use the reduced Hamiltonian to understand not only confinement but also the existence of a finite mass gap. In a Monte Carlo simulation based on Eq. (2), the discreteness of the spectrum is particularly welcome. In simulations based on the infinite-volume action, it is sometimes found necessary to enforce a projection on the zero-momentum sector by time-consuming summations. In the reduced model, this is done automatically.

Following the lines of Ref. 6, an infinite number of flavor-carrying fermions can be added to the pure gauge Hamiltonian. The resulting quantum matrix problem is expected to reproduce an impressive list of strong-interaction features: confinement in the absence of matter fields, approximately linear Regge trajectories, finite width resonances, glueball states, multiparticle production, flavor singlet-nonsinglet mass splittings, and chiral symmetry breaking.

In the infinite-volume theory, the idea that a Hamiltonian version of large N_c could reproduce glueball masses in addition to loop expectation values is implicit in the collective field formalism.[9,10] Recent numerical simulations[10] have demonstrated the necessity of adding constraints in the form of inequalities at weak coupling. The quenching of the Eguchi-Kawai Hamiltonian seems to be a simpler procedure and therefore might prove to be more advantageous.

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Oscillatory Scaling Violations and the Quantum Chromodynamic Coulomb Phase

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The exchange of gluons in the initial or final state of hard processes is shown to build a Q^2-dependent phase shift which may be observed as an oscillatory scaling violation in infrared-dominated processes. The non-Abelian structure of the phase can affect cross sections for experiments as diverse as pp elastic scattering at fixed angle and dilepton production in hadronic collisions, resulting in oscillatory components. These oscillations may already have been detected.

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Quantum chromodynamics (QCD) is a theory requiring interaction via the exchange of massless vector gluons. Most calculations within perturbative QCD inevitably encounter infrared divergences, which must be organized to extract meaningful predictions. A simple physical interpretation of the technical problem involved recognizes that, because of the long-range character of the interaction, the usual free asymptotic states are not appropriate. One effect of initial- and final-state interactions is a distortion of the propagation of color charges (e.g., quarks) and is described by a momentum-dependent phase shift. By analogy with quantum electrodynamics (QED) where this effect is well understood,[1] we call the leading component of the phase (describing nearly on-shell Green functions of charged particles in QCD) the quantum chromodynamic Coulomb phase shift (QCDCP). This phase shift is a very general feature of massless interaction in either perturbatively resummed calculations or nonperturbative (coherent-state[1,2]) approaches. The purpose of this Letter is to point out that when quarks are combined into hardons, the QCDCP introduces important effects. Measurement of the phase through interference effects in hadronic amplitudes thus provides a superb probe of the underlying dynamics.

In approximations based on a potential one can obtain a phase shift from familiar eikonal techniques which have generalizations in field theory. The perturbative approach we use, on the other hand, is based on the observation that techniques for resumming infrared (ir) divergences in a gauge theory (QED or QCD) can be applied to the QCDCP. The well-known exponentiation of leading double logarithms (LDL) in QCD[3] is accompanied by the QCDCP as follows: Order by order in perturbation theory, one sums terms of the form $\alpha_s^J \ln^{2J}[(-\tilde{s}-i\epsilon)/\lambda^2]$, where $\tilde{s}$ is a large scale and λ^2 is fixed, with α_s the QCD coupling constant. General (process-dependent) analyticity relations then require continuation via, e.g.,

$$\ln^2[(-\tilde{s}-i\epsilon)/\lambda^2)] \rightarrow \ln^2|\tilde{s}/\lambda^2| - 2\pi i \ln|\tilde{s}/\lambda^2|\,\theta(\tilde{s}) + O(1), \qquad (1)$$

which introduces imaginary parts in the usual way if $\tilde{s}$ refers to a (timelike) channel with a threshold. While the full details of analyticity and crossing are quite involved, in simple cases the imaginary terms of $O[i\alpha_s\pi \ln(|\tilde{s}|/\lambda^2)^{2J}]$ can be resummed into an exponential factor or phase. The behavior of the running coupling constant $\alpha_s(\mu^2)$ is an important aspect of the resummation, as will become clear below. Equally important is the dependence of the phase on a small scale λ, which can be interpreted as the inverse of the unscreened interaction time available for a phase to build up.

Since the generators of the quark color-charge algebra $(t_{ij}{}^a)$ are non-Abelian, it is not surprising that the QCDCP for quarks (q) requires a color-matrix representation. Although lowest-order (Born) amplitudes are known to be eigenvectors of real LDL (color matrix) corrections, the proof requires that different large scales be combined in the fixed-angle limit; for $qq \rightarrow qq$, e.g., one sets $\ln|s| \approx \ln|t|$ with s and t the usual Mandlestam variables. We must, however, distinguish the analytic properties of $\ln(-s)$ and $\ln(-t)$ to find the QCDCP, resulting in color-matrix exponentiation.

