with an exponential decay of correlation between the electron and the phonons. In the strong-coupling regime the polaron is superlocalized and the extension of the polaron is over one site only. It is this transition that we are observing in our Monte Carlo experiment. Of course finite temperatures will smear out the effects discussed so far because all states become thermally available. Indeed, simulations at more elevated temperatures reveal that the critical fluctuations decrease with increasing temperature. In the preceding discussion we have implicitly assumed that we were dealing with the one-dimensional case. In two and three dimensions we find the same features as for the one-dimensional polaron. The critical value λ_c grows with the dimension (see also Fig. 3). In the vicinity of λ_c the magnitude of $\Delta F_m{}^F$ increases with increasing dimension. A larger λ results in a smaller kinetic energy and the absolute value of the slope at λ_c increases with the dimension. Our observations are in qualitative agreement with the general principle that the critical region becomes smaller as the dimensionality of the system increases. In the strong-coupling limit the small polaron behaves effectively as a zero-dimensional system and the behavior of the system is insensitive to the lattice dimensionality.

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Construction of the Fermi Surface from Positron-Annihilation Measurements

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It is shown how the Fermi surface of metals and intermetallic compounds can be obtained from the two-dimensional angular correlation of positron-annihilation radiation. Results are given for both vanadium and V_3Si. The Fermi surfaces are compared with the results of band-structure calculations.

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Positron annihilation is a method widely used to study electron momentum distributions in solids.[1] Recent progress in two-dimensional position-sensitive γ-ray detectors has brought great improvement in two-dimensional angular correlation of positron-annihilation-radiation (2D ACPAR) measurements.[2-4] With the new machines, the measured distribution is given by

$$N(p_x, p_y) = \int_{-\infty}^{+\infty} \rho^{2\gamma}(\vec{p}) dp_z , \quad (1)$$

where $\rho^{2\gamma}(\vec{p})$ is the momentum distribution of the annihilated electron-positron pairs. These distributions contain information primarily on the electron and positron wave functions and electron-

positron and electron-electron correlations. The different ways to analyze them have been reviewed recently.[5]

In the present Letter, I discuss how the Fermi surface (FS) can be obtained from 2D ACPAR, by using an algorithm for the reconstruction of an object from several projections. In all previously published cases with similar methods,[6-9] the 2D ACPAR distributions were divided into a set of independent slices,

$$N_{p_y}(p_x)=\int_{-\infty}^{+\infty}\rho^{2\gamma}(\vec{p})dp_z, \tag{2}$$

and a two-dimensional (2D) reconstruction was performed for each value of the parameter p_y. I have developed a 3D reconstruction method which uses the complete measured 2D ACPAR distributions.

Another particularity of our method is the application of the Lock-Crisp-West (LCW) theorem[10] before the reconstruction. The LCW theorem considers the superposition $F(k_x,k_y)$ which is constructed from the 2D ACPAR as

$$F(k_x,k_y)=\sum_{H_x,H_y}N(p_x+H_x,p_y+H_y), \tag{3}$$

where H_x and H_y are the projections of the reciprocal-lattice vectors in the (p_x,p_y) plane defined by the crystal orientation in the laboratory frame. The LCW theorem states that the contribution of filled bands to $F(k_x,k_y)$ is a constant (which can be subtracted) and that the contribution of valence electrons is given only by $n_l(\vec{k})$, the occupation number, as

$$F(k_x,k_y)=\int_{BZ}dk_z\sum_l n_l(\vec{k}), \tag{4}$$

where the integration is over the Brillouin zone (BZ). There are two conditions for the LCW theorem to be valid: (1) The positron wave function is a constant, (2) there are neither electron-positron nor electron-electron correlations. I have shown that both conditions are fulfilled, to a good approximation, for niobium[5] as well as for vanadium.[11] Thus, we may consider a set of $F(k_x,k_y)$ distributions to be projections of the FS [described by $n_l(\vec{k})=1$] and consequently we should try to reconstruct the FS from such projections. To my knowledge, this is the first time that pure 3D reconstructions from $F(k_x,k_y)$ distributions have been performed. Moreover, as in Refs. 6–9 the momentum distribution is reconstructed from $N_{p_y}(p_x)$ curves, the domain where the function is nonzero extends over a large number of BZ's (10 to 100 typically) requiring many projections. However, in our case, fewer $F(k_x,k_y)$ are needed, conferring upon the method a real advantage since the determination of each $F(k_x,k_y)$ requires nearly one week of continuous measuring time.

