the prolate GDR appears in fact narrower than the oblate GDR. The quadrupole moment of the prolate states (+876 mb) is twice as big as that of the oblate states (−480 mb),[4] and all models coupling deformation degrees of freedom to the GDR would predict a significantly broader prolate giant resonance. These differences between prolate and oblate GDR's in ^{28}Si are quite unexpected, and indicate potential difficulties in extracting information on nuclear deformations from giant resonances seen in γ-ray inclusive experiments, such as those described by Newton *et al.*,[11] where individual final states are not resolved. Finally, no calculation has yet succeeded in accounting for the intermediate structure in the GDR of ^{28}Si, and the present experiment suggests that very little of it is due to deformation.

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Bernstein-Mode Quasioptical Maser Experiment

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Experimental observations of coherent, millimeter-wavelength, higher–cyclotron-harmonic oscillations are reported for a system of an electron beam in a magnetic field traversing a Fabry-Perot resonator. The parameters of the experiment tend to support the interpretation that the strong multiple-harmonic interaction results from mode conversion of short-wavelength electrostatic waves (Bernstein modes) to long-wavelength electromagnetic resonator modes at the beam boundary.

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Recently there has been a great deal of interest in electromagnetic gain mechanisms at millimeter wavelengths, because of potential applications to plasma heating in tokamaks, plasma diagnostics, radar, and far-infrared astronomy. One such mechanism is the electron cyclotron maser instability, in which the relativistic mass dependence of the electron cyclotron frequency results in azimuthal bunching of electrons gyrating in an external magnetic field.[1] A companion mechanism arises from the axial electron velocity modulation in the wave propagation direction.[2] Although electromagnetic gain has been predicted at the fundamental as well as the cyclotron harmonics, practical devices have been restricted to operations at the fundamental or second harmonic.[3]

On the other hand, it is well known that short-wavelength electrostatic waves (Bernstein modes) can propagate perpendicular to a magnetic field without loss in a collisionless plasma.[4] It is thought that the conversion of these modes to long-wavelength electromagnetic modes at a plasma boundary is responsible for multiple-harmonic interactions which have been observed in low-pressure discharges and ionospheric topside soundings.[5] A recent analysis of this problem suggests that when mode conversion occurs at an electron beam boundary, the instability growth rates at the higher harmonics can be much larger than for electromagnetic interactions alone.[6] This Letter reports the first experimental observation of coherent higher-cyclotron-harmonic oscillations for a system in which the aforementioned mode conversion can occur. The parameters of the experiment quantitatively support the previously published simplified model for the mode-coupling interaction.[6] The results suggest that a previously unexplored mechanism may now permit the development of higher-harmonic cyclotron masers for submillimeter and far-infrared application requiring modest magnetic fields.

The schematic of the experimental configuration is shown in Fig. 1. The solid laminar-flow electron beam from a space-charge-limited Pierce gun was injected along the axis of a cylindrical stainless steel vacuum vessel located in the bore of a superconducting solenoid. The gun,[7] designed to operate at a maximum of 20 kV, 5.6 A, 10^{-3} duty cycle, with a nominal beam radius of 0.3 cm, was driven with 5-μsec, 10-$\sec^{-1}$ pulses from a MIT model 9 modulator. The cathode and transmitted currents were measured with Pearson integrating transformers.

Immediately downstream of the gun exit, the beam electrons underwent nonadiabatic passage through a spatially localized transverse magnetic field ("kicker") provided by a pair of tailored Helmholtz coils which imparted controlled transverse momentum to the beam electrons. The subsequent passage of the beam through the increasing axial magnetic field resulted in an increased value of the transverse-to-axial momentum ratio α, if one assumes the motion to be adiabatic. Computer calculations of single-particle trajectories in combined axial and kicker fields showed that momentum ratios α of the order of 2 could easily be achieved in this apparatus for kicker fields of the order of 100 G in axial guide fields of up to 10 kG.

