solution of Eq. (1). Thus, n has an upper limit of the order of t_R. This is the reason why the number of the coexisting harmonic solutions is proportional to t_R.

The above consideration is valid for any delay-differential equation of the form (1), provided the corresponding difference equation (3) exhibits period-doubling bifurcations. To confirm this, we made the same numerical simulation for the following four functions $f(x;\mu)$: (i) $\mu x(1-x)$, (ii) $\mu x(1-x^2)$, (iii) $\mu x e^{-x}$, and (iv) $\mu x(1+x^4)^{-1}$. As was expected, structures of parameter domains which are topologically equivalent to Fig. 4 have been found for all of these functions. A remarkable fact is that the maximum degree of the harmonic solutions realized by increasing μ for fixed t_R, $n_{\max}$, is about the same in all cases. For example, we found $n_{\max} = 3-5$ for $t_R = 20$ and $n_{\max} = 7$ for $t_R = 40$ in all cases. This tells us that there may exist some quantitative universality in the parameter dependence of the number of the coexisting higher-harmonic solutions. Further investigations will be reported elsewhere.

The authors are very grateful to Professor F. A. Hopf for sending experimental data prior to publication and to Professor H. Hasegawa for continual encouragement.

[1] H. M. Gibbs, S. L. McCall, and T. N. C. Venkatesan, Phys. Rev. Lett. 19, 1135 (1976), and Opt. Eng. 19, 463 (1980), and references cited therein.

[2] *Optical Bistability*, edited by C. M. Bowden, M. Ciftan, and H. R. Robl (Plenum, New York, 1981).

[3] K. Ikeda, Opt. Commun. 30, 257 (1979); K. Ikeda, H. Daido, and O. Akimoto, Phys. Rev. Lett. 45, 709 (1980).

[4] K. Ikeda and O. Akimoto, Phys. Rev. Lett. 48, 617 (1982).

[5] H. M. Gibbs, F. A. Hopf, D. L. Kaplan, and R. L. Shoemaker, Phys. Rev. Lett. 46, 474 (1982).

[6] J. S. Griffith, *Mathematical Neurobiology* (Academic, London, 1971).

[7] R. May, in Proceedings of the Les Houches Summer School on Chaotic Behavior of Deterministic Systems, July 1981 (to be published).

[8] F. A. Hopf, D. L. Kaplan, H. M. Gibbs, and R. L. Shoemaker, Phys. Rev. A 25, 2172 (1982).

[9] In experiments, lockings to the ninth to thirteenth harmonics have been observed even by scanning μ for fixed t_R. The reason for this disagreement with the numerical results is not clear.

Bell's Theorem as a Nonlocality Property of Quantum Theory

Henry P. Stapp
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 3 May 1982)

Bell's theorem is formulated as a nonlocality property of quantum theory itself, with no explicit or implicit reference to determinism or hidden variables. A recent Letter on this subject is discussed.

PACS numbers: 03.65.Bz, 02.50.+s

In a recent Letter[1] related to Bell's theorem[2] Fine proved several propositions, and asserted the following conclusion: "Proposition (2) shows that, despite appearances, no significant generality is achieved by those derivations of the Bell/CH inequalities that dispense with explicit reference to hidden variables and/or determinism: The assumptions of such derivations imply the existence of deterministic hidden variables for any experiment to which they apply."

This conclusion consists of two assertions, which must be distinguished. The second is meant to be a rephrasing of proposition (2), and, as such, is technically correct. However, it is misleading because of two semantic irregularities: (1) Fine leaves the word "local" out of his name "deterministic hidden-variables models." Usually this word is inserted to remind the reader that the models in question have an important factorization property that normally arises from the idea that the deterministic hidden variables are separated into two local parts, each of which determines those results of the experiment that occur in one of two separated regions. (2) Fine leaves the word "model" out of the rephrasing. This creates the impression that what was proved was the actual existence of deterministic hidden variables, rather than the existence of a certain kind of factorizable model.

