Geometric Derivation of the Diffractive Multiplicity Distribution

Saul Barshay
III. Physikalisches Institut, Technische Hochschule Aachen, D-5100 Aachen, Germany
(Received 3 August 1982)

The charged-hadronic multiplicity distribution for high-mass diffractive states is derived from the distribution for e^+e^- annihilation, and is compared with a recent experiment. The result involves no parameters.

PACS numbers: 13.85.Ni, 12.40.Pp, 13.65.+i

In a recent Letter,[1] precision data have been presented on the multiplicity distribution of charged hadrons from diffractive states X in the mass range 2 GeV $\lesssim M_X \lesssim$ 6 GeV, produced in the following reactions in particular:

$$h+p \to X+p \quad (h=p,\pi^-). \tag{1}$$

The multiplicity distribution is approximately independent of M_X (and $\sqrt{s}$) when $\langle n\rangle\sigma_n{}^d/\sigma_d$ is plotted versus the scaled variable[2,3] $n/\langle n\rangle$, where $\langle n\rangle$ denotes the average charged multiplicity at M_X, and $\sigma_n{}^d$ and σ_d denote the n-particle diffractive cross section and the total (sum over n) diffractive cross section, respectively, at M_X. This confirms, with greater precision in the present mass range, the previous conjecture and the experimental evidence[4] for such approximate scaling behavior obtained from an analysis of pp and π^-p bubble-chamber data in the higher mass range 5 GeV $\lesssim M_X \lesssim$ 13 GeV. The experimental[1] multiplicity distribution is shown in Fig. 1. In this paper I show that this distribution can be derived, with no parameters, from the charged-hadronic multiplicity distribution observed[5,6] in e^+e^- annihilations.

The recently established[5,6] experimental multiplicity distribution for e^+e^- annihilations is shown in Fig. 2, plotted in the scaled form which shows little dependence upon the total center-of-mass energy $\sqrt{s}$ in the present range. The scaling function which provides a good, continuous[7] representation of the data is[8]

$$\psi_{e^+e^-}(z)=\tilde{\psi}(z)=(81\pi^2/64)z^3\exp[-(9\pi/16)z^2], \tag{2}$$

with

$$\int_0^\infty \tilde{\psi}(z)\,dz=\int_0^\infty z\tilde{\psi}(z)\,dz=2.$$

This particular functional form[8] contains no parameters since the overall numerical coefficient and that in the exponent are fixed by the two normalization conditions associated with Eq. (2). It can be shown[8,9] that simple geometric arguments[10] allow one to derive from the distribution in Eq. (2), with *no* parameters, the charged-hadronic multiplicity distribution for $p+p\to$ anything. The idea[10] of a (normalized) multiplicity distribution, $P_n(b_s)$ at each impact parameter[11] b, is introduced. The observed broad distribution for pp collisions is then given by summing the distributions from all impact parameters with the weight $\sigma_{\rm inel}(b_s)\propto 1-\exp[-2\Omega(b_s)]$, where $\Omega(b_s)$ is the eikonal:

$$\frac{\sigma_n(s)}{\sigma_{\rm inel}(s)}=P_n(s)=\frac{\int_0^\infty d(b^2)P_n(b_s)\sigma_{\rm inel}(b_s)}{\int_0^\infty d(b^2)\sigma_{\rm inel}(b_s)}. \tag{3}$$

The distribution for e^+e^- annihilation is that for a definite (near-zero) impact parameter[12] or, al-

