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Localization and Electron-Electron Interaction Effects in Submicron-Width Inversion Layers

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(Received 29 September 1982)

The localization and electron-electron interaction parts of the small conductivity variation with temperature have been extracted from data obtained on narrow field-effect transistors. One-dimensional behavior is observed and is compared with measurements on the two-dimensional region of the test samples. Magnetoconductance which selectively removes the localization part of the resistance has allowed a theoretical interpretation of the total temperature dependence.

PACS numbers: 71.55.Jv, 72.15.Lh, 73.40.Qv

The interpretation of magnetoconductance measurements performed on two-dimensional silicon inversion layers has been a significant triumph of localization theory.[1, 2] By comparison with a scale set by a magnetic length $l_B = (\hbar/2eB)^{1/2}$, experiments have deduced the electron diffusion length between inelastic collisional events, a quantity which previously has been difficult to measure with such directness.[3, 4] Concurrently, the calculation of perturbative corrections to the conductivity showed that small conductivity variations will arise of form similar to that predicted by localization considerations. The exchange and Hartree contributions making up this electron-electron interaction nearly cancel each other for the case of short-range impurity potentials applicable to inversion layers.[1] Thus, in comparison with localization the interaction effect is small.[5] In the transition from two dimensions to one dimension,[1] phase-space considerations prevent this near cancellation. Hence there is a high likelihood that both phenomena will be significant. We report observations on quasi one-dimensional silicon inversion layers where both contributions to the small conductivity changes are comparable and distinguishable by magnetoconductance measurements.

Long but narrow metal-oxide-semiconductor field-effect transistors (MOSFETs) have been fabricated where the width is comparable to l_{in}, the inelastic length. That such a structure ought to exhibit one-dimensional localization behavior can be seen by examining the theoretical expression for the weak-localization conductance applicable at a finite temperature,[6]

$$\delta\sigma = -\frac{Se^2}{\pi\hbar}\frac{D}{LW}\sum_{q_x}\sum_{q_y}\frac{1}{(Dq^2 + 1/\tau_{\text{in}})}. \tag{1}$$

Here L and W are the length and width, respectively, of the sample, τ_{in} is the inelastic time,

q is the momentum of an electron pair, D is the electron diffusivity, and $S=2$ is the degeneracy factor for [100] inversion layers.[7] The narrow boundaries defined by the width quantize q_x such that its values are restricted to 0 and $p\pi/W$ where p is an integer:

$$\delta\sigma = -\frac{Se^2}{\pi\hbar}\frac{1}{LW}\sum_{q_y}\sum_{p=0}^{\infty}\frac{1}{{q_y}^2+(D\tau_{\mathrm{in}})^{-1}+(p\pi/W)^2}\,.$$

When $l_{\mathrm{in}}=(D\tau_{\mathrm{in}})^{1/2}>W/\pi$, the $p>0$ terms make very small contributions to the sum. With q_y assuming quasicontinuous values, we have

$$\delta\sigma = \frac{Se^2}{2\pi\hbar}\frac{(D\tau_{\mathrm{in}})^{1/2}}{W} = -\frac{Se^2}{2\pi\hbar}\frac{l_{\mathrm{in}}}{W} \tag{2}$$

in mks units, where $\delta\sigma$ is expressed as a two-dimensional conductivity. The localization conductance per unit length, $\delta G = W\delta\sigma$, for a small magnetic field applied perpendicular to the plane of the sample, has been given by Al'tshuler and Aronov[8] as

$$\delta G(B) = -\frac{Se^2}{2\pi\hbar}\left(\frac{1}{{l_{\mathrm{in}}}^2}+\frac{W^2}{12{l_B}^4}\right)^{-1/2}. \tag{3}$$

We contrast this expression to that for the two-dimensional system where the lengths are compared according to the digamma function.[1,4] Thus in both dimensions a perpendicular field removes the localization part of any resistance change, a change induced by the temperature dependence of τ_{in}.

