rho}) = (\varphi_{1m_d'}(\xi_1)\varphi_{1m_d}(\xi_2),\ V\varphi_\alpha(\xi_1\xi_2,\rho)), \tag{1}$$

where the φ_{1m} are the normalized internal wave functions of the emitted deuteron and the transferred $(n+p)$ pair, the ξ's represent their internal coordinates, and φ_α is the normalized α-particle wave function. Also, V represents the interaction between the two "deuterons" and $\vec{\rho}$ is the separation of their centers of mass. From general considerations of parity and angular momenta, $U(\vec{\rho})$ can be expanded as

$$U(\vec{\rho}) = \sum_{L'M'} 3^{-1/2}(-)^{1-m_d}\langle L'1M'm_d' \,|\, 1-m_d\rangle U_{L'}(\rho)Y_{L'}^{M'}(\hat{\rho}), \tag{2}$$

with $L'=0$ and 2. The $U_0(\rho)$ and $U_2(\rho)$ arise from the S and D states of relative motion of the two deuterons in the α particle. The structure of the target and residual nucleus appears through the overlap

$$(\psi_{I_BM_B}, \psi_{I_AM_A}\varphi_{1m_d'}) = \sum_{LJ} \alpha_{LJ}(I_BI_A)\langle I_AJM_AM \,|\, I_BM_B\rangle\langle L1Mm_d' \,|\, JM_J\rangle R_L(R)Y_L^M(\hat{R})^*, \tag{3}$$

where R is the vector between the centers of mass of the captured "deuteron" and the target nucleus. When the target has closed shells, the reaction feeds those residual states that are two-particle states coupled to the spin-0 closed-shell core. Then the spectroscopic amplitude $\alpha_{LJ}(I_B I_A)$ is proportional to an LS-jj transformation coefficient $\langle(l_p\frac{1}{2})j_p(l_n\frac{1}{2})j_n;J|(l_pl_n)L(\frac{1}{2}\frac{1}{2})S;J\rangle$, where (l_p,j_p) and (l_n,j_n) are the quantum numbers of the captured proton and neutron. The DWBA amplitudes,[6] for a given orbital angular momentum transfer l and a total angular momentum transfer J and corresponding to the S and D state ($L'=0$ and 2) terms in Eq. (2), are

$$\beta_{1J}{}^{lm}(L'=0)=\alpha_{lJ}[12\pi(2l+1)]^{-1/2}\int\chi_d{}^{(-)}(\vec{R}_d)^*R_l(R)Y_l{}^m(\hat{R})^*U_0(\rho)\chi_\alpha{}^{(+)}(\vec{R}_\alpha)d^3R_\alpha d^3\rho \tag{4a}$$

and

$$\beta_{1J}{}^{lm}(L'=2)=\sum_L\alpha_{LJ}W(12JL;1l)\int\chi_d{}^{(-)}(\vec{R}_d)^*R_L(R)U_2(\rho)\{Y_L(\hat{R})\times Y_2(\hat{\rho})\}_l{}^{m*}\chi_\alpha{}^{(+)}(\vec{R}_\alpha)d^3R_\alpha d^3\rho, \tag{4b}$$

where $\vec{R}_\alpha$ is the separation of the centers of mass of the α particle and the target nucleus, while $\vec{R}=\vec{R}_\alpha-\frac{1}{2}\vec{\rho}$ and $\vec{R}_d=[A/(A+2)]\vec{R}_\alpha+[(A+2)/(A+4)]\vec{\rho}$. Also, the χ_i are the usual distorted waves.