The coefficients needed for the QCDCP can be interpreted as matrix-valued imaginary anomalous dimensions that can be obtained from the differential equation method of Refs. 3 and 4. Letting $M_B{}^{i_1 \cdots i_N}(q_1,\cdots,q_N)$ be the Born amplitude for a Green function with N quark legs having momenta $q_\alpha{}^\mu$, color index i_α, in the limit $q_\alpha{}^2$ fixed, $Q^2 \rightarrow \infty$, where $Q^2 = |s_{\alpha\beta}|$ and $s_{\alpha\beta} \equiv (q_\alpha + q_\beta)^2$, one obtains a resummed amplitude $M^{i_1 \cdots i_N}(q_1,\ldots, q_N,\mu)$ given by

$$M^{i_1\cdots i_N} = \exp[-(NC_F/2)B(Q^2,\mu^2)]\,[\exp(i\,\overline{\varphi}(Q^2,\mu^2))]_{j_1\cdots j_N}{}^{i_1\cdots i_N} M_B{}^{j_1\cdots j_N}, \qquad (2)$$

where $B(Q^2,\mu^2) = 6\ln(Q^2/\mu^2)\ln\ln(Q^2/\mu^2)/(33-2N_f)$ is the LDL factor in QCD with $N_f = 4$ flavors, $C_F = \frac{4}{3}$, and μ is the scale introduced in dimensional regularization. The phase $\overline{\varphi}$ then satisfies

$$\left(\mu^2 \frac{\partial}{\partial\mu^2} - \frac{\alpha_s(\mu^2)}{2\pi} \sum_{\substack{\alpha,\beta=1 \\ \alpha<\beta}}^{N} i\pi\,\theta(s_{\alpha\beta})\overline{\gamma}_{\alpha\beta}\right) e^{i\overline{\varphi}} = 0, \qquad (3)$$

where $[\overline{\gamma}_{\alpha\beta}]_{i_\beta j_\beta}{}^{i_\alpha j_\alpha} = \lambda_{i_\alpha j_\alpha}{}^a \lambda_{i_\beta j_\beta}{}^a$, with $\lambda^a = -t^a$ $(+t^{aT})$ for outgoing (incoming) quark fermion number and T denotes transpose. (We can replace the quark mass dependence of Ref. 4 with μ in the limit considered. All $q_j{}^\mu$ are directed inwards.)

To illustrate the QCDCP, and the point that the QCDCP need not cancel in a simple manner, we present the example of $qq' \rightarrow qq'$ scattering. Among the one-loop graphs giving LDL corrections in Feynman gauge (Fig. 1) only one graph [Fig. 1(a)] has a threshold; the color structure is that of a t-channel octet plus singlet. Straightforward application of Eqs. (2) and (3) yields

$$M^{i_1\cdots i_4}(q_1,\cdots,q_4,\mu) = \exp(-2C_F B)\exp[i\omega(\overline{1} + \overline{A}_{12})]M_B{}^{j_1\cdots j_4}, \quad (4)$$

where $\omega = (6\pi/25N_c)\ln\ln(Q^2/\mu^2)$, $\overline{A}_{12} = -N_c\,\delta_{j_2}{}^{i_1}\delta_{j_1}{}^{i_2}$,

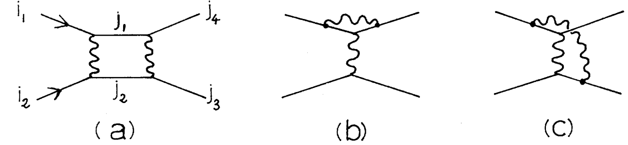

FIG. 1. One-loop graphs illustrating LDL corrections for $qq' \rightarrow qq'$ in Feynman gauge.

$N_f=4$, $N_c=3$ in QCD, and color indices are shown in Fig. 1.

Equation (4) illustrates two significant features: (a) A phase of order $\ln\ln(Q^2/\mu^2)$ is asymptotically characteristic of the QCDCP including running-coupling-constant effects. This is contrasted with QED, where replacing $\alpha_{\rm em}(\mu)\to\frac{1}{137}$ gives a phase of order $\ln(Q^2/\mu^2)$. (b) The color matrix $\exp(i\omega\bar{A})$ is not unimodular. Letting $b^2=\langle\bar{A}_{12}{}^2\rangle/\langle\mathrm{I}\rangle$, $a^2=\langle\bar{A}_{12}\rangle^2/b^2\langle\mathrm{I}\rangle^2$, where the angular brackets denote color traces with a particular state, one finds

$$|\langle\exp(i\omega\bar{A}_{12})\rangle|^2\propto 1+a^2+(1-a^2)\cos(2b\omega). \tag{5}$$