Following a similar argument, one can show that when $(\hbar D/K_BT)^{1/2}>W/\pi$ the interaction mechanism leads to the one-dimensional result,[1,8]

$$\delta G = -(e^2/2\pi\hbar)(4-2F)(\hbar D/KT)^{1/2}, \tag{4}$$

when $K_F l_e \gg 1$ where K_F is the Fermi vector. Here F is a theoretical screening parameter equal to 0.8 for the case at hand.[7] Theoretically there is some controversy over the effect of a small magnetic field upon the interaction part.[8-10] Experimentally in narrow metal wires the effect of a small magnetic field is unobservable.[11]

Devices have been fabricated which allow the simultaneous measurement of both a two-dimensional and a narrow channel. Fabrication used a combination of optical macro- and micro-photolithographic techniques on an initially wide-gate structure.[12] The design is schematically shown in Fig. 1. Each segment can be probed potentiometrically so as to avoid magnetic end effects as well as contact resistances. Since both sections are subjected to the same oxide growth and annealing procedures, we expect uniformity over the complete channel. Comparison of measured channel geometries and conductivities implies that any nonuniformity between differing widths on the same device is less than 5%. The devices

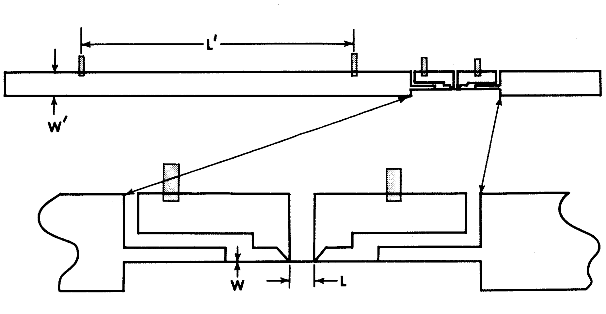

FIG. 1. A schematic representation of the gate structure of devices used in these experiments. The darkened areas indicate locations of diffused $n+$ probe regions. Here $W'=100\times10^{-6}$ m, $L'=1050\times10^{-6}$ m, and $L=25\times10^{-6}$ m. The data reported here were from device AU310 where the mean value of $W=0.40\times10^{-6}$ m with a mean deviation of 0.02×10^{-6} m.

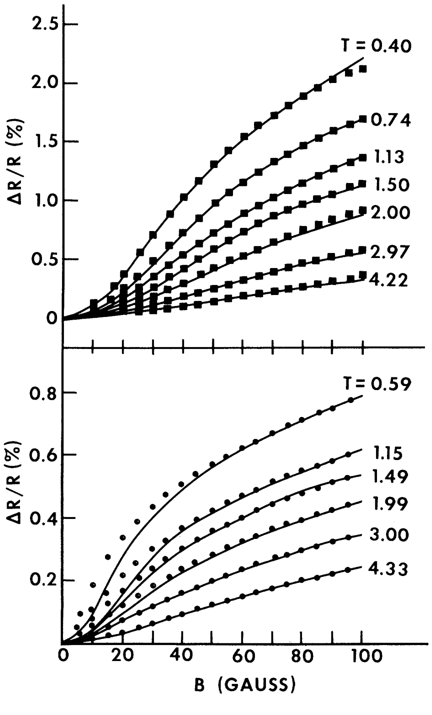

FIG. 2. Magnetoconductance measurements, $\Delta R/R = [\delta R(0) - \delta R(B)]/R$, against magnetic field for differing temperatures displayed for the narrow channel (upper part) and the wide channel (lower part). Fits to the theoretical expressions for a single parameter l_{in} are indicated at 5-G intervals. Here $n_s = 5.98\times10^{16}/\mathrm{m}^2$ for the narrow channel and $n_s = 5.93\times10^{16}/\mathrm{m}^2$ for the wide channel, and $D = 1.4\times10^{-2}$ m^2/s, implying a mean free path of $\sim10^{-7}$ m.

had oxide thickness about 3.5×10^{-8} m with maximum mobilities between 17 000 and 20 000 cm^2/V s. The size of the narrow region was measured by scanning electron microscopy after all electrical measurements were completed.