If one considers a reaction where the energy of the α particle is not too far above the Coulomb barrier, one may invoke a no-recoil approximation[7,8] by putting $\vec{R}_d\approx\vec{R}'=[A/(A+2)]\vec{R}_\alpha$ in the distorted wave χ_d. {The alternative approximation of putting $\vec{R}_\alpha\approx[(A+2)/A]\vec{R}_d$ in the other distorted wave χ_α leads to somewhat smaller estimates of the effect.} This allows the integration over $\vec{\rho}$ to be done and one consequence is a parity selection rule, $L+l=$even, for the $L'=2$ amplitude which may be embodied in the Clebsch-Gordan coefficient $\langle L200|l0\rangle$. When the target has zero spin, $I_A=0$, so that $J=I_B$, this and the selection rules contained in the Racah coefficient result in $l=L=J$ for transitions to natural-parity states for which $(-)^L=(-)^J$, while for transitions to unnatural-parity states we have $L=J\pm1$ and $l=J$, $J\pm1$ or $l=L,L\pm2$.

The D state will be most important when the corresponding $L'=2$ amplitude acts coherently with that of the $L'=0$, S-state amplitude so that interference terms appear. Thus, the important contribution to the cross section arises from $l=L$ for both natural- and unnatural-parity states. Then an angular momentum coefficient

$$C_{lJ}=[30(2l+1)]^{1/2}\langle l200|l0\rangle W(12Jl;1l) \tag{5}$$

may be factored from the D-state amplitude. This has the values unity for $l=J$, $-(J+2)/(2J+1)$ for $l=J+1$, and $-(J-1)/(2J+1)$ for $l=J-1$. This change of sign between natural- and unnatural-parity states means the S-D-state interference will also change sign and determines the staggering of the cross sections for odd or even J. We may also make the further approximation[7] of replacing $R_l(R)$ by the asymptotic spherical Hankel function $h_l{}^{(1)}(i\kappa R)$, where κ is the local wave number of the relative motion of the transferred "deuteron" and the target nucleus in the important region. Then we obtain the simple relation

$$\beta_{1J}{}^{lm}(L'=2)\approx\lambda_{lJ}\beta_{1J}{}^{lm}(L'=0), \tag{6}$$

where

$$\lambda_{lJ}=\kappa^2D_2C_{lJ}/2^{1/2}. \tag{7}$$

Here D_2 is defined by[8]

$$D_2=(15)^{-1}\int_0^\infty\rho^4U_2(\rho)d\rho\Big/\int_0^\infty\rho^2U_0(\rho)d\rho. \tag{8}$$

Since the reaction occurs mainly in the vicinity of the distance of closest approach for grazing collisions, the effective wave number κ may be estimated by adding the Coulomb potential at that radius to the deuteron separation energy. Equations (6) and (7) predict that the contribution from the D state of the (α,d) overlap is proportional to that of the S state with a proportionality factor λ_{lJ} that depends upon l and J through Eq. (5). Thus, the transfer cross section for a given l and J will be given by

$$\sigma_{lJ}=\sigma_{lJ}(L'=0)(1+\lambda_{lJ})^2. \tag{9}$$

Daehnick *et al.*[2] present in their Fig. 1 values for the one-step DWBA cross sections for the reaction ^{208}Pb$(\alpha,d)^{210}$Bi exciting states of the $(h_{9/2}g_{9/2})$ multiplet, using the S state alone and with an empirical normalization.[1] We have applied Eq. (9) to these values, using $D_2=-0.20$ fm^2, and estimating $\kappa^2\approx2.0$ fm^{-2} for an interaction radius of 8 fm. Further, since the largest allowed l transfer is usually dominant in (α,d) reactions, we assume that $l=L=J+1$ for the unnatural-parity states. We then obtain the predictions shown in Fig. 1 along with the measured cross sections. The diminution of the cross sections of odd-J states and the enhancement of those for even-J states, especially for $J=0$, reproduces the trend of the experimental results. While our numerical estimates were based on several simplifying assumptions such as the use of the no-recoil approximation and the use of asymptotic forms of the "deuteron" wave function in the re-

