Cosmological Effects of Primeval Fermion Degeneracy in the Grand Unified Theories

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Generalized grand unified big-bang models which include initial fermion degeneracies are considered, and limits are obtained on specific entropy and lepton number valid for any theory which contains SU(5). Lepton degeneracies may occur which significantly affect element production. These results affect inferences about the early universe based on the standard model.

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One of the more interesting consequences of grand unified cosmologies is that a universe which initially contains pure entropy may go sufficiently out of equilibrium to produce a small net baryon charge.[1] Although this simple picture is attractive, it is not unique; pure entropy is only one extreme of a continuum of possible initial conditions. In this Letter we investigate the consequences of the opposite assumption—a universe containing initially only cold, degenerate fermionic matter (i.e., chemical potential $\mu \gg T$ vs $\mu \ll T$).[2] By doing this, we are able to show that there is an upper limit to the baryon charge and lepton charges which can be produced, for arbitrary initial conditions, in any theory which contains SU(5).

This limit translates directly into an upper limit on the neutrino degeneracy parameters $\xi_\nu \equiv \mu_\nu / T_\nu$ used in cosmological nucleosynthesis calculations.[3-5] For completely degenerate initial conditions, $\xi_{\nu e}$ is predicted to lie in a range where it significantly affects element production.[6] Such an "alternative" cosmology offers several possible departures from conventional scenarios with $\xi_\nu \ll 1$; for example, it may allow a much lower entropy at nucleosynthesis, or a different mix of primordial abundances, or a larger number of neutrino flavors, without violating observational constraints.[7-13] At the same time it is no longer entirely *ad hoc* to postulate ξ_ν's in this interesting range, because they are predicted by the arguments below. In these models the entropy is produced at late times ($t > 10^5$ y) by stars, quasars, or other astrophysical sources, and thermalized by grains or molecules.

Let us suppose then that matter shortly after the Planck time consists entirely of cold fermions. Invariance under a universal gauge group G [e.g., G = SU(5), SO(10), E(6), E(7), ...] requires equal asymmetries (equal chemical potentials) for all components within an irreducible representation of G. It is not possible to maintain a cold degenerate state in equilibrium unless there is some conserved quantity carried only by the fermions; and it is not obvious in an arbitrary grand unified model, with both fermions and antifermions present in the same representations and where at high energy baryon and lepton number are not conserved, that this initial condition will not relax to pure entropy. Nevertheless, initial degeneracy has interesting consequences for the following reason: Gauge interactions do not mix L and R Weyl projections, while helicity-mixing fermion-scalar (Yukawa) interactions are not effective until $\mu \lesssim \alpha_H^2 m_{Pl} \ll M_X$, when interactions violating baryon and lepton number conservation are frozen out. Thus, net baryon and lepton numbers are established at high energy, when left-handed fermion number is approximately conserved; and when scalar interactions do become important, allowing left-right transitions and fermion-antifermion annhilations to produce entropy, baryon and lepton number *are* conserved.

In G = SU(5) the usual fermions inhabit three generations of the representations $\underline{5}_L{}^* = \{\bar{d}_i, (e^-, \nu)\}_L$ and $\underline{10}_L = \{\bar{u}_i, (u_i, d_i), e^+\}_L$, where weak doublets are paired and the subscript i indicates color triplet. The SU(5) gauge interactions do not mix different fermion representations. Thus, in the absence of other couplings, the net number of particles in each $\underline{5}^*$ and $\underline{10}$ are conserved individually (as is the net $B-L$ charge, which is also conserved by the Yukawa interactions; see below), and the ratio of net lepton (minus antilepton) to baryon (minus antibaryon) density,

$$L \equiv \frac{n_L}{n_B} = \frac{2n(5^*) - n(10)}{n(10) - n(5^*)} \qquad (1)$$

[where $n(I) = \mu_I{}^3/6\pi^2$ is the number density in any one species in representation I], is a parameter which must be specified in the initial conditions.