The first part of Fine's conclusion is incorrect. Fine's proposition (2) limits in no way the generality achieved in the cited works. The aim of those works was to identify a locality property that does not depend on deterministic hidden variables but is incompatible with the statistical predictions of quantum theory, and thereby to extend Bell's result, that no local deterministic hidden-variables theory can agree with quantum theory, to the much larger class of all local theories. In this context Fine's proposition (2) entails, as he correctly asserts in the second part of his conclusion, that the statistical predictions of any local theory can be reproduced or simulated by the predictions of some local deterministic hidden-variables model. This nontrivial result permits the general conflict between quantum theory and local theories to be proved by reduction to the special case of local deterministic hidden-variables theories considered already by Bell. However, a nontrivial reduction of a general theorem to a special case does not render that generalization insignificant. Moreover, in this case the results of Fine and of Bell taken together are not sufficient to obtain the general result. The crucial input is precisely the identification of the locality property that does not depend on deterministic hidden variables but is incompatible with the statistical predictions of quantum theory. Nothing in the works of Fine or Bell identifies this locality property.

To make this point absolutely clear a concrete example of a generalization of the kind under discussion is needed. Rather than restating an existing work, I use the opportunity to present a modified, and intrinsically interesting, version of a previous theorem[3] that makes weaker assumptions and shows quantum theory itself to be nonlocal in a physically reasonable sense that is formulated with no explicit or implicit reference to determinism or hidden variables.

The point of departure is Bell's theorem, which says that any theory compatible with the statistical predictions of quantum theory is nonlocal, provided the theory is a deterministic hidden-variables theory. The aim of the generalization is to remove this proviso.

The experiment used to demonstrate the result is well known.[2,3] I add one extra feature. The particles entering the original scattering experiment are monitored by fast electronics that allow the individual pairs to be identified. Those scattered pairs i that pass through two polar escape holes in a spherical array of counters are numbered $i=(1,\ldots,n)$. The fast electronics and known geometry allow the individual arrival times t_i at two Stern-Gerlach devices A and B to be placed in separate and known time windows.

The *result* of the experiment is specified by

$$r=(r_A;r_B)=(r_{A1},\ldots,r_{An};r_{B1},\ldots,r_{Bn}), \qquad (1)$$

where each r_{Ai} and r_{Bi} takes a value of either $+1$ or -1, corresponding to a deflection along the direction D_A or D_B, or against this direction, respectively.

There are two alternative possible settings D_A' and D_A'' of the direction D_A, and two alternative possible settings D_B' and D_B'' of D_B. The experiments are set up so that both the choice between D_A' and D_A'' and the subsequent deflections and recordings of the results r_{iA} $[i=(1,2,\ldots,n)]$ will occur in a space-time region R_A, and similarly for B, where R_A and R_B are spacelike separated.

The four alternative possible experiments are labeled by the four values of (D_A,D_B). For each alternative value of (D_A,D_B) there are $(2^n)^2$ conceivable results r. To each conceivable result r of each of the four alternative possibilities (D_A, D_B) quantum theory assigns a probability P.

Consider the set S consisting of all conceivable combinations of the conceivable results of all four alternative possible experiments. The different elements of S correspond to the different possible functions

$$r(D_A,D_B)=(r_{A1}(D_A,D_B),\ldots,r_{An}(D_A,D_B);r_{B1}(D_A,D_B),\ldots,r_{Bn}(D_A,D_B)), \qquad (2)$$

where the possible values of each function $r_{Ai}(D_A, D_B)$ and $r_{Bi}(D_A,D_B)$ are $+1$ and -1.

A general theory T that makes statistical predictions for all four possible experiments of the kind under consideration here will be said to entail a *nonlocal connection* (or be *nonlocal*) if, as n tends to infinity, there is no conceivable combination of conceivable results of the four alternative possible measurements that is compatible with both the statistical predictions of T and the locality conditions that the results in each region be independent of the choice made in the other:

$$r_{Ai}(D_A,D_B)=r_{Ai}(D_A), \qquad r_{Bi}(D_A,D_B)=r_{Bi}(D_B). \qquad (3)$$

Quantum theory predicts that, whichever of the four experiments (D_A, D_B) is performed, the correlation parameter

$$c[r(D_A,D_B)] = \frac{1}{n}\sum_{i=1}^{n} r_{iA}(D_A,D_B) r_{iB}(D_A,D_B) \tag{4a}$$

will, as n tends to infinity, come to satisfy

$$|c[r(D_A,D_B)] - \bar{c}(D_A,D_B)| < 0.03, \tag{4b}$$

where $\bar{c}(D_A,D_B)$ is a number specified by quantum theory. But Bell's arithmetic shows[3] that there is no conceivable combination of conceivable results that satisfies both (3) and (4). Thus any theory T that gives the prediction (4) is nonlocal. One such theory is quantum theory itself.

