sponding one-dimensional Ising model with $1/r^2$ interactions. Taking over the result from the latter model,[3] we predict that as a function of η, $\langle q \rangle$ will at a critical value $\eta_c \sim h/4q_0^2$ change discontinuously from zero to a nonzero value. (This means that for $\eta > \eta_c$ the ground state is twofold degenerate.) Equivalently, we can say that a particle initially localized in, e.g., the left-hand well has a greater than 50% chance of being found in the same well after infinite time.

The most likely system for an experimental test of these ideas is the SQUID,[10] in which the flux through the ring plays the role of the coordinate q. For the case where the two minima differ by nearly a whole flux quantum, it seems likely that tunneling rates will be too small to observe, but by varying the system parameters it may be possible to arrange for the two minima to be separated by a small fraction of a flux quantum, making observation much more likely.

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Axions and Family Symmetry Breaking

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Possible advantages of replacing the Peccei-Quinn U(1) quasisymmetry by a group of genuine flavor symmetries are pointed out. Characteristic neutral Nambu-Goldstone bosons will arise, which might be observed in rare K or μ decays. The formulation of Lagrangians embodying these ideas is discussed schematically.

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In order to understand why the strong interaction does not exhibit large CP-invariance violations, it is desirable to postulate that conservation of axial baryon number[1] $U(1)_A$ is violated spontaneously, except for (soft) instanton effects.[2-4] Phenomenological considerations then require that this symmetry be broken at a very large scale,[5] leading among other things to the emergence of exceedingly light, exceedingly weakly coupled particles ("invisible axions")[6] which are essentially the Nambu-Goldstone bosons associated with the quasisymmetry $U(1)_A$.

Reflecting on this scheme, I think we can identify two unsatisfying features:

(i) Axial baryon number is only one small part of a very large flavor symmetry group that emerges when quark masses are neglected. Why should it be treated on such a special footing?

(ii) The requirement that the theory exhibit a symmetry "except for instanton effects" seems an artificial one. After all, instantons refer to a method of calculation and not to an intrinsic element of the theory. Although one can partially justify the separation of instanton effects on the basis that they are very soft (disappearing rapidly at high momentum scales),[4] it would seem more satisfactory to have the offensive interaction terms banished for real symmetry reasons.

These points reinforce one another, since enlargement of the $U(1)_A$ symmetry might automatically forbid the dangerous terms which previously were banished by appeal to the $U(1)_A$ quasisymmetry.

Actually, point (i) should be viewed in a more general context. A main problem of particle physics is to understand the origin of the quark and lepton mass matrix which in present theories is simply parametrized in terms of many "fundamental" coupling constants. A more economical description could emerge if the apparently chaotic distribution of quark and lepton mass parameters is limited by some symmetry, presumably a spontaneously broken one. In the past fear of massless Nambu-Goldstone bosons derivatively coupled to exotic currents frightened us away from considering such symmetries, or at least tempted us to limit ourselves to discrete symmetries.[7] Discrete symmetries of course do not lead to Nambu-Goldstone bosons, but there are so many possible groups and representations that any particular choice was never very convincing. Our recent experience in making axions invisible has reminded us that Nambu-Goldstone bosons connected with symmetries broken at a very large scale will couple weakly and need not necessarily be feared, thus reopening the possibility of considering continuous flavor symmetries.

(Another option is to gauge some of the flavor symmetries, absorbing the Nambu-Goldstone bosons mentioned above through the Higgs mechanism.[8] This is a not unappealing possibility; however, we will not pursue it further here except to note that some of the flavor symmetries cannot be consistently gauged because of anomalies, and that it is often awkward to insure that all the gauge bosons introduced become massive.)

In this paper the possibility that flavor symmetries are spontaneously broken is discussed in a very general way; I hope to present more specific models in the future.

Familons.—Spontaneous breakdown of family symmetries will lead to characteristic neutral massless Nambu-Goldstone bosons, which I will call familons. Familons will couple to the divergences of currents changing flavor quantum numbers, and therefore can be emitted in flavor-changing decays. Observation of rare decay modes seems to be the most sensitive method for detecting familons.