The millimeter resonator was a confocal Fabry-Perot resonator with a fixed intermirror spacing $L = 3.125$ cm, mirror radii of curvature 11 cm, and intermode frequency spacing $\Delta f = c/2L = 4.8$ GHz. The mirrors were made of polished oxygen-free high-conductivity copper with a coupling hole and WR-10 waveguide machined into the output mirror to couple the millimeter-wave output from the system. In cold tests of the Fabry-Perot with a sweep oscillator, the complete longitudinal mode spectrum between 62 and 105 GHz was measured, with typical power coupling into the resonator of approximately −3 dB at 91 GHz. The resonator was designed for operation in the lowest transverse mode with the polarization of the electric field vector transverse to the axial guide magnetic field.

Table I summarizes the conditions under which coherent millimeter-wave output signals were observed in the frequency range 62–105 GHz. The low-frequency limit was determined by the low-frequency cutoff of the WR-10 waveguide used in these experiments which was about 59 GHz. The upper frequency was limited by the maximum value of the axial magnetic field used in these experiments which was about 40 kG. Since coherent microwave output at several cyclotron harmonics was expected for a given value of the axial magnetic field at which the cyclotron frequency coincided with a resonant mode of the Fabry-Perot, the axial magnetic field was varied slowly until oscillations were established in the cavity. The output frequency was then estimated by use of a series of waveguide cutoff filters. Thus, for the Fabry-Perot mode at 62.4 GHz, four cyclotron harmonics were observed corresponding to the fundamental mode at an axial magnetic field of 23.2 kG ($f_0 = 62.6$ GHz), second harmonic at 11.5 kG ($f_0 = 62.1$ GHz), third har-

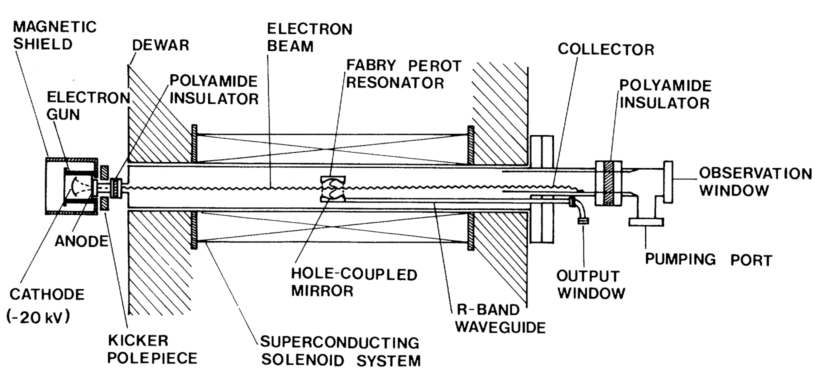

FIG. 1. Schematic diagram of the experimental arrangement.

TABLE I. Conditions under which coherent output signals were observed and the frequency of the observed output. Intermirror spacing $L = 3.125$ cm, intermode frequency $\Delta f = 4.8$ GHz. $\gamma = 1.0371$, $f_0 = N\Omega_0/2\pi\gamma$, $\Omega_0 = |e|B_0/m_0$. $f_{\text{F-P}}$ are the resonant modes of the Fabry-Perot. B_0 is in kilogauss and all frequencies are in gigahertz. N is the harmonic number.