What do Fine's arguments and results show? As delicate issues are involved it is best to state things precisely. Consider a couple (E, T) consisting of an experiment E, and a theory T that makes predictions about E. Each experiment E consists of a set of four alternative possibilities of the kind being discussed.

Some theories predict probabilities and some predict individual results. Let $P(E,T)$ represent the probabilities predicted for E by T, if such predictions are made. Let (F) represent the conditions imposed on $P(E,T)$ by the requirements on Fine's class of deterministic hidden-variables models. Let $R(E)$ represent a conceivable combination of conceivable results of E.

Two classes of couples (E,T) may now be defined:

$$C_{\rm FD} \equiv \{(E,T);\ P(E,T) \text{ is defined and satisfies } (F)\}, \tag{5}$$

$$C_{\rm NL} \equiv \{(E,T);\ P(E,T) \text{ is defined, and no conceivable } R(E) \text{ is compatible with both } P(E,T) \text{ and } (3)\}. \tag{6}$$

The subscripts FD and NL stand for factorized deterministic (as defined by Fine's equations) and nonlocal (as defined by the present work). Two semicomplementary classes $C_{\rm NFD}$ and $C_{\rm NNL} \equiv C_{\rm LOC}$ are defined by changing "satisfies" to "does not satisfy" and "no" to "some," respectively.

Two conceivable definitions of nonlocal theories are identified by the following two classes of theories:

$$\tau_{\rm NFD} \equiv \{T;\ \text{for some } E,\ (E,T) \in C_{\rm NFD}\}, \tag{7}$$

$$\tau_{\rm NL} \equiv \{T;\ \text{for some } E,\ (E,T) \in C_{\rm NL}\}. \tag{8}$$

The final class is the one defined in this work. The other possibility uses the equations of Fine.

Fine's argument claims that $C_{\rm LOC}$ is contained in $C_{\rm FD}$: $C_{\rm LOC} \subset C_{\rm FD}$. This result is true: It follows immediately from the fact that if a set of conceivable results $R(E,T)$ satisfies the independence property (3), then the probabilities generated by those results will satisfy the crucial factorization property imposed by Fine's equations.[4] {This is the property that each of the four four-valued functions $[AB(\lambda), AB'(\lambda), A'B(\lambda)$, and $A'B'(\lambda)]$ normally required to model such an experiment be factorized into a product of two two-valued functions: $AB(\lambda) = A(\lambda)B(\lambda)$, $AB'(\lambda) = A(\lambda) \times B'(\lambda)$, $A'B(\lambda) = A'(\lambda)B(\lambda)$, and $A'B'(\lambda) = A'(\lambda) \times B'(\lambda)$.} It is easy to prove also that $C_{\rm FD} \subset C_{\rm LOC}$, and thus derive $C_{\rm LOC} = C_{\rm FD}$, and hence conclude that $\tau_{\rm NL} = \tau_{\rm NFD}$. Thus the definition of nonlocal theories introduced in this work is equivalent to a similar one that could be defined by using Fine's equations.

The equivalence of these two alternative possible definitions of nonlocality, which is the essential basis of Fine's claim, has no effect on the generality achieved by definition (8). Simply defining a theory T to be nonlocal if it belongs to class $\tau_{\rm NFD}$ would not permit any claim of having derived a nonlocality property of, say, quantum theory, with no explicit or implicit reference to determinism or hidden variables. This definition depends on the concept of deterministic hidden variables. What is needed is a conception of nonlocality that makes no explicit or implicit reference to determinism or hidden variables, and which leads, via the conflict between (3) and (4) discovered by Bell, to a conflict between locality and any theory that gives the quantum predictions (4). Such a concept of nonlocality is embodied in definition (8).