In Fig. 2 we display the magnetoconductance observed for both channels at a number of different temperatures for nearly the same electron surface density n_s, where $K_F l_e = 46$. The inelastic lengths are extracted from the data by fits to the theoretical localization expressions. We have been *unsuccessful* in fitting the magnetoconductance data with the two forms suggested by Al'tshuler and Aronov, attributing all magnetoconductance either to the interaction part or to a sum of interaction and localization parts.[8] We conclude that the interaction part is insensitive to a weak magnetic field. Similar data and quantitatively equivalent fits at other electron surface densities and on a second similar device have been obtained.

The temperature dependence of these inelastic lengths deduced from the magnetoconductance, shown in Fig. 3, gives further credence that one-dimensional localization is being observed. We follow the current understanding that electron-

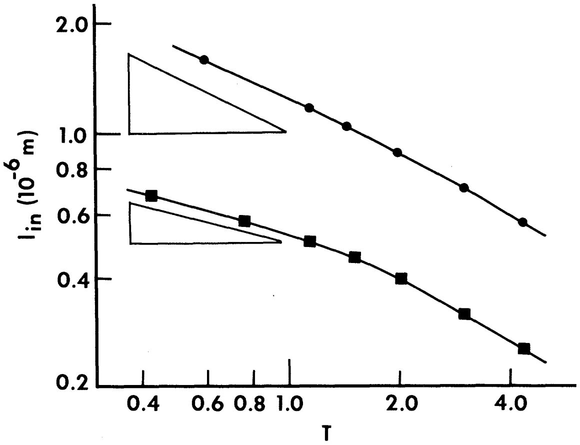

FIG. 3. Plots of l_{in} deduced from the magnetoconductance data (Fig. 2) against temperature. The upper curve is that associated with the wide channel, the lower curve with the narrow channel. These results for the narrow channel demonstrate that the condition $l_{in} > W/\pi$, as well as $(\hbar D/kT)^{1/2} > W/\pi$, are satisfied at all temperatures investigated. Numerically, Eqs. (5) and (6) predict that the ratio of inelastic lengths is 3.6 at 0.5 K. We believe these results are in satisfactory agreement, considering a possible error of 10% in the absolute magnitudes, as discussed in Ref. 4. The triangles are drawn as an aid to the eye, where the slopes are proportional to $T^{-1/2}$ and $T^{-1/4}$.