Yukawa couplings mix $\underline{5}_R$ and $\underline{10}_L$ representations, and $\underline{10}_R{}^*$ and $\underline{10}_L$, to generate masses m_u, m_d, and m_e via the Higgs mechanism. Virtual exchanges lead to only a small damping of the initial asymmetries before baryon-number-violating interactions freeze out[1,2]; however, at later times, these interactions convert remaining fermion-antifermion pairs to entropy, leaving behind only net (uncompensated) quantities.

The number of fermion degrees of freedom which thermalize in each generation [in units of $n(10)$] is $6+8(1+L)/(2+L)$ for $L>-1$ (this includes 6 from $u,\bar{u}$; 8 from $d,\bar{d}+e,\bar{e}$; and similarly for other L). By conservation of energy, $\rho=\sum(1/8\pi^2)\mu^4 \to g_{\text{eff}}(\pi^2/30)T^4$, where the sum is over annihilating species and $g_{\text{eff}}=\sum g_{\text{boson}} + \frac{7}{8}\sum g_{\text{fermion}} = 106.75$ is the effective number of (boson) spin degrees of freedom. With entropy density $s=\frac{4}{3}\rho/T$ this results in an entropy per baryon of

$$S \equiv \frac{s}{n_B} = \left(\frac{4\pi^4 g}{15N_0}\right)^{1/4}[6(2+L)^{4/3}+8(1+L)^{4/3}]^{3/4} \tag{2}$$

(for $L>-1$, and N_0 initial degenerate generations with equal μ).

For nucleosynthesis, it is primarily the electron-type lepton number, $L_e=L/N_0$, which determines the neutron-proton ratio; per unit entropy this is

$$L_e/S \leq (15/4\pi^4 g)^{1/4}(14N_0)^{-3/4}, \tag{3}$$

where the maximum limit is $L_e/S \leq 0.019\,040$, for $N_0=1$, only first-generation fermions present initially, and $|L|\to\infty$. At nucleosynthesis $g_{\text{eff}}'=2+6(\frac{7}{8})(\frac{4}{11})=3.9091$; the fraction of total entropy in photons is $S_\gamma/S=2/g'=0.511\,63$.

One case of interest requires closer attention: When $\mu(5^*)=\mu(10)$ initially, Eq. (1) indicates $L^{-1}\equiv 0$ ($n_B=0$)—but here the "negligible" dampings by scalars before baryon-number-violating interactions freeze out become important. The Yukawa couplings lead to dampings of (comoving) $n(5^*)$ and $n(10)$ (Refs. 1 and 2) but in such a way as to preserve $B-L=2n(10)-3n(5^*)$; the result of this is a small net baryon number and large antilepton asymmetry,[2]

$$L^{-1} \sim \exp[-(\alpha_{\text{H}}{}^2 m_{\text{Pl}}/\pi m_{\text{H}})]-1. \tag{4}$$

If $\alpha_{\text{H}}{}^2 m_{\text{Pl}}/m_{\text{H}} \gtrsim 1$, then only $B-L$ conservation prevents complete degeneration into entropy, and the initial $B-L$ is distributed approximately equally among all baryons and leptons, $L\sim-1$.

Other grand unification gauge groups [SO(10), E(6), ...] do not allow the freedom we have in SU(5): With typically all fermions of a generation in a single irreducible representation [$\underline{16}$ in SO(10); $\underline{27}$ in E(6)], the final value of L will not be free but will be determined by microscopic interactions. In these models the global accidental $B-L$ symmetry of SU(5) becomes instead a gauge symmetry, and thus $B-L$ charge must also be zero initially, by adding the SU(5) singlet $N_L{}^*$ ($B-L=+1$).