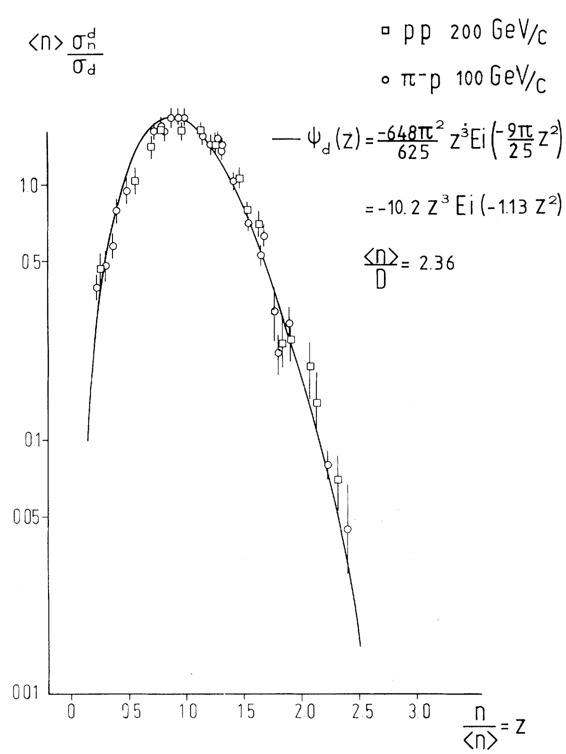

FIG. 1. The charged-hadronic multiplicity distribution from diffractive states X produced in $p(\pi^-)+p\to X+p$. The data are from Ref. 1.

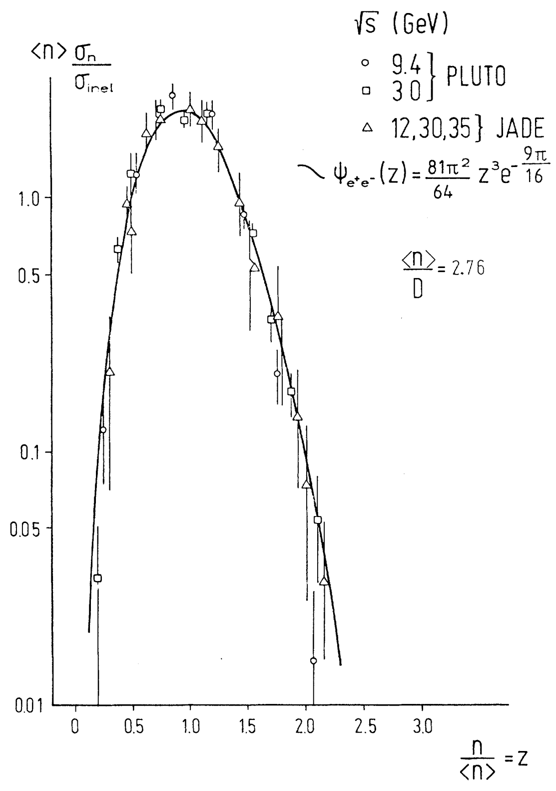

FIG. 2. The multiplicity distribution for $e^+e^- \to n$ charged hadrons. The initial data are from the PLUTO Collaboration (Ref. 5); the most recent are from the JADE Collaboration (Ref. 6) and the points are the same at the three energies, within present errors.

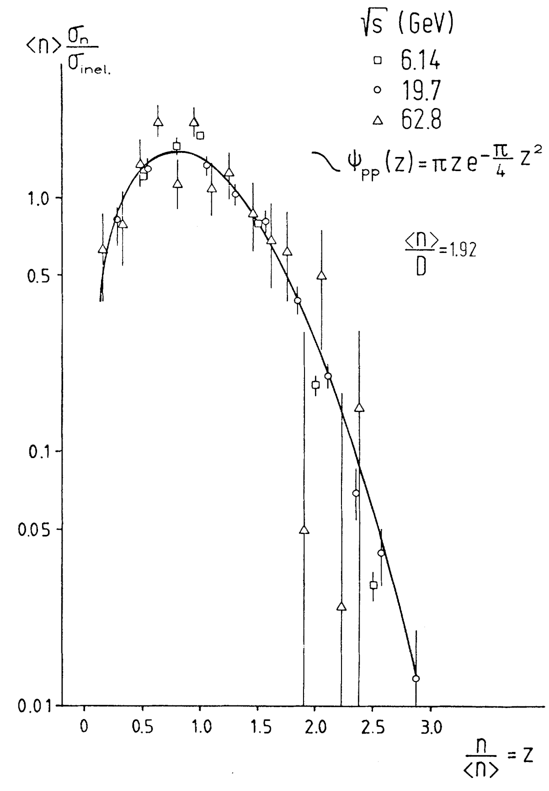

FIG. 3. The multiplicity distribution for $pp \to n$ charged hadrons. The data are from Refs. 3 and 18.