This indicates that the matrix QCDCP can have observable interference consequences even when there is no other interfering amplitude. To extend this conclusion to the hadron level, one must combine a complete set of graphs; indeed qq', $q\bar{q}'$, and $\bar{q}\bar{q}'$ multiple elastic scattering in QCD are all characterized by matrix exponentiation of the type displayed in Eq. (5). Of course, some model is required to translate relations for quark amplitudes into phase dependence of hadron amplitudes, and at this point the intricacies of screening arising from the color-singlet nature of hadrons is crucial. Although a complete understanding of such problems is still under development,[5] it is now accepted that corrections of LDL order at the quark level produce important phenomenological effects in many hadronic observables.[6] We claim that the QCDCP introduces an important feature here. Specifically an experimental signal of interference of the QCDCP is an additive, oscillating component with argument ω in cross sections. At first sight an oscillation proportional to $\ln\ln(Q^2/\mu_0{}^2)$ may appear to be unobservably slow, but if the scale $\mu_0{}^2$ is small (e.g., $\mu_0\simeq\Lambda_{\rm QCD}=100$ MeV) one finds $\omega\sim$ const $+\ln(Q^2/\mu^2)\ln(\mu^2/\mu_0{}^2)+\ldots$, i.e., almost logarithmic oscillations. Oscillations of exactly logarithmic order due to the QCDCP can also be found, in general, in terms suppressed by powers of $\exp(-\ln s\ln\ln s)$.[7,8]

In the following we discuss three diverse experimental situations whose common link is LDL sensitivity and the presence of a timelike scale:

(1) *Fixed-angle elastic scattering.*—Mueller[6] has recently shown that the LDL-suppressed hard-scattering contributions of Landshoff[9] give a leading asymptotic power-law dependence ($d\sigma/dt\sim\bar{s}^P$). The QCDCP effect in this case yields an additive component to $d\sigma/dt$ which oscillates with $\ln\ln s$ period.[8] In fact, data for $pp\to pp$ at fixed angle ($\theta_{\rm c.m.}=90°$) are known to oscillate substantially about the power-law relation and indeed with roughly logarithmic period. Although the available data certainly do not correspond to asymptotic energies, the qualitative agreement is very encouraging.[8]

(2) *The Q_T distribution of dileptons.*—Many authors[10] have concluded that $d\sigma/d^4Q$ (for dileptons of momentum Q^μ produced in hadronic collisions) can be calculated in the small-transverse-momentum (Q_T) region by resumming terms of LDL origin. In this case, Q_T serves as a small scale reminiscent of a cutoff. If the various QCDCP's of hard-scattering subprocesses do not cancel, then oscillations will occur with respect to $Q_T{}^2$ in $d\sigma/d^4Q$ (at fixed Q^2 and rapidity). Although the data available are not yet sufficiently precise to be conclusive, they are consistent with roughly logarithmic oscillations about recent theoretical calculations[11] (which include the LDL effects but not the QCDCP). Such a signal could be of great importance if it persists in more precise data.

(3) $e^+e^-\to\gamma+\pi^{(\pm)}+X$.—Recently Collins and Soper[12] have demonstrated that the small-transverse-momentum distribution of hadrons in oppositely oriented jets produced in e^+e^- collisions can be calculated in QCD. Since all scales are timelike in this case, the QCDCP cancels exactly for the cross section proposed in Ref. 12. An interesting (but experimentally difficult) variation, however, is to choose one of the hadrons to be a photon. In that case, interference between final-state [Fig. 2(a)] and initial-state [Fig. 2(b)] bremmstrahlung will make the QCDCP observable. Letting $x_\gamma=2E_\gamma/(Q^2)^{1/2}$, where $(Q^2)^{1/2}$ is the total energy and E_γ the photon energy in the e^+e^- c.m. frame, one finds a mismatch of hadronic scales Q^2 [Fig. 2(a)] and $Q^2(1-x_\gamma)$ [Fig. 2(b)] in the interference term [Fig. 2(c)]. This produces a charge-asymmetric term which oscillates like $\ln[\ln(1-x_\gamma)Q^2/\ln Q^2]\sim\ln(1-x_\gamma)$ at fixed Q^2. Let us remind the reader that related QED charge-

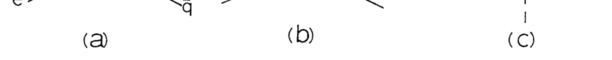

FIG. 2. (a) Final-state and (b) initial-state bremsstrahlung contributions to the reaction $ee\to\gamma\pi\chi$ discussed in the text. (c) Interference leading to the QCDCP oscillations. Ovals represent higher-order interactions.

asymmetry calculations[13] show that such observables are large enough for experiments to detect.

In conclusion, we have shown that the QCDCP is a prediction of perturbative QCD with observable consequences. The non-Abelian aspects of its behavior at the hadron level are complicated and constitute a challenging problem. Data for pp elastic scattering indicate that the QCDCP has been seen. While nonleading effects are undoubtedly important, theoretical techniques that are becoming available[14] promise to make the QCDCP an attractive and sensitive tool for probing hadron dynamics.

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Note added.—Recent work by A. Sen [Fermilab Report No. Fermi-pub 82/66 THY (to be published)] confirms that the QCDCP is computable for qq scattering in QCD, and goes on to specify $\mu_0 = \Lambda_{\rm QCD}$ [in agreement with the leading-order prediction of Eq. (3)] when nonleading logarithms are summed.

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