Nevertheless, in SO(10) we do end up with net baryon and lepton numbers. The SU(5) singlet N is given a large Majorana mass M_N to make the ordinary neutrino light, $m_\nu=m_u{}^2/M_N$.[14] This leads to a damping of n_N, while X'-mediated scatterings redistribute the asymmetries among other fields [the X' couples SU(5) representations $\underline{10}+\underline{10}\to\underline{5}^*+\underline{1}$] until they freeze out at $\mu\sim M_X$, leaving[2]

$$L^{-1}=\exp[-(\text{const})(3\pi/f)^{1/2}m_{\text{Pl}} \times M_N{}^2/M_{X'}{}^3]-1. \tag{5}$$

[If there is an SU(5) phase between SO(10) and SU(3)⊗SU(2)⊗U(1), then the Higgs damping, Eq. (4), also operates.] For "reasonable" parameter values, this gives L large and negative.[2] With the extra fermion degree of freedom (the N) contributing, $S_{\min}\to(4\pi^4 g/15N)^{1/4}(15L^{4/3})^{3/4}$ as $|L|\to\infty$.

It seems that this kind of result will be typical of any model: Low-energy phenomenology has an uncompensated lepton, which then ($B-L$ symmetry) leads to $B/L<0$. To have $B>0$ requires $L<0$—an excess of antineutrinos unless there exist additional, undetected particles, which do not participate in nucleosynthesis, which carry baryon and/or lepton charges.

We now compare these limits with constraints imposed by nucleosynthesis. The production of elements in universes with arbitrary S_γ and L_e has never been calculated exactly, but a fairly accurate picture can be drawn by using the following approximate scheme, which agrees with numerical results where they are available. We assume that (i) the temperature T_N of nucleosynthesis depends only on S_γ; (ii) the mass fraction Y of helium produced depends only on $\eta(S_\gamma,\xi_e)$, the neutron-to-proton ratio at T_N, as $Y=2\eta/(1+\eta)$; and (iii) η depends on the electron-neutrino degeneracy parameter $\xi_e=\mu_{\nu e}/T_{\nu e}$ as $\eta=\eta_0\times\exp(-1.2\xi_e)$ (valid to $\sim 1\%$ for $5\times10^8<T_N<3\times10^9$ K; Ref. 3), where $\eta_0(S_\gamma)$ is the value of η for ξ_e

$=0$. We need also $L_e/S_\gamma = (15/22\pi^4)(\pi^2\xi_e + \xi_e^3)$ (valid for $L_e \gg 1$, $S_\gamma \gg 1$, but otherwise arbitrary ξ_e; Ref. 4). The limit (3) corresponds to $\xi_e < 0.524$. It is straightforward from (i) and (ii) that

$$Y = Y_0[1 + (1 - Y_0/2)(\eta_0 - \eta)/\eta]^{-1}, \tag{6}$$

where Y_0, the helium produced for $\xi_e = 0$, has been calculated exactly in the range of interest (Ref. 3, Table 3A). These formulas have been used to calculate the contours of helium production shown in Figs. 1 and 2.

Inspection of these figures shows that there is considerable, but not unlimited, scope for modification of the standard cosmology by starting the universe in a degenerate state. We offer comments on several possibilities: (1) It is possible to produce "cold" ($S \ll 1$) matter only if $L = -2$. The composition of matter at nuclear density on this adiabat is a nearly pure neutron solid; the medium shatters into low-mass neutron stars ("polyneutrons") which evaporate on a half-life time scale ($\tau \simeq 1000$ sec). The resulting abundances consist almost entirely of r-process elements.[15] (2) Other negative values of L_e yield more acceptable abundances; but in the range allowed by grand unified theories, antineutrino degeneracy only *increases* η. Thus there are no interesting $L_e < 0$ alternatives to the standard model. (3) For positive values of L_e, a wide range of adiabats fall into the "interesting" zone of helium production, $Y \approx 20\%-30\%$. The lowest entropy compatible with SU(5) and with the strict 24% limit on Y is $S_\gamma = 10^4$, a factor of 10^6 below the standard model with the same limit.[16] (4) The most interesting cases fall into a zone where Y increases with S_γ; entropy production between $T = M_X$ and $T = T_N$ dilutes the effect of the leptons. If the pre-T_N entropy production is *very* large,[17] then we return to the standard model with $\xi_e \ll 1$. (5) The discussion above might be of interest even if the standard high-entropy model of nucleosynthesis turns out to be essentially correct. Some of the most carefully studied and best understood extragalactic H II regions appear to have helium abundances slightly, but significantly, lower than allowed by the standard model[7] with $\xi_e = 0$. While this discrepancy might be due to unknown systematic errors in analyzing the data, it might also be telling us that the true primordial helium abundance is indeed somewhat lower than is generally assumed. Previously, $L \simeq S$ had to be introduced *ad hoc*[6]; the above discussion shows that in some theories it arises naturally if the universe began in a degenerate state. Note that this scheme offers a way to reduce ^{4}He production without overproducing ^{3}He or D (Refs. 3 and 6). (6) It is also apparent in Fig. 1 that no universe which has the minimum allowable present baryon density (as given in Ref. 5) can now be

