# Isotope Effect in Electron-Capture Differential Cross Sections at Intermediate Energies

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The isotope dependence in the angular distribution of electron-capture cross sections for protons and deuterons with equal velocity ($E = 40$ keV/u) colliding with atomic hydrogen or deuterium targets is predicted theoretically and observed experimentally. A projectile-dependent effect is observed at small scattering angles. No target dependence was detected in the differential cross sections. A scaling relationship is determined which permits the prediction of differential cross sections for hydrogen-isotope projectiles.

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The fundamental interest in collisions dealing with hydrogen and its isotopes is magnified by the practical applications of the cross sections involving hydrogen and deuterium in the design of high-intensity ion sources for controlled thermonuclear fusion reactors and for directed particle-beam applications.[1,2] The assumption is frequently made that both the total and differential cross sections are essentially identical for protons and deuterons of the same velocity.[2] In fact, this assumption is used in ion-atom collision physics to extend total cross-section measurements to lower velocities than would otherwise be possible with the available equipment.[3] This assumption is based on the fact that at high velocities the projectiles follow almost straight-line trajectories so that the total cross sections are virtually identical for protons and deuterons at the same velocity.[4] However, the assumption has not been tested for differential cross sections which in general are more sensitive than the total cross sections to weaknesses in theoretical approximations and/or experimental methods. Therefore, differential cross sections for particular collision systems are expected to be more affected by isotope effects than the corresponding total cross sections.

This Letter reports for the first time an isotope effect in both experimentally measured and theoretically calculated differential electron-capture cross sections for the intermediate-energy projectiles $p$ and $d$ at very small scattering angles in the laboratory system. We studied the systems

$$p + \mathrm{H} \rightarrow \mathrm{H}(\theta) + p,$$

$$p + \mathrm{D} \rightarrow \mathrm{H}(\theta) + d,$$

$$d + \mathrm{H} \rightarrow \mathrm{D}(\theta) + p,$$

$$d + \mathrm{D} \rightarrow \mathrm{D}(\theta) + d,$$

at a collision energy of 40 keV/u ($v = 1.26$ a.u.).

The experimental arrangement is basically the same as that recently described[5-7] so that only a brief description of the apparatus will be given here. The measurements were made using the University of Missouri–Rolla (UMR) ion-energy-loss spectrometer with an angular resolution of 120 $\mu$rad and an angular precision of 3.3 $\mu$rad in the laboratory system. Protons or deuterons are produced in a Colutron model G2 ion source which includes a Wien filter to provide mass selection prior to acceleration.

The incident ion beam ($p$ or $d$) is focused on the center of a tubular, coaxial, tungsten furnace which has been specially constructed for cross-section measurements differential in the laboratory scattering angle. The furnace is maintained at a constant temperature by Joule heating. Hydrogen or deuterium gas entering this furnace is thermally dissociated to provide the atomic gas target.

Identical gas mixtures of either $Ar/H_2$ or $Ar/D_2$ were fed into the ion source for producing the respective projectile ions. We do not expect the small difference in the ionization potentials of $H_2$ and $D_2$ to affect the intermolecular energy transfer between the excited particles and the molecular species in the ion source discharge. Although in the case of deuteron projectiles we used a high-purity deuterium gas (99.9 at.% deuterium), the deuteron beam could be contaminated with $H_2^+$ ions formed from residual water vapor and hydrogen gas. To estimate the amount of $H_2^+$ ions in the primary ion beam, a procedure described in the literature was used.[8] Ion currents $I_1$, $I_2$, and $I_4$ corresponding to the masses 1, 2, and 4 u were measured. The ratio of the molecular-ion to the atomic-ion current, $C$, is assumed to be the same for proton or deuteron production for identical ion source parameters. If $I_1$ is a pure

 

proton current, and $I_4$ is a current of $D_2^+$ ions only, the currents of $H_2^+$ and $d$ are related to the measured currents $I_1$, $I_2$, and $I_4$ as follows:

$$I(H_2^+)+CI_1, \quad I(d)=I_4/C,$$

with

$$C=[I_2-(I_2{}^2-4I_1I_4)^{1/2}]/2I_1.$$

When the $Ar/D_2$ gas mixture was introduced into the ion source, 30 min elapsed before $C$ assumed a steady-state value, which indicated that the contamination of the deuteron beam by $H_2^+$ was less than 15%.

An extrapolation to 80-keV impact energy of the total electron-capture cross-section data for $H_2^+ + H$ from Ref. 8 indicates a cross-section value which is 2–4 times smaller than our determined total electron-capture cross section for $d+H$ at 80-keV impact energy. The maximum possible $H_2^+$ contamination combined with the extrapolated value of the total electron-capture cross section implies an impurity contribution of less than 10% to the deuteron cross sections.

