Rev. Lett. 42, 1698 (1979).

[3]S. A. Brazovskii, Zh. Eksp. Teor. Fiz. 78, 677 (1980) [Sov. Phys. JETP 51, 342 (1980)].

[4]H. Takayama, Y. R. Lin-Liu, and K. Maki, Phys. Rev. B 21, 2388 (1980).

[5]D. Wohrle, Adv. Polymer Sci. 10, 35 (1972).

[6]The situation is similar to that encountered by W. P. Su and J. R. Schrieffer, Phys. Rev. Lett. 46, 738 (1981), and J. T. Gammel and J. A. Krumhansl, to be published, in their respective studies of the soliton excitations of an $M$-fold commensurate Peierls insulator. There, $\sigma = (M-2)/M$ is a rational fraction.

# Gravitation, Phase Transitions, and the Big Bang

Lawrence M. Krauss [(a)]

*Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

(Received 7 June 1982)

Introduced here is a model of the early universe based on the possibility of a first-order phase transition involving gravity, and arrived at by a consideration of instabilities in the semiclassical theory. The evolution of the system is very different from the standard Friedmann-Robertson-Walker big-bang scenario, indicating the potential importance of semiclassical finite-temperature gravitational effects. Baryosynthesis and monopole production in this scenario are also outlined.

PACS numbers: 98.80.Bp, 04.60.+n, 11.30.Qc

The rapproachement between particle physics and cosmology cannot be complete until quantum gravity is fully understood, when it will be possible to trace quantitatively the big bang to times $\sim t_{\mathrm{Planck}}$ ($= 5.4\times10^{-44}$ s). Developments in particle theory, however, have motivated a consideration of periods shortly thereafter. Not only might one explain such fundamental quantities as the observed baryon-to-photon ratio,[1] but the early universe may have undergone phase transitions during which its dynamics may have differed greatly from that of the adiabatic Robertson-Walker model.[2] Thus the early universe can serve as a laboratory in which to test our models of particle interactions at high energies. In particular, the resolution of various problems of cosmology may be tied to understanding the peculiarities of gravity as a field theory.

The model I present, based on treatment of classical gravity as a remnant of a phase transition, is somewhat speculative and preliminary, but illustrates several important aspects of such an approach: (1) The attempt to couple quantum mechanics and general relativity is strongly tied to thermodynamics. Resulting effects will be important in the early universe, and need further investigation. (2) Quantum, or semiclassical, gravitational effects may be relevant at temperatures below the Planck temperature.

Specifically this model indicates that after such a transition the temperature of space may have always been lower than the critical temperature for restoration of grand unified gauge symmetries. At the same time it may be possible to generate the observed baryon excess while suppressing monopole production. I here briefly outline these results, leaving more detailed discussions to a future paper.

Although they present some problems, first-order transitions may play a crucial role in early-universe dynamics, perhaps resolving several paradoxes of the standard Friedman-Robertson-Walker adiabatic model. Indeed, given the possibility that baryon number may not be conserved, all the observed matter and entropy of the present universe may have been generated in such a transition.[2] Thus the big-bang explosion itself may have been the result of a first-order phase transition. In an earlier article[3] I suggested that it may be feasible to connect such a possibility to the nature of classical gravity. The gravitational Lagrangian with its dimensional coupling $K = (16\pi G)^{-1} \sim O(m_{\mathrm{Planck}}^{-2})$ has the form of a nonrenormalizable low-energy effective interaction in an expansion in inverse powers of a large mass scale at which some heavy degree of freedom is frozen out. In this sense it resembles the Fermi weak effective Lagrangian.

Also, Weinberg demonstrated on general grounds that any such effective interaction, in order to have detectable macroscopic effects at large distances, might reasonably have long-range dynamics governed by a Lagrangian like that of gravity.[4]

Whether it is possible to deduce explicitly from an effective theory the existence of a transition and the nature of a fundamental high-energy theory is not clear, although renormalization-group techniques may offer some possibilities.[5] A more intuitive approach involves investigating the classical theory for instabilities which may signal the onset of a transition and may characterize the relevant physics of the transition region. This is the approach of the present work. I thus produce an *Ansatz* for the physics of a state immediately following a transition to a vacuum effectively describable by a semi-classical coupling of gravity to quantized matter fields. It is then possible to evolve this state with use of the equations of general relativity, in order to investigate alternative early-universe behavior and relevant semiclassical gravitational effects therein.

Classical gravity is beset by instabilities. Even Newtonian gravity involves the Jeans instability.[6] In general relativity instabilities lead to gravitational collapse and the formation of singularities in space-time,[7] which are particularly relevant for studies of the early universe as they indicate points where the predictive power of the classical theory breaks down. If such naked singularities are cloaked behind an event horizon[8] this results in the formation of black holes (BH's). Since such singularities imply the incompleteness of the classical theory, the formation of associated BH's may be important in the region of a gravitational phase transition. Indeed, if classical gravity is self-consistently coupled to quantized matter fields, BH's exhibit thermodynamic behavior relevant to the description of a transition. Associated with their finite event horizon, BH's have finite entropy[9]:

$$S = kc^3(4G\hbar)^{-1}A_H = 4\pi k GM^2/\hbar c\,, \tag{1}$$

where $A_H$ is the area of the event horizon. Thus BH's radiate at a temperature

$$T_H^{-1} = \partial S/\partial E_H = (\hbar c^3/8\pi k GM)^{-1} \tag{2}$$

and thus have negative specific heat.