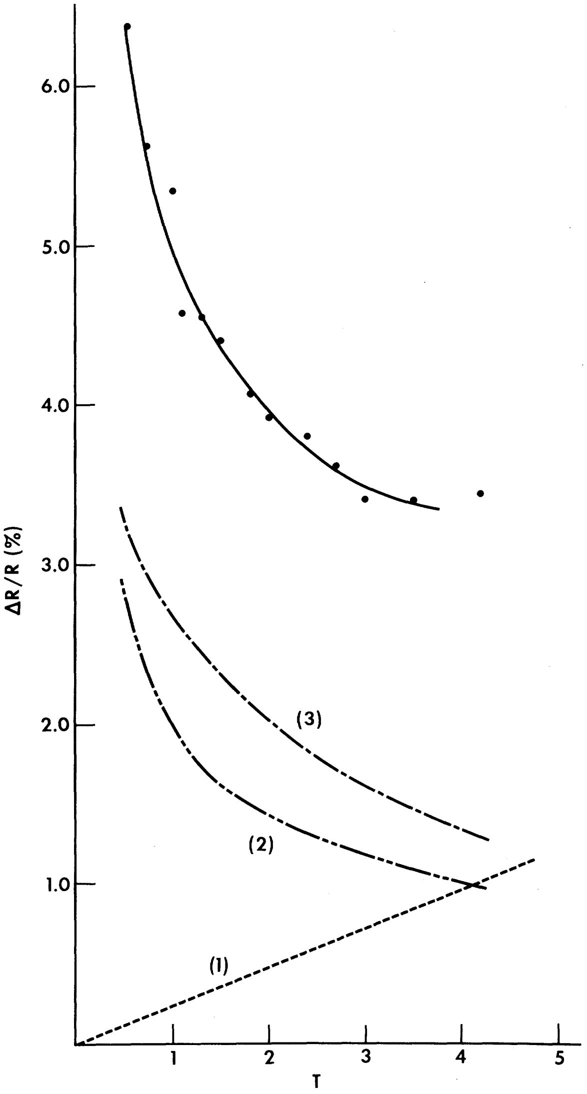

FIG. 4. The individual contributions to the resistance change plotted as a function of temperature. Curve (1) is the contribution due to the temperature-dependent Coulomb scattering, based on the empirical formula in the text. Curve (2) is the contribution due to electron-electron interaction, calculated theoretically from Eq. (5). Curve (3) is the localization contribution, where l_{in} has been determined from the magnetoconductance measurements. The sum of these contributions is indicated by the solid curve. The zeros of the data points $\Delta R/R$ relative to R at $T = 4.2$ K are positioned to agree best with the sum curve. The total change in the data from 4 to 0.5 K is nearly 3%, larger than the sum of the temperature-dependent screening part plus either the localization or the interaction part over this temperature range. At lower electron surface densities the temperature-dependent screening part is larger than here such that $\Delta R/R$ data clearly show a minimum well below 4 K.

electron scattering processes are the dominant inelastic mechanisms in high-mobility MOSFETs. The rates are for momentum-conserving scattering $1/\tau_{\text{in}} \sim (T/T_{\text{F}})^2$.[13] For impurity-mitigated scattering at low temperatures, the predicted rates are for two-dimensional systems[13]

$$\frac{1}{\tau_{\text{in}}} = \frac{a}{4\pi N D \hbar^2} KT, \tag{5}$$

and, on extension of the calculation to the one-dimensional case,

$$\frac{1}{\tau_{\text{in}}} = \frac{a\sqrt{2}}{\pi\hbar W N}\left[\frac{kT}{D\hbar}\right]^{1/2} = \frac{a e^2 \sqrt{2} D}{\pi\hbar W \sigma}\left[\frac{kT}{D\hbar}\right]^{1/2}, \tag{6}$$

where N is the density of states, a has a theoretical value of 1 at $T=0$, and experimentally $a \sim 10$ for both cases. As the temperature is reduced, we expect that the latter process will dominate, leading toward an l_{in} dependence upon temperature as $T^{-d/4}$ where d is the dimensionality.

For the narrow channel it is now tempting to ascribe any resistance change with temperature to be solely a sum of the localization and interaction parts. Unfortunately a temperature-dependent screening conspires to make this difficult.[14,15] The actual temperature dependence is, in the spirit of a Matthiessen's rule, the sum of these three contributions. Since the mean free path is less than the sample width, the latter can be represented, for high-mobility devices, as a temperature-dependent elastic scattering rate $1/\tau_T = (1.5 \pm 0.5)\times 10^{14}\, T/n_s \tau_e$ s^{-1}. In Fig. 4 we plot each contribution and the sum. Also indicated are the measured values of $\Delta R/R$ relative to R at 4.2 K. We take these data to be in excellent agreement, confirming the existence and magnitude of all three contributions to the temperature dependence.

We have constructed a quasi one-dimensional electron system where $K_{\text{F}} l_e \gg 1$ from a silicon inversion layer by fabricating a channel width less than the inelastic length. Resistance and magnetoconductance measurements have allowed us to extract both localization and interaction contributions to the differential conductance.

We conclude by noting that while the localization mechanism is dominant in two dimensions, the situation is quite different in one dimension. If we desire to have the localization resistance larger than the interaction resistance then $l_{\text{in}} > [\hbar D/kT]^{1/2}$. If l_{in} is limited by the scattering time given by Eq. (6), we obtain $W/[\hbar D/kT]^{1/2} \geqslant (e^2/\pi\hbar\sigma)\sqrt{2}a$. The left side of this inequality is less than π; therefore the conductivity must be larger than $\sigma_c = (e^2/\pi^2\hbar)\sqrt{2}a$. This result implies that the localization resistance is not easily observed in silicon inversion layers when $\sigma < 4 \times 10^{-4}\ (\Omega/\square)^{-1}$.

This work has been supported in part by the National Science Foundation under Grant No. DMR-7820071 and by the U. S. Office of Naval Research under Grant No. N00014-76-C-1083.

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