renormalized by the interaction among more strongly interacting spins.[5,8]

In the reentrant spin-glass Fe:Cr, Shapiro *et al.*[14] may have observed the analog of the field-induced dispersion reported here for ferromagnetic coupling.

In conclusion, the amorphous antiferromagnet shows no exchange stiffness in the absence of a field but does show increasing exchange stiffness and resultant dispersion as magnetization is induced with increasing field. The high resolution of SFRS should find application in probing the dispersion of the excitations of other random systems including a variety of spin-glasses.

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Nonlinear Mixing of Bulk and Surface Magnetostatic Spin Waves

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The first discussion of the nonlinear mixing of bulk and surface magnetostatic spin waves is presented. The unusual property of these waves—that the frequency is dependent on the direction of propagation and not the magnitude of the wave vector—allows resonant generation of a new wave if the propagation directions of the two incident waves are properly chosen. In addition to the mixing of two bulk (or surface) waves to produce a new bulk (or surface) wave, the mixing of two bulk waves to produce a resonantly enhanced surface wave is discussed.

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Nonlinear mixing of bulk waves and of surface waves of various types has received a great deal of attention. For instance, the mixing of acoustic waves and the mixing of polaritons has a considerable literature.[1-3] In contrast, the topic discussed here—the nonlinear mixing of magnetostatic bulk and surface spin waves—has not been treated up to the present time, and shows some very interesting results.

Magnetostatic spin waves are long-wavelength modes which propagate in magnetic systems. The properties of these modes are governed, not by the short-range exchange interaction, but by macroscopic dipole fields set up by the motion of the spins precessing around the magnetic field. These dipole fields are calculated through the use of the magnetostatic form of Maxwell's equations. The properties of these magnetostatic spin waves have recently been extensively studied through the use of Brillouin scattering from ferromagnets.[4-7]

The equations governing the spin system, Bloch's equations, are inherently nonlinear, and nonlinear problems in magnetic systems have received considerable study. The nonlinear problems studied so far fall into two main classes: (1) domain-wall–soliton problems,[8] and (2) the decay of the uniform precession mode into spin waves in magnetic resonance experiments.[9]

Here we will deal with a different type of problem—the mixing of two waves to produce a resonantly enhanced third wave. Within our formalism, one can also deal with second-harmonic generation, but this is not as interesting an effect since resonant enhancement is not possible for magnetostatic waves when one considers second-harmonic generation.[10]

We begin by obtaining the conditions under which a magnetostatic wave described by a wave vector $\vec{k}$ and frequency $\omega(\vec{k})$ can interact nonlinearly with a second magnetostatic wave of wave vector $\vec{k}'$ and frequency $\omega(\vec{k}')$ to produce a wave of wave vector $\vec{k}+\vec{k}'$ and frequency $\omega(\vec{k})+\omega(\vec{k}')$, whose amplitude is resonantly enhanced.[11] In general, this requires that the frequency and wave vector correspond to a point on the dispersion curve for the linear medium. Thus the resonance condition is

$$\omega(\vec{k})+\omega(\vec{k}')=\omega(\vec{k}+\vec{k}')\,. \tag{1}$$

Because of the unusual property of the magnetostatic spin waves considered here—the frequency depends only on the direction of propagation—this condition may be satisfied in an unusual way. If the directions of $\vec{k}$ and $\vec{k}'$ are fixed, varying the *magnitude* of $\vec{k}$ relative to that of $\vec{k}'$ varies the *direction* of the output wave. By varying the direction of the output wave, one varies the frequency until the resonance condition (1) is satisfied.

An alternative method to find a geometry for resonance is to fix the direction of one of the input waves and of the generated wave, and to find the direction of the second input wave for which the resonance condition holds. In this case, the direction of the generated wave may be fixed because the magnitudes of the propagation wave vectors may be arbitrarily changed until the vector sum lies in the proper direction. We will study nonlinear mixing using the geometry.