The couplings of familons at low energies will be described by effective Lagrangians of the type

$$\Delta\mathcal{L} = F^{-1} j_\mu \partial_\mu f, \tag{1}$$

where j_μ is the appropriate current and F is the scale at which the flavor symmetry is spontaneously broken. For phenomenological purposes, the two most interesting possibilities appear to be

$$\Delta\mathcal{L} = F_{\mu e}^{-1} \bar{\mu} \gamma_\rho e \partial_\rho f_{\mu e}, \tag{2}$$

$$\Delta\mathcal{L} = F_{K\pi}^{-1} \bar{K} \overleftrightarrow{\partial}_\rho \pi \partial_\rho f_{K\pi}. \tag{3}$$

These interactions will lead to the decays

$$\mu \to e + f, \tag{4}$$

$$K^+ \to \pi^+ + f \tag{5}$$

with branching ratios

$$\frac{\Gamma(\mu \to e f)}{\Gamma(\mu \to e \nu \bar{\nu})} = \frac{2.5 \times 10^{14}\ \mathrm{GeV}^2}{F_{\mu e}^{\,2}}, \tag{6}$$

$$\frac{\Gamma(K^+ \to \pi^+ f)}{\Gamma(K^+ \to \pi^+ \pi^0)} = \frac{1.3 \times 10^{14}\ \mathrm{GeV}^2}{F_{K\pi}^{\,2}}. \tag{7}$$

These estimates are given some urgency by recent considerations on the cosmological implications of axions.[9] These considerations suggest that the scale for the breakdown of the Peccei-Quinn quasisymmetry, which should be of the same order as $F_{\mu e}$ or $F_{K\pi}$, is likely less than $\sim 10^{12}$ GeV (but greater than 10^9 GeV, to avoid problems with stellar evolution[5]). In this context, it should also be mentioned that continuous flavor symmetries provide a natural method for avoiding the problem of axion domain walls.[10] (There are also other proposals for avoiding domain walls.[11]) If $F \lesssim 10^{12}$ GeV, it may be practical to detect these decays. Clearly, great interest attaches to improved experimental searches for these rare decay modes.

One might worry that truly massless particles, as proposed here, might mediate corrections to the gravitational force or might be copiously radiated by the binary pulsar. Indeed, since we are contemplating $F \ll M_{\mathrm{Pl}}$ the Planck mass, familons in some sense couple more strongly than gravitons. Some of them should couple in part to nonexotic currents, like axial isospin. What saves the situation is that the derivative coupling reduces the effectiveness of familons in mediating coherent interactions. The static force mediated by familons falls as $1/R^4$ and for $F = 10^{10}$ GeV only dominates gravity for $R \lesssim 10^{-5}$ cm. Similarly, although the binary pulsar would emit dipole familon radiation, this radiation is suppressed relative to gravitons by a factor $(m_p R)^{-2} \lesssim 10^{-48}$ (here m_p is the proton mass and R the size of orbit).

The K_L-K_S mixing amplitude involves *two* powers of $1/F$ and is utterly negligible, as (it would seem) are all virtual processes involving familons.

Massive neutrinos could decay, one into another, by familon emission. The effective Lagrangian

$$\Delta\mathcal{L} = F_{\nu'\nu}^{-1}\bar{\nu}'\gamma_\mu\nu\partial_\mu f_{\nu'\nu} \qquad (8)$$

for decay of a massive neutrino ν' into a much lighter one ν and a familon f leads to a lifetime

$$\tau(\nu' \to \nu f) = 3\times 10^9 \left(\frac{F}{10^{10}\ \text{GeV}}\right)^2 \left(\frac{100\ \text{keV}}{m_{\nu'}}\right)^3 \text{sec.} \qquad (9)$$

This process could destabilize heavy neutrinos on cosmological time scales, thus circumventing the mass limit devised by consideration of their present mass density.[12] Unstable massive neutrinos could be very effective in triggering galaxy formation.[13]

Lagrangians.—Several options are available for implementing the idea of spontaneously broken flavor symmetries at the Lagrangian level. I will discuss three possibilities, the last of which appears to me to be the most economical.

(a) *Many Higgs doublets.* The flavor symmetry might be implemented by having many light SU(2)⊗U(1) Higgs doublets coupling to the quarks and leptons. If we are putting the quarks and leptons into multiplets of a flavor symmetry, in general we will need Higgs mesons in rather large multiplets of this symmetry in order to generate an acceptable mass matrix.

Let us suppose that all these particles survive down to low energy, i.e., that they do not acquire large masses when the flavor symmetry breaks. [The opposite case will be considered below, (c).] The asymmetric quark and lepton masses will arise because different Higgs fields acquire different vacuum expectation values.

What has happened to the Nambu-Goldstone bosons? Symmetry breaking at large mass scales can lead to asymmetric scalar self-couplings at small mass scales, by the mechanism shown in Fig. 1. So although the Higgs-boson–fermion couplings are symmetric, the Higgs boson self-couplings are not and there are no Nambu-Goldstone bosons emerging at the SU(2)⊗U(1)-breaking scale. Of course in this type of scheme many charged Higgs bosons with mass of order at most $\sim m_w$ are expected, and we must wonder why they have escaped detection.