Another concern was the target purity. Energy-loss spectra taken before and after data acquisition showed that the residual molecular hydrogen or deuterium was less than 5% of the gas in the target furnace.[7]

Ions exiting from the collision region pass through the exit collimator, which consists of a fixed exit slit and a pair of movable collimating slits, via an analyzing magnet, then to a decelerating column for ion energy-loss determination. A fast-atom detector with defining slits to establish an appropriate solid angle mounted on the zero-angle port of the analyzing magnet is employed to measure the scattered fast atoms resulting from electron-capture processes.[7]

The differential cross sections from scattered-ion angular distributions are obtained by using the methods discussed by Park and co-workers.[6,9] Because the detection efficiency of the neutral-particle detector for hydrogen atoms was not measured, the experimental results were normalized to our classical-trajectory Monte-Carlo (CTMC) calculation[10,11] for capture into all bound states. This cross-section value of $\sigma_{tot} = 1.76 \times 10^{-16}$ cm$^2$ is in agreement with the measurements of McClure.[12]

In Fig. 1 the experimentally determined differential cross sections for electron capture in collisions of protons and deuterons with both atomic hydrogen and atomic deuterium are presented for an incident projectile velocity of 1.26 a.u. in the laboratory system. The experimental data for capture into all bound states are compared to our CTMC calculation. The agreement is good and both experimental data and theoretical results show a crossing of the differential cross sections for the two systems at a scattering angle of about 0.18 mrad in the laboratory system.

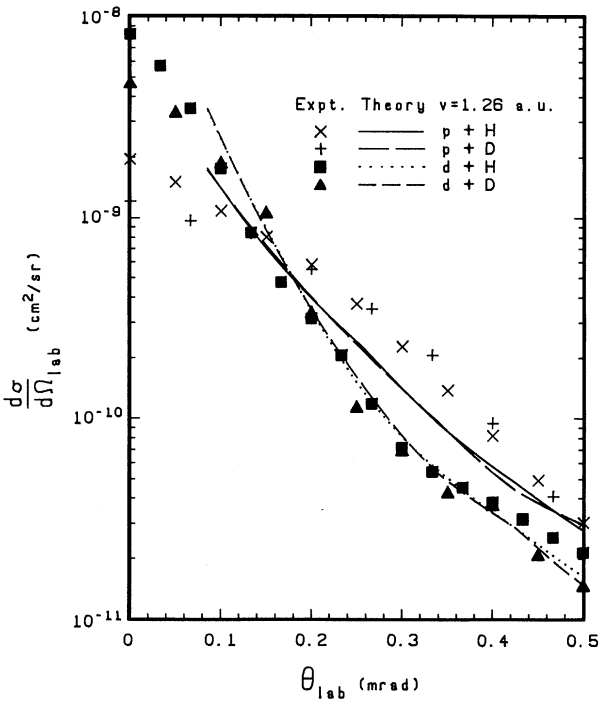


FIG. 1. Comparison of calculated and measured differential electron-capture cross sections, in the laboratory frame. The experimentally determined differential electron-capture cross sections (errors are typically ± 30%) are normalized to the calculated total electron-capture cross section into all bound states, $\sigma_{tot} = 1.76 \times 10^{-16}$ cm$^2$.

Integration of the differential electron-capture cross sections to obtain total cross sections yields essentially identical total cross sections for protons and deuterons at the same impact velocity. While the experimental error bars (omitted in Fig. 1 for clarity) on the differential cross-section measurements overlap at some points, the angular dependence of the differential cross sections for equal-velocity projectiles is markedly different. However, there is no statistically significant difference between the differential cross sections for electron capture from an atomic-hydrogen and an atomic-deuterium target. This feature is also observed in our CTMC calculations.

We are unaware of any intermediate-energy

quantum mechanical calculations of isotope effects in proton/deuteron electron capture. It is possible to predict the results of such a calculation in which typical approximations are employed. The formal general quantum mechanical expression for the differential cross section for a rearrangement collision is defined by[13]

$$\frac{d\sigma}{d\Omega}=\frac{\mu_i\mu_f}{4\pi^2}\frac{k_f}{k_i}|T_{if}|^2,$$

where $\mu_i$ and $\mu_f$ are the initial and final reduced masses, and $k_i$ and $k_f$ are the initial and final relative momenta of the colliding systems. The $T_{if}$ is the transition matrix element from the initial state to the final state. This expression can be used to obtain an approximate scaling relation for the differential cross sections.

The general expression for the differential cross section for electron capture can be simplified by considering approximations relevant to our range of incident energies and scattering angles. If we neglect the mass of the electron relative to the mass of the proton (or deuteron), then $\mu_f\simeq\mu_i=\mu$. The energy defect is small compared to the initial incident relative kinetic energy; therefore, $k_f\simeq k_i=k$ or $v_f\simeq v_i=v$, where $v_i$ and $v_f$ are the initial and final relative velocities of the colliding systems. The momentum transfer given by $\vec{q}=\vec{k}_i-\vec{k}_f$ becomes $q\simeq 2k\sin(\theta/2)$ with these assumptions, where $\theta$ is the center-of-mass scattering angle. For very small scattering angles, $q\simeq k\theta=\mu v\theta$.