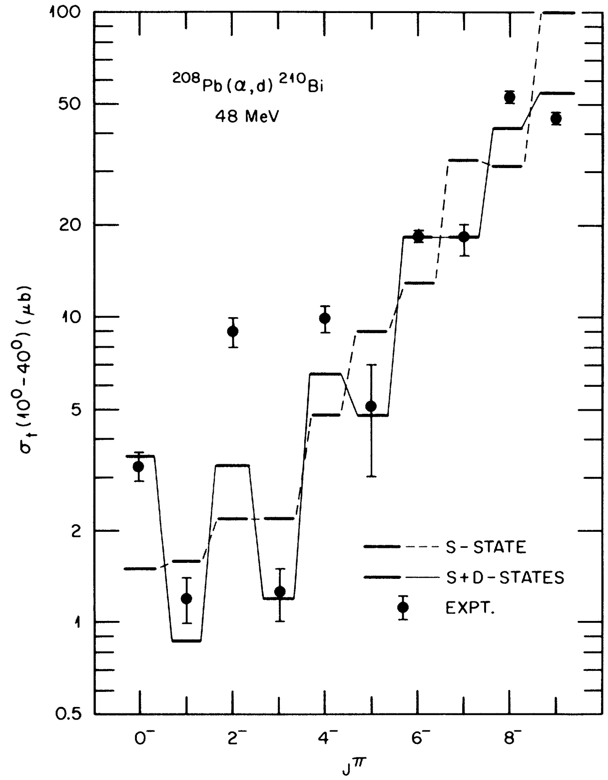

FIG. 1. Integrated cross sections for exciting the $h_{9/2}g_{9/2}$ multiplet. Values joined by dashed lines are the one-step results from Ref. 2 with only the S state, while those joined by solid lines include the effect of the D state by use of Eq. (9).

sidual nucleus, the main effect of the D state is manifested through the Racah coefficient in Eq. (5) which has opposite signs for $l = J$ and $l = J \pm 1$. This will persist even in a full finite-range calculation. The magnitudes of the corrections predicted here may, however, change. Finite-range calculations with S and D states are currently in progress[9]; preliminary results provide support for our conclusions.

The J-dependent effects arising from the D state should also be observed in reactions such as $^{48}Ca(\alpha,d)^{50}Sc$ for the states of the configuration $(f_{7/2}p_{3/2})$. In this reaction, the even-J states have natural parity and should be suppressed while the odd-J states should be enhanced. Further, these J-dependent features should also appear in (^{6}Li, α) reactions close to the Coulomb barrier and indeed measurements of these would provide a way to determine the sign and magnitude of the D-state parameter D_2 for ^{6}Li.

Finally, we note that the calculations of Refs. 1 and 2 imply that contributions from the two-step sequential transfers are not negligible. However, extending the simple arguments of Ref. 2, one may show that the effect of the D state on the two-step amplitudes is similar to that discussed above for the one-step, although the relative importance of the S and D states will, in general, be different. Consequently, the J dependence discussed here will still be present when the two-step processes are included.

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[1]W. W. Daehnick, M. J. Spisak, and J. R. Comfort, Phys. Rev. C 23, 1906 (1981).

[2]W. W. Daehnick, M. J. Spisak, J. R. Comfort, H. Hafner, and H. H. Duhm, Phys. Rev. Lett. 41, 639 (1978).

[3]W. T. Pinkston and G. R. Satchler, Nucl. Phys. A383, 61 (1982).

[4]F. D. Santos, S. A. Tousfelt, T. B. Clegg, E. J. Ludwig, Y. Tagishi, and J. F. Wilkerson, Phys. Rev. C 25, 3243 (1982).

[5]A. D. Jackson and D. O. Riska, Phys. Lett. 50B, 207 (1974).

[6]G. R. Satchler, Nucl. Phys. 55, 1 (1964).

[7]P. J. A. Buttle and L. J. B. Goldfarb, Nucl. Phys. A115, 461 (1968), and 78, 409 (1966).

[8]R. C. Johnson and F. D. Santos, Part. Nucl. 2, 285 (1971).

[9]C. F. Maguire, private communication.