(b) *One Higgs doublet.* Another possibility is that only one Higgs SU(2)⊗U(1) doublet gives mass to all the quarks and leptons. Asymmetric couplings could arise by spontaneous symmetry breakdown from initially symmetric ones by the mechanism shown in Fig. 2. Of course, this mechanism invokes ultraheavy leptons *ad hoc*.

Both (a) and (b) represent, in my opinion, interesting supplements to the decoupling theorem, demonstrating how dimensionless couplings among light particles can get shifted as well as renormalized by effects of arbitrarily heavy particles.[13]

(c) *Many called, few chosen.* More economical models of spontaneous flavor symmetry breakdown can be built along the following lines. At high mass scales several Higgs doublets couple symmetrically to the quarks and leptons. When the flavor symmetry breaks, all but one of these doublets acquire large masses. One doublet remains light and becomes the usual doublet of the Weinberg-Salam model. It is the orientation of this light doublet in the full original space of Higgs degrees of freedom which determines the form of the quark and lepton mass matrices. The mass terms are generated by the mechanism shown in Fig. 3.

A vastly oversimplified toy model of this type may be constructed as follows. Consider an SU(2)⊗U(1) theory of the standard type with two families of quarks. With the quark masses (and associated Higgs couplings) turned off, the model has a $U(2)_L \otimes U(2)_\uparrow \otimes U(2)_\downarrow / U(1)_A$ family symmetry. [The U(1) factor is extracted because one symmetry is spoiled by the strong anomaly. The weak anomaly is ignored.] The symmetry operations involve rotating left-handed doublets into

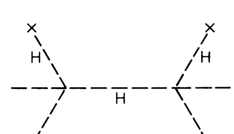

FIG. 1. Couplings of very heavy Higgs bosons shift the quartic self-couplings among light bosons, by this sort of tree graph. Lines with H next to them represent heavy particles or fields acquiring large expectation values.

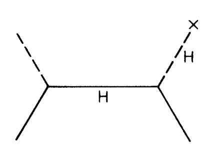

FIG. 2. Couplings of very heavy Higgs bosons and fermions shift the Yukawa couplings among light Higgs bosons and fermions, by this sort of tree graph.

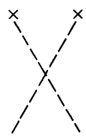

FIG. 3. Large vacuum expectation values for some scalar fields generating large mass terms for other scalar fields.

one another, etc. Let us see how the diagonal SU(2) subgroup of this can be broken spontaneously, at a large scale. To this end consider three Higgs multiplets $\vec{\rho}_\alpha, \vec{\rho}_1, \vec{\rho}_2$, all vectors under the flavor SU(2). $\vec{\rho}_\alpha$ is a gauge $SU(2)\otimes U(1)$ doublet and $\vec{\rho}_1, \vec{\rho}_2$ are singlets. We can suppose that $\vec{\rho}_1$ and $\vec{\rho}_2$ acquire large vacuum expectation values. This will split the flavor components of $\vec{\rho}_\alpha$ into three doublets of different mass. The bare-mass term $\mu^2(\vec{\rho}_\alpha)^2$ can be adjusted so that one doublet is nearly massless, and forms the usual Weinberg-Salam doublet. This particular model has no special virtues, but does, I hope, illustrate the principles of model building. The gauge hierarchy problem is present, neither better nor worse than in the standard model. A more interesting model, with the quarks acquiring unequal masses, can be obtained by adding a flavor-singlet, $SU(2)\otimes U(1)$-doublet field ρ_α and supposing that it is a linear combination of ρ_α and one component of $\vec{\rho}_\alpha$ which forms the Weinberg-Salam doublet.

Note added.—Reiss[15] has also recently considered the possibility that flavor symmetries are broken spontaneously, without, however, discussing rare decays for the connection with axions.

Although as argued above long-range forces mediated by familons are of little practical interest, it is conceivable that similar effects involving axions could be observable. The point is that axions are not really derivatively coupled, because of anomalies. With $F = 10^{12}$ GeV, the Compton wavelength of the axion is ~1 cm. Since axions are P and T odd, their coherent exchange will be considerably suppressed, but might still compete with gravity at distances $\lesssim 1$ cm. This possibility could be checked by an appropriate version of the Cavendish experiment. I am very grateful to J. Preskill for discussions and for showing me his unpublished work on this subject.

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