For many approximations[13] the absolute value of the transition matrix element squared is only a function of the relative velocity $v$ and the momentum transfer $q$ between the colliding systems. If we assume that this is the only dependence, then the differential cross section will be given by

$$d\sigma/d\Omega=\mu^2F(v,q),$$

where $F=|T_{if}|^2/4\pi^2$ is some function of $v$ and $q$. With the assumptions discussed above and for very small scattering angles the differential cross section assumes the following form:

$$d\sigma/d\Omega=\mu^2F(v,\mu v\theta).$$

This differential cross section can be transformed to the laboratory system with the small-angle approximation. We obtain the result

$$d\sigma/d\Omega_{\rm lab}=M_i^2F(v,M_iv\theta_{\rm lab}),$$

where $M_i$ is the mass of the incident ion.

In the laboratory system for a given relative velocity this yields electron-capture differential cross sections which do not depend upon the target atom, but depend only upon the incident ion. CTMC calculations at 25 and 100 keV/u verify such a procedure to be valid over a larger energy regime. The above-discussed scaling relationship gives differential electron-capture cross sections for deuterons which are 4 times larger than the differential cross sections for protons at a scattering angle which equals half the scattering angle for protons. Application of both the derived scaling relationship for the differential electron-capture cross sections and the small-angle approximation in integrating over all scattering angles produces the same total electron-capture cross sections for $p$ and $d$.

In Fig. 2 the experimental data for all investigated projectile-target combinations are plotted on one graph by use of our scaling relationship. The differential electron-capture cross sections for projectiles with equal velocity are transformed by this scaling to a common curve.

As we have clearly demonstrated in this Letter, the experimentally observed proton/deuteron iso-

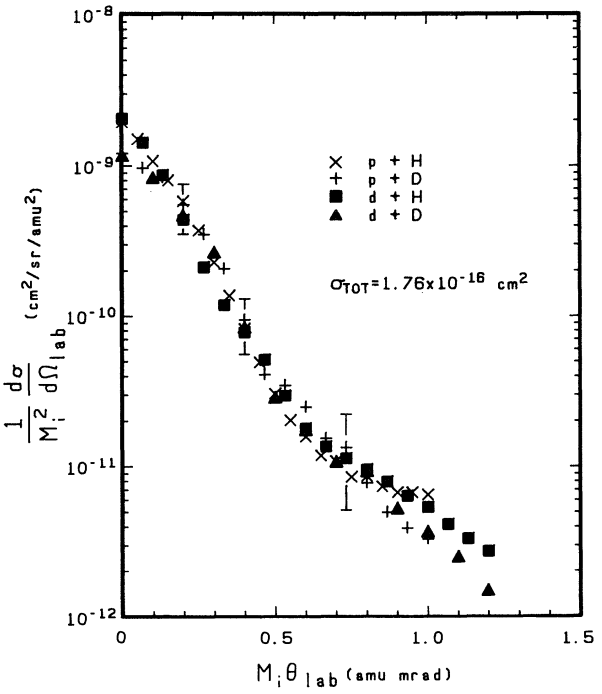


FIG. 2. A plot of $M_i^{-2}\,d\sigma/d\Omega_{\rm lab}$ against $M_i\theta_{\rm lab}$. The projectile velocity is $v=1.26$ a.u. ($E=40$ keV/u). All the experimental data appear to lie on a common curve. The error bars represent one standard deviation from the mean.

tope effect is consistent with accepted classical and quantum mechanical models. This Letter also provides a scaling relationship accurate at intermediate energies for differential electron-capture cross sections which is expected to be applicable to other types of differential cross sections involving hydrogen-isotope projectiles.

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## Resistive-Wall Destabilization of Diocotron Waves

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The growth of the negative-energy $l = 1$ diocotron wave has been studied on a column of pure electron plasma. An external $RC$ circuit connecting sectors of the bounding wall is used to induce growth of the wave, which is otherwise essentially neutrally stable. Such a circuit also causes small shifts in the real part of the wave frequency. The experimentally measured growth rates and frequency shifts are in close agreement with the predictions of theory over a wide range of experimental parameters.

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The stability properties of diocotron waves affect the operation of many magnetized charged-particle systems, such as electron beams,[1,2] magnetrons,[3,4] and Penning discharges.[5] The diocotron waves are $\vec{E}\times\vec{B}$ drift waves propagating in a nonneutral plasma. Some of these waves lower the electrostatic potential energy of the system and may grow if energy is dissipated by the image currents induced in boundary walls. The release of potential energy by this mechanism is analogous and complementary to the release of kinetic energy seen in the resistive-wall instability of negative-energy plasma waves in a beam.[6] In our case, growth rates can be predicted on the basis of energy conservation.[7] Further, a calculation considering a complex wall impedance as a boundary-value perturbation on the wave predicts a complex wave-frequency shift proportional to the impedance.

Here, we report experimental data on the resistive-wall destabilization of diocotron waves with azimuthal mode number $l = 1$. The waves are observed on a confined, quiescent pure-electron-plasma column.[8-10] In this system, the $l = 1$

 