The reconstruction of an object from its projections is a well-known and well-studied problem.[12] It is encountered in many fields, e.g., in electron microscopy, radio astronomy, and tomography. I have used a so-called "filtered back-projection" algorithm,[13] extended to 2D projections, and have taken full advantage of the symmetry of the crystal lattice. This algorithm has been applied to the reconstruction of the Fermi surface of V and V_3Si from [100] and [110] projections.

The result obtained for V is shown in Fig. 1. For comparison, I have also plotted the well-known FS of this metal.[14] This comparison illustrates clearly the possibilities offered by positron annihilation. The jungle gym and the ellipsoidal hole pockets are well observed and it should be possible to increase the agreement further if the projections are corrected for a residual effect of the positron wave function. This effect has already been studied for some $N_{p_y}(p_x)$ curves.[11]

The reconstruction results in a density distribution over the BZ, which, according to the LCW theorem, is a distribution of the number of occupied states. Taking this value from a band-structure calculation, we can define a threshold in the reconstruction yield to differentiate between empty states and occupied ones. This threshold can be used as a Fermi energy in a modified version the program[15] FERMI which is used to draw the final result.

In Fig. 2 I show the reconstruction yield along

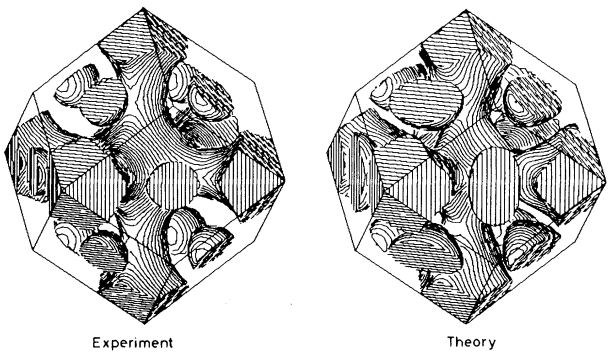

FIG. 1. A hole sheet of the vanadium Fermi surface. Left-hand side: positron-annihilation measurements; right-hand side: band-structure calculation (Ref. 14).

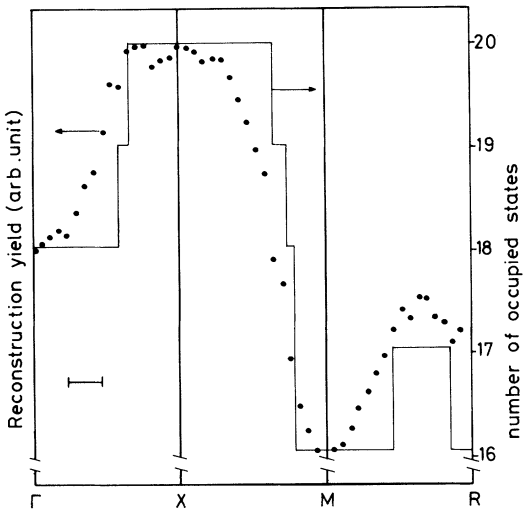

FIG. 2. The reconstruction yield and the calculated number of occupied states for V_3Si along three high-symmetry lines of the Brillouin zone.

three symmetry lines for V_3Si, a high-transition-temperature superconductor ($T_c = 16$ K). This figure shows also the number of occupied states, obtained from the linear muffin-tin orbital (LMTO) calculation by Jarlborg, Manuel, and Peter.[16] The two sets of data are normalized to the same amplitude. The experimental resolution of the measured 2D ACPAR distributions[17] is equal to 0.3 of the ΓX distance. This large value (which is 0.1 for V) is due to the small size of the BZ; in this sense, V_3Si is an extreme case and makes a good test for the reconstruction algorithm. Figure 2 shows that, despite the small number of projections used, the final resolution equals that of the 2D ACPAR distributions themselves. In Fig. 3, I show a sheet of the FS obtained using the procedure described above. The band-structure calculation result[16] is also shown for this sheet. The hole structure along the zone edges is in good agreement with the previous positron-annihilation measurement[18] and with the partial result given by de Haas–van Alphen measurements.[19] The hole pocket observed in the center of the BZ has not been obtained previously but is well reproduced by the LMTO calculation. Complete results on V_3Si will be published elsewhere.[16]