HARMONIC NUMBER (N)											
	$f_{\text{F-P}}$	62·4	67·2	72·0	76·8	81·6	86·4	91·2	96·0	100·8	105·6
	Mode No. m	13	14	15	16	17	18	19	20	21	22
1	B_0	23·2	24·9	26·7	28·6	30·4	32·2	34·0	36·2	38·1	40·0
	f_0	62·6	67·2	72·1	77·2	82·1	86·9	91·8	97·7	102·9	108·0
2	B_0	11·5	12·4	13·3	14·3	15·2	16·1	17·1	18·0	18·8	19·8
	f_0	62·1	67·0	71·8	77·2	82·1	86·9	92·3	97·2	101·5	106·9
3	B_0	7·7	8·3	8·9	9·6	10·1	10·7				
	f_0	62·4	67·2	72·1	77·8	81·8	86·7				
4	B_0	5·7									
	f_0	61·6									

monic at 7.7 kG ($f_0 = 62.4$ GHz), and fourth harmonic at 5.7 kG ($f_0 = 61.6$ GHz). The small differences between the estimated cyclotron-harmonic frequencies and frequency of the resonant mode of the Fabry-Perot could be due to the accuracy of the axial magnetic field measurements (on the order of 2%) or to slight pulling of the resonator frequency by the electron beam. Between 67 and 86 GHz observations were made of up to the third cyclotron harmonic for every resonant mode of the Fabry-Perot cavity, and between 91 and 105 GHz we have observed up to the second harmonic. At the high values of the axial magnetic field, the lower electron-beam currents could only sustain up to the second harmonic. It should be noted that the estimated quality factor Q of the empty Fabry-Perot is slightly greater than 10^4 over this frequency range but that Q drops at higher frequencies because of increased Ohmic and coupling losses. It is estimated that microwave output power levels of 1–2 W have been observed at the fundamental frequency at 86 GHz although the system was operated at conditions which were far from optimum.

The linear and nonlinear theory[8] for the cyclotron maser instability has recently been formulated for a quasioptical system of very similar geometry to that employed in the present experiment. With use of the linear small-signal efficiency at the nth harmonic it can be shown that the start oscillation conditions for the beam cur- are given by

$$I_s^{(n)} = (2^{n-1} n!\, r_0 B_0)^2 \omega L (1 - 1/\gamma_0) \exp[\xi_0^2(\Delta\omega/\omega)^2/2]\{\xi_0^2(kp_\perp/m_0\Omega_0)^{2(n-1)}[\xi_0^2\beta_\perp^2(\Delta\omega/\omega)/2 - n](2\pi QV)\}^{-1}, \tag{1}$$

$$(\Delta\omega/\omega)_{\min} = [n + (n^2 + \xi^2\beta_{\perp0}^4)^{1/2}]/\xi^2\beta_{\perp0}^2, \tag{2}$$

where $\xi_0 = (r_0\omega/c)/\beta_{z0}$, $\Omega_0 = |e|B_0/m_0c$, $\gamma = (1 + \vec{p}\cdot\vec{p}/m_0^2c^2)^{1/2}$, $\vec{p}$ is the electron momentum vector, ω is the angular frequency, r_0 is the radiation spot size, B_0 is the axial magnetic field, $\beta_{\perp0}$ is the normalized transverse component of the electron velocity, β_{z0} is the normalized parallel component of the electron velocity, V is the electron energy, Q is the Fabry-Perot quality factor $\sim 10^4$, and $\Delta\omega = \omega - n\Omega_0/\gamma_0$ is the frequency mismatch. We have assumed a value of $\Delta\omega/\omega$ which gives a minimum value for the starting current I_s.

By contrast, from the analysis of unstable Bernstein modes and their coupling to Fabry-Perot modes presented in Ref. 6, one can determine the starting current from the requirement for steady state, $\mathrm{Im}\,\omega/\mathrm{Re}\,\omega = 1/2fQ$, where f is a geometric filling factor. This reduces to

$$I_s^{(n)} = 6.6 \times 10^4 (nDB_0/Q)^2 V^{-1/2}(1 + \alpha^2)^{1/2}\alpha^{-2} \text{ A}, \tag{3}$$

where D is the radiation waist radius in centimeters, B_0 is in kilogauss, V is in kilovolts, and f is taken as r_b/D with r_b the electron beam radius.