The geometry of the system we consider is as follows. An externally applied dc magnetic field, H_0, and the saturation magnetization, M_s, are directed parallel to the x_3 axis. For bulk waves we consider a ferromagnetic medium that occupies all of space. For the surface waves, we consider a semi-infinite geometry where the ferromagnet occupies the region $x_1<0$.

We first consider two bulk waves propagating in the x_2-x_3 plane as shown in Fig. 1. The general equation for the frequency of bulk spin waves as a function of the angle of propagation θ with respect to the x_2 axis is

$$\omega_b(\theta)=[\,\omega_H(\omega_H+4\pi\omega_M\cos^2\theta)]^{1/2}, \tag{2}$$

where $\omega_H=\gamma H_0$ and $\omega_M=\gamma M_s$. Note that γ, the gyromagnetic ratio, is negative, so that ω_H and ω_M are negative quantities. For propagation along the x_2 axis the frequency is thus $(\omega_H\omega_B)^{1/2}$, where $\omega_B=\omega_H+4\pi\omega_M$. For convenience, we set the geometry of the two incident waves so that the output wave is always directed along x_2. Thus we let $k_3=-k_3'$ (but $k_2\neq k_2'$).[10] The condition relating θ and θ' may then be found from Eqs. (1) and (2):

$$\omega_b(\theta)+\omega_b(\theta')=(\omega_H\omega_B)^{1/2}. \tag{3}$$

This condition may be easily achieved for typical values of H_0 and M_s in ferromagnets as we will see later.

The condition for resonance in the mixing of surface waves may be derived similarly. We again take the geometry shown in Fig. 1, but now the mixing is of two surface waves. In this case, the frequency of the surface waves as a function

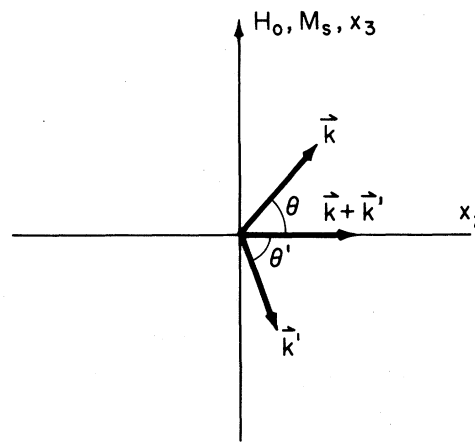

FIG. 1. The geometry considered in this paper. The two incident waves with wave vectors $\vec{k}$ and $\vec{k}'$ propagate at angles θ and θ' with respect to the x_2 axis. The generated wave with wave vector $\vec{k}+\vec{k}'$ is always directed along the x_2 axis.

of θ is given by

$$\omega_s(\theta) = -\tfrac{1}{2}(\omega_H/\cos\theta + \omega_B\cos\theta). \quad (4)$$

The surface modes are restricted to propagate only for angles $|\theta| < \theta_c$ where $\cos\theta_c = (\omega_H/\omega_B)^{1/2}$. With this restriction in mind, the resonance condition relating the angles θ and θ' of the two surface waves is given by

$$\omega_s(\theta) + \omega_s(\theta') = -\tfrac{1}{2}(\omega_H + \omega_B). \quad (5)$$

Again, this condition may be achieved for typical values of H_0 and M_s for ferromagnets.

We also note the possibility of using two bulk waves to generate a resonantly enhanced surface wave. We again restrict $\vec{k}$ and $\vec{k}'$ to lie in the x_2-x_3 plane. The resonance condition is now

$$\omega_b(\vec{k}) + \omega_b(\vec{k}') = \omega_s(\vec{k} + \vec{k}'). \quad (6)$$

We note that the resonance conditions have been obtained above in a simple, almost intuitive, manner. These same conditions may also be derived in a more formal way by using Green's functions as described below.

We have calculated the form of the wave generated by the nonlinear mixing of two bulk or two surface waves. The details of the calculation will be presented elsewhere,[10] but we outline the method of solution here.