The results presented in Figs. 1 and 3 demonstrate that positron annihilation is now a powerful method for the study of FS topologies. Moreover, it is one of the few methods which works for intermetallic compounds and for alloys. Electron cyclotron resonance methods are efficient when the mean free path of the electrons is greater than the length of a cyclotron orbit (pure metals and dilute alloys), but these methods are drastically limited in the other cases. On the contrary, positron annihilation does not suffer from this limitation and, when it is used in close connection with band-structure calculations, it is a useful tool for the study of the electronic properties of solids.

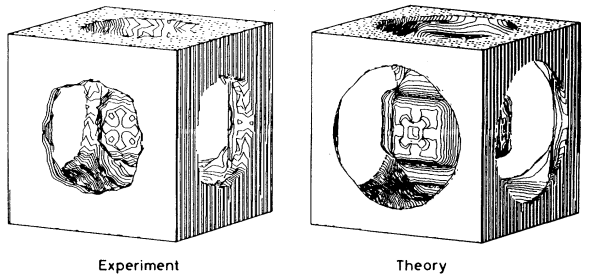

FIG. 3. A hole sheet of the V_3Si Fermi surface. Left-hand side: positron-annihilation measurements; right-hand side: band-structure calculation (Ref. 16).

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Density Dependence of an Electron-Hole–Liquid Correlation Factor in Ge: Experiment

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This paper reports the first absolute measurement of the density dependence of the enhancement factor $g_{eh}(0)$ for the electron-hole liquid in Ge. This factor $g_{eh}(0)$ is a measure of the electron-hole spatial correlation function, and provides a valuable and sensitive test for the predictions of various many-body approximations. A strain-confined system of electron-hole liquid and free excitons is used. The data reported here agree quantitatively with the results of two different many-body–theory approximations.

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A uniquely useful physical system for testing the results of many-body–theory approximation schemes is the electron-hole liquid (EHL). The EHL is a two-component (electrons and holes) Fermi liquid existing in optically excited semiconductors at liquid-helium temperatures. Other Fermi liquids such as neutron stars, nuclear matter, and electrons in metals have various disadvantages when used to test many-body–theory approximations: The experimenter cannot alter parameters; the number of constituent particles is small; many-body effects are small corrections; or the system's characteristics are not known accurately for the purpose of theoretical calculation. We measure the enhancement factor $g_{eh}(0)$ which is the electron-hole (e-h) spatial correlation function evaluated at zero e-h separation and normalized to the average plasma density. This measurement of the correlation function provides a sensitive and valuable test for the predictions of many-body approximation schemes. Several calculations[1-3] for the EHL have yielded varying predictions for the density dependence of $g_{eh}(0)$ while still being in reasonable agreement with experiment for EHL densities and ground-state energies.

In this experiment we determine the enhancement factor of the EHL as a function of EHL density n_l. A free-exciton (FE) gas and at most one EHL droplet are confined to a strain-induced potential well[4,5] at temperature $T = 2.16$ K in an ultrapure Ge crystal. The EHL density is varied by stressing the crystal. The only prior experiment dealing with the density dependence of $g_{eh}(0)$ in stressed Ge is due to Chou and Wong.[6] To "estimate semiquantitatively"[6] the density dependence of a quantity proportional to $g_{eh}(0)$, they assumed a model for EHL decay dependent only on EHL density n_l. Work exists supporting other models.[7] At a stress and density at which they measure an EHL lifetime $\tau_l \approx 0.5$ ms we measure $\tau_l \approx 0.75$ ms. This runs counter to the expectations of their model.

Our method of determining $g_{eh}(0)$ has the advantage of being independent of EHL recombination models. The enhancement factor is related to the probability of an electron being at the site of a hole and thus to the radiative decay rate.