ternatively, for the lowest total angular momenta even at large s, since the annihilation proceeds through the single-photon intermediate state. Thus[10]

$$P_n(b_s) = \frac{\tilde{\psi}(\tilde{z})}{\langle n(b_s,s)\rangle} \text{ with } \tilde{z} = \frac{n}{\langle n(b_s,s)\rangle}. \qquad (4)$$

The physical assumption is made[8,10] that more particles are produced on the average in hard collisions at small b, and, in particular

$$\langle n(b_s,s)\rangle = N(s)[\Omega(b_s)]^{1/2}. \qquad (5)$$

This is reasonable since the eikonal may be interpreted[13] as an overlap on the impact-parameter plane of two colliding matter distributions. In a Gaussian approximation in which each distribution is proportional to $\exp[-2\lambda(b_s^2)]p$ (for pp collisions) we have $\Omega(b_s) \propto \exp(-\lambda b_s^2)$, where λ is an inverse-size parameter.[14] With use of the Gaussian approximation also for $\sigma_{inel}(b_s) \approx \sigma_0 \times \exp(-\lambda b_s^2)$ and Eqs. (4) and (5), the integration over impact parameter in Eq. (3) becomes elementary with a change of variable $u = \exp(\lambda b_s^2)$, leading to

$$P_n(s) = \frac{81\pi^2}{64N}\left(\frac{n}{N}\right)^3 \int_1^\infty du \exp\left[-\frac{9\pi}{16}\left(\frac{n}{N}\right)^2 u\right]$$

$$= \frac{\pi}{\langle n(s)\rangle} z \exp(-\tfrac{1}{4}\pi z^2) \qquad (6)$$

with $z = n/\langle n(s)\rangle$, and $\langle n(s)\rangle = \frac{2}{3}N(s) = \frac{2}{3}\langle n(b=0,s)\rangle$.

In Fig. 3 we show the strikingly broad charged-hadronic multiplicity distribution for pp collisions and the function $\psi_{pp}(z)$ from Eq. (6). In fact it was long known[15,16] that such a functional form gave a simple, good fit to the pp data, and this fact was used with the geometric interpretation to predict[8] the e^+e^- multiplicity distribution in Eq. (2) some years ago. The further prediction[8] from Eq. (6), that the average number of charged hadrons produced in e^+e^- annihilation (like a head-on pp collision) should be about 1.5 times the overall average (from all $b \geq 0$) observed in pp collisions, is also in accord with recent measurements[17,18] at the highest energies [$N(30$ GeV$) \cong \langle n(30)\rangle_{e^+e^-} \cong 13$; $\langle n(30)\rangle_{pp} \cong 9$].

Now, in order to obtain the geometrical prediction for the diffractive multiplicity distribution in Fig. 1, we need two new elements. (1) In place of $\sigma_{\rm inel}(b_s)$ in Eq. (3) we need an impact-parameter representation of σ_d. Empirically[19] $d\sigma_d/dM_X^2 \propto 1/M_X^2$; in the geometrical picture we expect M_X to be proportional to the amount of overlapping matter. In fact this naive assumption is itself empirically grounded in the data of Ref. 1. In the mass range studied the observed average multiplicity is found to be approximately proportional to $M_X^{1/2}$. This fact, together with the assumption about impact-parameter dependence in Eq. (5), is naturally accomodated by $M_X(b_s)\propto\Omega(b_s)$. Therefore we take $\sigma_d(b_s)\propto[\Omega(b_s)]^{-2}\propto\exp(2\lambda b_s^2)$. (2) We still need an additional factor in $\sigma_d(b_s)$ which reflects the fact that the proton target must "hang together" in the diffractive reactions of Eq. (1). Indeed, the fact that the proton recoils predominantly[19] at small four-momentum transfers t means, of course, that different b must contribute coherently in the amplitude. However, let us make the gross approximation that the measured t interval[1] $[0.025<|t|<0.095\ ({\rm GeV}/c)^2]$ is complete, that is we neglect events outside this interval so that integration over "all" t removes the coherence. Then $\sigma_d(b_s)$ must be multiplied by something like the *square* of the b-space form factor of an individual proton, i.e., by approximately $[\exp(-2\lambda b_s^2)]^2$. With these changes in Eq. (3) and replacing s by M_X^2, we obtain immediately in place of Eq. (6) the distribution