The nonlinear equations of motion are given (to lowest order in the nonlinearity) by

$$\begin{bmatrix} \omega_H & \partial/\partial t & \omega_m(\partial/\partial x_1) \\ -\partial/\partial t & \omega_H & \omega_m(\partial/\partial x_2) \\ 4\pi(\partial/\partial x_1) & 4\pi(\partial/\partial x_2) & -\nabla^2 \end{bmatrix} \begin{bmatrix} m_1(\vec{x}, t) \\ m_2(\vec{x}, t) \\ \varphi(\vec{x}, t) \end{bmatrix} = \begin{bmatrix} \gamma m_1(\vec{x}, t)(\partial/\partial x_3)\varphi(\vec{x}, t) \\ \gamma m_2(\vec{x}, t)(\partial/\partial x_3)\varphi(\vec{x}, t) \\ (2\pi/M_S)(\partial/\partial x_3)[m_1^2(\vec{x}, t) + m_2^2(\vec{x}, t)] \end{bmatrix}. \quad (7)$$

Here $m_1(\vec{x}, t)$ and $m_2(\vec{x}, t)$ are the fluctuating transverse components of magnetization, and $\varphi(\vec{x}, t)$ is the magnetic scalar potential. The upper two equations are found from Bloch's equations and the lower equation is found from the magnetostatic Maxwell equations. Note that the linear terms are on the left-hand side of this equation and the nonlinear terms are on the right-hand side. We solve these equations by successive approximation. First the solutions to the linear equations are found, using the appropriate boundary conditions. These zero-order solutions will be the usual bulk or surface spin waves. Then one substitutes these solutions into the right-hand side (the nonlinear terms) and solves for the new waves generated by the nonlinear mixing. This may be conveniently accomplished by using the Green's functions for the linear equations (for the infinite or semi-infinite geometry as appropriate) and considering the nonlinear terms as driving the system. The result for the nonlinearly generated wave can be expressed in the form

$$u_\alpha^{(g)}(\vec{x}, t) = \sum_\beta \int d^3x' \int dt'\, G_{\alpha\beta}(\vec{x}, \vec{x}'; t - t') F_\beta^{\rm NL}(\vec{u}^{(i)}(\vec{x}'; t')), \quad (8)$$

where, to simplify the notation, we have defined $u_1 = m_1$, $u_2 = m_2$, and $u_3 = \varphi$. Explicit expressions for the Green's function $G_{\alpha\beta}(\vec{x}, \vec{x}'; t-t')$ are presented elsewhere.[10,12] The vector $\vec{F}^{\rm NL}$ is given by the right-hand side of Eq. (7) with m_1, m_2, and φ replaced by the solutions of the linear equations.

In order to obtain a feeling for the efficiency of the generation of the new wave, we calculate the time average of the square of one of the components of magnetization for both the generated wave and the sum of the input waves. If we take the component of magnetization along x_1, then these values will be denoted by $\langle (m_1^2)_g \rangle$ and $\langle (m_1^2)_i \rangle$ for the generated and incident waves, respectively. Examination of the ratio $\langle (m_1^2)_g / \langle (m_1^2)_i \rangle$ allows one to see the resonant behavior very clearly.

We consider the geometry shown in Fig. 1 for both bulk and surface waves. For the ferromagnet, we take yttrium iron garnet with $M_s = 140$ G and apply a field $H_0 = 50$ G. We fix $\theta = 50°$ and plot the ratio $\langle (m_1^2)_g \rangle / \langle (m_1^2)_i \rangle$ as a function of θ' in Fig. 2. In order to calculate these curves we have added a phenomenological linewidth to the problem by adding to $\omega(\vec{k}) + \omega(\vec{k}')$ a term $i\Delta\omega$ in the Green's functions. $\Delta\omega$ is related to the linewidth ΔH (the full width at half maximum in ferromagnetic resonance) by $\Delta\omega = \gamma\Delta H/2$. For the curves here we take a linewidth of 1 G. We see that for $\theta' = 67.6°$ there is a clear resonance for the generation of bulk waves. For surface waves there is a resonance at $\theta' = 71.4°$. We note that for the surface waves both θ and θ' are within the critical angle θ_c, since $\theta_c = 80.40°$ here. It is