The fact that this conflict between (3) and (4) can also be formulated, as it originally was, by using deterministic hidden variables has no bearing on the fact that is essential for the kind of generalization being sought, namely that it is not *necessary* to invoke determinism or hidden variables in order to exploit the conflict between (3) and (4).

The essential point is that there are no actual mathematical conditions on the equations of Bell from which the contradiction with quantum theory arises that demand that the functions $r_{Ai}(D_A,D_B)$ and $r_{Bi}(D_A,D_B)$ in his proof represent the results of the alternative possible experiments *determined beforehand* by some *invisible* variables. Thus, from a mathematical point of view, the content of his result is not well represented by the words "deterministic hidden variable": these words are present, but there are no corresponding mathematical conditions of "beforehandedness" and "invisibility." The aim of the generalization is to exploit this fact, and show how to use Bell's mathematics without getting embroiled with these irrelevant concepts of determinism and hidden variables.

The formulation of nonlocality used here avoids having to introduce the concept that all four alternative possible results of the experiment be determined beforehand by hidden variables. This concept assigns definite results to experiments that "could have been performed but were not." The need to use this contrafactual concept severely limits the scope of the theorem, in the form originally put forth by Bell. The present formulation asserts that a theory entails a nonlocal connection if it makes statistical predictions, and these predictions, by themselves, entail (in some cases) that there is no way within the set of all conceivable combinations of conceivable results for the results in each region to be independent of the choice made in the other region. Quantum theory has such a nonlocal connection: That is what Bell actually discovered. Tying this discovery to the mathematically irrelevant concepts of beforehandedness and invisibility obscurs its logical essence, and needlessly curtails its significance. The functions $r_{Ai}(D_A,D_B)$ and $r_{Bi}(D_A,D_B)$ can more rationally be viewed as defining the set of all conceivable combinations of conceivable results.

The nonlocality property of quantum theory discussed here does not conflict with the microcausality property of quantum theory, which prevents faster-than-light communication by means of quantum observables.

As stressed in Ref. 3 the nonlocality property of quantum theory does not necessarily entail nonlocal influences: There appear to be two alternatives. The first is a superdeterminism, in which the choice of the experimenter is not effectively free: Some tight connection from their common past binds the results in one region to the choice of experiment in the other. The second alternative, exemplified by the many-worlds (or many-minds) interpretation of quantum theory, exploits the fact that *experienced* worlds in which the results in both regions are definite are confined to the intersection of the forward light cones from the two regions. A third alternative is that the manifestly nonlocal character of von Neumann's process 1, unlike that of its counterpart in classical statistical mechanics, reflects the existence of subtle nonlocal influences that are not evident at the level of probabilities and averages normally dealt with by pragmatic quantum theory and classical mechanics.

Note added.—Fine's Comment[5] contains a defense of his statement quoted at the beginning of this paper. He certifies that the first part of that statement means nothing more than the second. He also admits that the second part contains semantic irregularities. The reader is thus well advised to examine carefully what was actually proved in Fine's paper, rather than trust impressions created by this summary sentence.

Fine defends his position regarding locality by claiming the existence of physically local models that violate his condition (2). The violations of (2) in the models he cites arise from faulty experimental design: The particles detected in coincidence are not the ones produced in coincidence. Consequently the experiments do not measure the quantities that are the basis of Bell's theorem. They measure, instead, certain nonlocal observables that depend critically on the relative times of the detections of particles in the two spacelike-separated regions. The factorization property (2) cannot be expected to hold for these nonlocal observables. In properly designed experiments, such as the one described herein, the particles produced in coincidence are correctly tagged by signals originating from the region where these particles are produced, and there is, consequently, no dependence on the nonlocal time-difference variables.

To prove nonlocality it is sufficient to consider experiments that correctly measure the theoretical quantities dealt with by Bell's theorem. The experiments considered by Fine are therefore irrelevant, and Fine's discussion of the obviously false converse is likewise irrelevant. Thus no rational justification of his position on "locality" has been provided.

I thank Philippe Eberhard for many very useful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U. S. Department of Energy under Contract No. DE-AC03-76SF00098.