$$P_n^{\,d}(M_X^2)=\frac{81\pi^2}{32N}\left(\frac{n}{N}\right)^3\int_1^\infty\frac{du}{u}\exp\left[-\frac{9\pi}{16}\left(\frac{n}{N}\right)^2u\right]$$

$$=-\frac{648\pi^2}{625\langle n(M_X^2)\rangle}z^3\,{\rm Ei}\left(-\frac{9\pi}{25}z^2\right)\qquad(7)$$

with $z=n/\langle n(M_X^2)\rangle$ and $\langle n(M_X^2)\rangle=\frac{4}{5}N(M_X^2)$. Here $-{\rm Ei}(-x)$ is the exponential integral function; the corresponding scaling function $\psi_d(z)$ is shown in Fig. 1. Considering the drastic approximation (2) above, which removed coherence, there is a remarkable resemblance of this theoretically motivated and parameter-free function to the data: In addition to the maximum height of the curve, the essential numbers are $\langle n\rangle/D=2.36$ ($D=[\langle n^2\rangle-\langle n\rangle^2]^{1/2}$), and the peak value of $z_p\approx0.9$; the experimental numbers are stated[1] as $\langle n\rangle/D\cong2.2$ and $z_p\lesssim1$. Also, we have experimentally[5,6,17] $N(6\ {\rm GeV})=\langle n(6)\rangle_{e^+e^-}\cong5$ and[1] $\langle n_d(6)\rangle_{pp}\cong4$, in agreement with the further prediction in Eq. (7).

In Figs. 1–3 there are three different, parameter-free functions which give good first-approximation representations of the charged-hadron multiplicities from three physically different high-energy collision processes. Through Eqs. (6) and (7) the hadron-induced distributions are related, with no parameters, to that for e^+e^- annihilation via weighted sums over impact parameter. There is a striking correlation of data achieved within the geometrical picture. Dynamical questions for further experimental study at the e^+e^- and $\bar pp$ colliders include the following: (1) Does the distribution for e^+e^- have a tendency to broaden with increasing $\sqrt{s}$, as does[18] that for pp? (This is the residual energy dependence not removed by the scaled variable z.) Or does it narrow (a statistical process)? (2) Is the average multiplicity function $N(s)$ gradually changing form in different high-energy domains? (3) The distribution for $\bar pp$ annihilation (directly measured only at relatively low $\sqrt{s}$) definitely looks like[5,6,20] that for e^+e^-, in particular $\langle n\rangle/D\sim2.8$ and the average multiplicity is about 50% higher[21] than for pp (at the same nominally available energy[21]). Can one study this small-b multiquark process at high energies for comparison with e^+e^-?

I wish to thank Professor H. Faissner and Professor R. Rodenberg for their warm hospitality at the III. Physikalisches Institut. This work was supported in part by the Deutsche Forschungsgemeinschaft.

[1]R. L. Cool *et al.*, Phys. Rev. Lett. 48, 1451 (1982).

[2]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[3]P. Slattery, Phys. Rev. D 7, 2073 (1973).

[4]S. Barshay *et al.*, Phys. Rev. Lett. 32, 1390 (1974).

[5]Ch. Berger *et al.* (PLUTO Collaboration), Phys. Lett. 95B, 313 (1980).

[6]R. Felst (JADE Collaboration), Deutsches Elektronen-Synchrotron Report No. 81-075, 1981 (to be published).

[7]The discrete variable z is approximated as continuous, leading to the normalization conditions (= 2) of Eq. (2).

[8]S. Barshay and Y. Yamaguchi, Phys. Lett. 51B, 376 (1974).

[9]S. Barshay, Phys. Lett. 116B, 193 (1982).