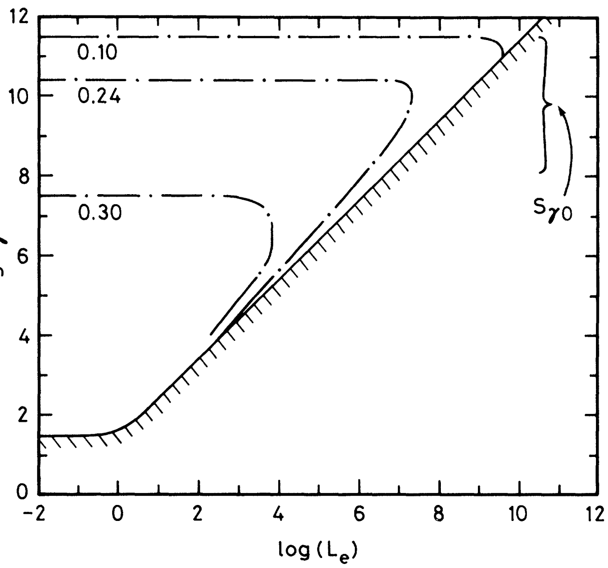

FIG. 1. Specific photon entropy S_γ at nucleosynthesis, plotted against specific electron lepton number. *Solid line* represents minimum entropy produced in SU(5); more general theories and entropy production move matter upwards in this diagram, so that hatched region is inaccessible. *Dot-dashed lines* represent approximate contours of helium production labeled by mass fraction Y. Bracket labeled $S_{\gamma 0}$ indicates the present-day entropy of the universe (Ref. 5).

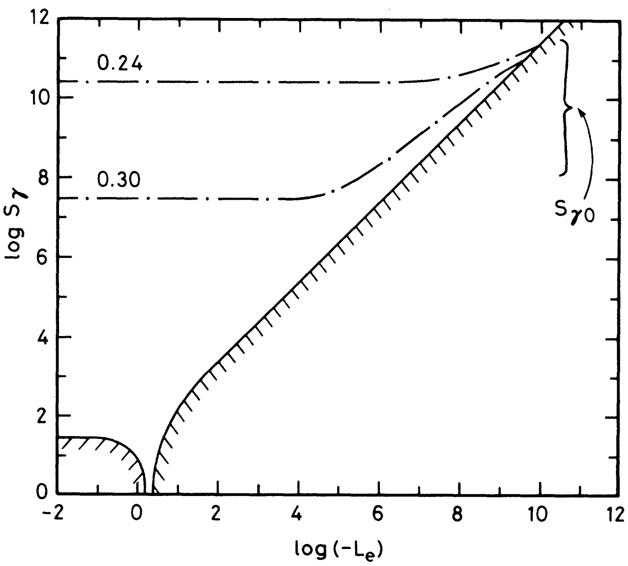

FIG. 2. Same as Fig. 1, for negative electron lepton number.

set up which produces less than about ~10% ^{4}He, because of the restriction imposed by grand unified symmetry on the lepton number.

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Note added.—After this work was completed we learned of a parallel study of limits on L/S by Langacker, Segrè, and Soni.[18] They find a fixed point with $\xi_e \sim 1$ in certain models with spontaneous Majorana-type lepton number violation.

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