[1]Aurthur Fine, Phys. Rev. Lett. 48, 291 (1982).

[2]J. S. Bell, Physics (N.Y.) 1, 195 (1964).

[3]H. P. Stapp, Phys. Rev. D 3, 1303 (1971).

[4]Here n is assumed to be large enough so that $1/n$ is negligible.

[5]A. Fine, this issue [Phys. Rev. Lett. 49, 1536(C) (1982)].

Constraints of Determinism and of Bell's Inequalities Are Not Equivalent

Philippe H. Eberhard
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 6 May 1982)

Arguments are given against considering Bell's inequalities to be equivalent with determinism. Possible misinterpretations of the conflict between quantum mechanics and these inequalities are pointed out. With use of results obtained in previous papers on this subject, it is shown that locality rather than determinism is the issue.

PACS numbers: 03.65.Bz

In a recent Letter,[1] the relationship between hidden variables, joint probability, and the Bell inequalities[2] was discussed. It was claimed that an equivalence exists between the requirement that the inequalities hold and the existence of a deterministic hidden-variables model. It was concluded that the inequalities impose requirements to make well defined quantities whose rejection is the very essence of quantum mechanics. It is the intent of this paper to show that such claims are likely to be misleading.

The context is a well-known quantum correlation experiment.[3] There are two well-separated regions of space, R_A and R_B. In R_A (R_B) two noncommuting observables $\underline{A}_0$ and $\underline{A}_1$ ($\underline{B}_0$ and $\underline{B}_1$) are defined.[4] It is possible to measure simultaneously any of the four combinations of two commuting observables, $\underline{A}_0$ and $\underline{B}_0$, $\underline{A}_0$ and $\underline{B}_1$, $\underline{A}_1$ and $\underline{B}_0$, or $\underline{A}_1$ and $\underline{B}_1$, corresponding to probability distributions $P_{00}(A_0B_0)$, $P_{01}(A_0B_1)$, $P_{10}(A_1B_0)$, or $P_{11}(A_1B_1)$, respectively. These distributions are functions of the values A_0, A_1, B_0, and B_1 which $\underline{A}_0$, $\underline{A}_1$, $\underline{B}_0$, and $\underline{B}_1$ can take. Central to the discussion is whether or not at least one positive-definite function $p(A_0A_1B_0B_1)$ exists with the following properties:

$$P_{JK}(A_JB_K) = \sum_{\substack{A_{1-J}\\ B_{1-K}}} p(A_0A_1B_0B_1)\,. \qquad (1)$$

In Ref. 1, whenever such a function $p(A_0A_1B_0B_1)$ exists, it is interpreted as a joint probability for the four observables $\underline{A}_0$, $\underline{A}_1$, $\underline{B}_0$, and $\underline{B}_1$. Then it is correctly demonstrated that the existence of one or several such functions $p(A_0A_1B_0B_1)$ implies that the functions $P_{JK}(A_JB_K)$ satisfy Bell's inequalities and vice versa, if each of the observables A_J and B_K is two-valued.[5] However, other statements were also made.

Statement (a).—"The existence of a deterministic hidden-variables model is strictly equivalent to the existence of a joint probability distribution $p(A_0A_1B_0B_1)$."

Statement (a) is correct only if an *unusually* restrictive meaning is given to the word "deterministic."[6] In general, determinism means that the evolution of a system is determined by its initial state and by its environment. Then, the outcome of any experiment depends only on some variables which specify the state of the system, on the interactions with other systems, and on all the apparatus that are connected to make measurements. In this general sense, any probability distribution can be reproduced by a deterministic hidden-variables model,[7] whether or not Bell's inequalities hold. Any computerized Monte Carlo simulation is a deterministic hidden-variables model for the theory it simulates and there is no limit to the kind of quantum mechanical probability distribution the Monte Carlo technique can reproduce. The Monte Carlo generation of the results depends on the preparation of the system, on its law of evolution, and on the entire measuring apparatus. The generation of $\underline{A}_0$ in R_A when $\underline{B}_0$ is measured in R_B may have to be different from the generation of $\underline{A}_0$ when $\underline{B}_1$ is measured in R_B. However, if the algorithm that generates the data is allowed to have enough mathe-