The start oscillation conditions for the two instabilities for the first four harmonics at 62.4 GHz are shown in Fig. 2, for several different values of the momentum ratio α. The operating values of the beam current observed experimentally when oscillations were sustained in the cavity are also shown for comparison. For reasonable experimentally achievable values of the momentum ratio the low values of starting current favor the Bernstein-mode instability as the gain mechanism in these experiments. The experimentally observed values of the beam current are larger than those predicted for the Bernstein-mode coupling because the experimental values are operating values rather than the minimum values required to start oscillations. In the present setup the beam current cannot be easily varied independently of other beam parameters. Although higher values of $\alpha(\sim 10)$ lower the starting conditions quite significantly, it is extremely unlikely that a momentum ratio of 10 was obtained in these experiments. Orbit calculations for the experimental conditions gave a value of α of 1.6. Furthermore, measurements of the collector current as a function of the kicker magnetic field showed no abrupt cutoff in collector current, suggesting that these experiments were not conducted in a parameter regime close to the mirroring point of the beam electrons, where large values of α might be expected.

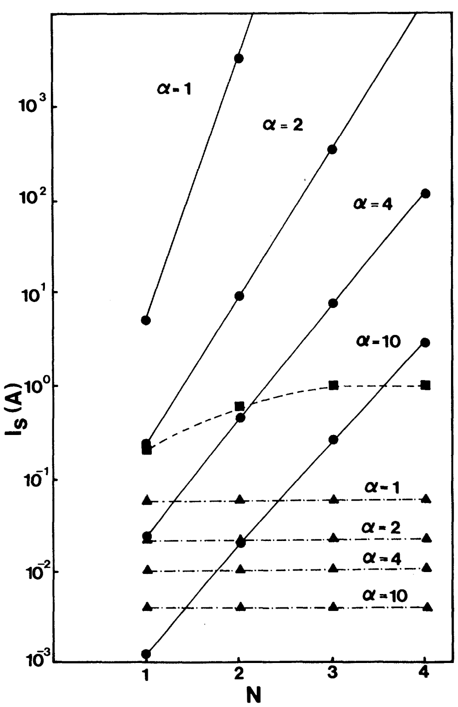

FIG. 2. Start oscillation current vs the harmonic number. Solid curves calculated from Eq. (1). Triangles, calculated from Eq. (3). Squares, experimental measurements of the operating current.

With use of the linear theory of the cyclotron maser instability, to calculate the small-signal efficiency, it can be shown that the output power at the nth harmonic normalized to the output power at the fundamental is given by

$$P_{0n}/P_{01} = 2(n/2)^{2n}(n!)^{-2}\beta_{\perp 0}^{2(n-1)}, \tag{4}$$

where n is the harmonic number and $n \geq 2$. The normalized power according to Eq. (4) for the first four harmonics shows a very rapid decrease with harmonic number. For instance, for $\alpha = 2$, $(\beta_{\perp 0} = 0.243)$, the power output at the second harmonic is 2.8% of the fundamental power, at the third harmonic it is 0.2%, and at the fourth harmonic it is approximately 0.02% of the power at the fundamental. On the other hand, the experimentally observed detector output normalized to the fundamental showed a much slower decrease than that predicted by the cyclotron-maser instability theory. Experimentally observed signal at the second harmonic is 40% of the fundamental signal, at the third harmonic it is 6%, and at the fourth harmonic it is 1%. These comparisons are consistent with the observations that the present experiments have been in a regime where collective effects and mode conversion at the beam boundary determine the electromagnetic gain.

The results reported in this Letter confirm that strong interaction at the higher cyclotron harmonics is possible for very modest values of electron-beam currents. Our present interpretation is that this is due to the much larger instability growth rates at the higher harmonics for the electrostatic modes (Bernstein modes) compared to the case of electromagnetic interaction alone. This previously unexplored mechanism may have practical applications in the development of high-harmonic cyclotron masers operating at modest magnetic fields.

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