[10]S. Barshay, Phys. Lett. 42B, 457 (1972), and Lett. Nuovo Cimento 7, 671 (1973).

[11]Henceforth, the scaled impact parameter, $b_s=b/R(s)$, is used. In the present energy domain, this is required by the recent exhaustive analysis of the data on the increasing pp cross section and the shrinking

diffraction peak by U. Amaldi and K. R. Schubert, Nucl. Phys. B166, 301 (1980), and references therein.
[12]This can also be seen by viewing the time-reversed collision between practically pointlike quark and antiquark.
[13]T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1968).
[14]The derived distributions, for collisions of hadrons of similar size, will not depend upon this parameter in first approximation.
[15]G. Bozóki *et al.*, Nuovo Cimento A 64, 881 (1969).
[16]A. J. Buras and Z. Koba, Lett. Nuovo Cimento 6, 629 (1973).
[17]R. Brandelik *et al.* (TASSO Collaboration), Phys. Lett. 89B, 418 (1980).
[18]W. Thomé *et al.*, Nucl. Phys. B129, 365 (1977).
[19]R. L. Cool *et al.*, Phys. Rev. Lett. 47, 701 (1981).
[20]J. Salava and V. Simák, Nucl. Phys. B69, 15 (1974).
[21]S. Barshay, A. Fridman, and P. Juillot, Phys. Rev. D 15, 2702 (1977).

Observation of *P*-Wave $b\bar{b}$ Bound States

K. Han, T. Böhringer,[a] P. Franzini, G. Mageras, D. Peterson, E. Rice, and J. K. Yoh[b]
Columbia University, New York, New York 10027

and

J. E. Horstkotte, C. Klopfenstein, J. Lee-Franzini, R. D. Schamberger, Jr., M. Sivertz, L. J. Spencer, and P. M. Tuts
The State University of New York at Stony Brook, Stony Brook, New York 11794

and

S. W. Herb
Cornell University, Ithaca, New York 14853

and

R. Imlay, G. Levman, W. Metcalf, and V. Sreedhar
Louisiana State University, Baton Rouge, Louisiana 70803

and

G. Blanar,[c] H. Dietl, G. Eigen, E. Lorenz, F. Pauss, and H. Vogel
Max-Planck-Institut für Physik, D-8000 Munich 40, Federal Republic of Germany

(Received 3 August 1982)

The existence of P-wave $b\bar{b}$ bound states χ_b' is demonstrated by observation of photons from the transition $\Upsilon'' \to \gamma + \chi_b'$ in the inclusive photon spectrum from Υ'' decays. The center of gravity of the observed photon energies is 98 MeV and the branching ratio for the transition of the Υ'' to the χ_b' states is $(34\pm3)\%$ (statistical).

PACS numbers: 14.40.Gx, 13.40.Hq, 13.65.+i

The discovery[1] of the J/ψ and its explanation by Appelquist and Politzer as the bound state of a charmed quark and antiquark[2] opened a new field of experimental and theoretical physics, the spectroscopy of heavy "quarkonia."[3] The discovery of four Υ states[4-6] has further enriched this field by the addition of a fifth quark, the b quark, of mass ~ 5 GeV. In typical quarkonium potentials the bound $b\bar{b}$ quarks are considerably less relativistic than the lighter $c\bar{c}$ quarks and the $b\bar{b}$ states are therefore more amenable to calculations by nonrelativistic potential methods.[7,8] In any model of quark-antiquark bound systems one expects the existence of singlet and triplet S-wave, P-wave, etc., states. Most of these states have been found in "charmonium."[9] Only triplet S states are typically produced in e^+e^- annihilations. Other states can be reached via electromagnetic or hadronic transitions.

We report in the following the first evidence for the existence of P-wave $b\bar{b}$ states, obtained from the observation of a strong, quasimonochromatic photon signal in Υ'' decays. We interpret this signal as being due to the electric dipole ($E1$) radiative transition $3^3S_1(b\bar{b}) \to \gamma + 2^3P_J(b\bar{b})$ because (i) the large observed branching ratio is in good