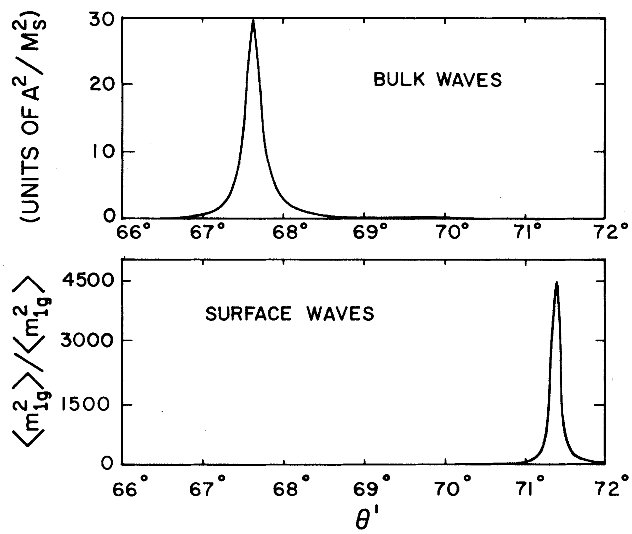

FIG. 2. $\langle(m_1^2)_g\rangle/\langle(m_1^2)_i\rangle$ vs θ'. θ is fixed at 50°. Note that the resonance for the surface waves occurs at a different frequency from the resonance of the bulk wave. In the units used A is the amplitude of $h_2(\vec{x}, t)$ $[=-\partial\varphi(\vec{x}, t)/\partial x_2]$ for one of the incident waves.

interesting to note that the resonance for the bulk waves is clearly broader than the resonance for the surface waves. The positions of these resonances for the bulk waves and for the surface waves agree with Eq. (3) and with Eq. (5).

We comment briefly on the possibility of experimental verification of our predictions. First, for the equations used here to hold, one must have wave vectors k large enough so that if L is the minimum sample dimension $kL \gg 1$. This ensures that the material is effectively infinite (or semi-infinite). Typical dimensions for yttrium iron garnet films[13] grown on a GGG substrate are 1.5×1.5 cm^2 in surface area and 10^{-2} cm thick. Thus k must be greater than 10^2 cm^{-1}. Exchange effects have also been neglected in our calculation. This is valid for $k<10^5$ cm^{-1}. Magnetostatic waves with the allowed range of k vectors, 10^{+2} cm$^{-1}<k<10^5$ cm^{-1}, can be generated by several different methods: interdigital transducers,[14] short-circuited wire couplers,[15] or microwave resonance.[13]

A further experimental consideration is whether the generated wave is strong enough to be detected. We note than nonlinear mixing of magnetostatic bulk spin waves *off resonance* has already been observed.[15] Thus it is reasonable that the mixing of two waves to produce a resonantly enhanced wave is also observable.

In summary, we have seen that the nonlinear terms in the equations of motion can lead to the mixing of both bulk and surface spin waves. The generated waves from this mixing can be resonantly enhanced if some simple conditions on the directions of the incident waves are met.

We note that the calculation presented here is merely the first example of a class of physical systems where resonant enhancement of the nonlinearly generated wave may be obtained in the novel way described here. One would expect similar behavior from other elementary excitations in gyromagnetic media (magnetostatic spin waves in antiferromagnets,[16] or plasmons in the presence of an applied magnetic field). In addition, experimental work on this problem could give insight into the extent to which the nonlinear Bloch equations describe nonlinear interaction of excitations in ferromagnetic materials.

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