It can easily be shown[10] that this implies that BH's can exist in equilibrium with radiation in a box with fixed total energy if

$$T_H = T_{\text{space}}; \quad \partial T_{\text{space}}/\partial E_{\text{space}} > \partial T_{\text{hole}}/\partial E_{\text{hole}}.$$

For radiation, $T_{\text{space}} \propto E_{\text{space}}^{1/4}$ and with use of (2) this then implies

$$4E_{\text{space}} \leq E_{\text{hole}}. \tag{3}$$

Thus if one raises the energy density in a fixed volume the equilibrium state will eventually be that of a black hole and radiation at a temperature which is less than the equilibrium temperature of pure radiation with the same energy density. [For a similar result, using a fixed-temperature ensemble, see Ref. 6.]

This suggests that BH configurations should become more important in the early universe, where energy density and temperature are increased. Moreover, a "black-hole gas" would have nonstandard thermodynamic properties reminiscent of a system near a first-order transition, being dominated by the negative specific heat of the BH's. [Note: The possibility of an abundance of primordial BH's has been considered elsewhere for other reasons.[11]]

Let us next consider how such a state may arise out of a first-order transition. On the basis of semiclassical calculations in model field theories,[12] such a transition occurs locally at random sites via the nucleation of "bubbles" of fixed size and energy density which then evolve classically until the phase transition to a new equilibrium state is completed via percolation. If the transition is to a state described by semiclassical gravity coupled to quantized matter, and if the bubble size and energy density are within the proper range, then the state which is tunneled to inside the bubbles will involve a BH surrounded by radiation.

I will assume here that such a situation describes to some approximation the universe shortly after a transition. After it is completed we are left with a remnant "gas" of BH's with a mean mass and volume per hole (with which each hole is in thermal contact). While a fundamental theory is needed to calculate the parameters of such a transition, we can take this *Ansatz* as an initial-state condition and investigate its consistency and the consequences of its evolution in time.

The pretunneling state may have had an arbitrarily long time to relax into a metastable equilibrium (since we are only measuring expansion time from the point when the phase transition is completed). Then, if the nucleation rate is sufficiently fast, we may imagine that on a scale large compared to the volume per hole the universe is sufficiently isotropic and homogeneous to describe its evolution by the Einstein equations for the Robertson-Walker metric scale factor $R(t)$:

$$(\dot{R}/R)^2 + Kc^2/R^2 = 8\pi G\rho/3\,, \tag{4}$$

$$d(\rho c^2 R^3)/dt = -p\, dR^3/dt\,, \tag{5}$$

where $\rho c^2$ is the total energy density, $p$ is the pressure, and Eq. (5) represents the statement of energy conservation. To solve these we must supplement them by an equation of state $p(\rho, T)$ for a BH-radiation mixture. If we assume the standard equation of state for radiation ($p = \rho/3$), and that BH's act like massive dust particles ($p = 0$), (5) implies

$$\dot{\rho}_{\rm tot}/\rho_{\rm tot} = -(3 + \rho_{\rm space}/\rho_{\rm tot})\dot{R}/R\,, \tag{6}$$

where I will henceforward refer to the quantity in parentheses as $K(t)$, which smoothly goes from the matter value $K(t) = 3$ to that of radiation, $K(t) = 4$, as the BH's decay.

One can also show that the universe expansion, combined with Eq. (2), implies that BH's lose mass at a rate

$$\dot{M}/M = -3.8\times10^{15} N/M^3 + 2.0\times10^{-45} M\rho_{\rm space} \tag{7}$$

(in mks units), where $N$ is the standard helicity factor dependent on the number of massless fermionic and bosonic degrees of freedom; $N = \frac{1}{2} \times (N_B + \frac{7}{8} N_F)$.

Then, using Eqs. (4) (choosing $K = 0$) and (6), we have

$$\dot{\rho}_{\rm tot} = -K(t) c^{1/2} \rho_{\rm tot}^{3/2} \quad (c^{1/2} = 2.3\times10^{-5})\,. \tag{8}$$

Finally, the time behavior of $\rho_{\rm BH}$ and $\rho_{\rm space}$ (when we use the fact that $\rho_{\rm space} = \rho_{\rm tot} - \rho_{\rm BH}$) is given by

$$\dot{\rho}_{\rm BH}/\rho_{\rm BH} = \dot{M}/M - 3\dot{R}/R\,, \tag{9}$$

$$\dot{\rho}_{\rm space}/\rho_{\rm space} = -4c^{1/2}\rho_{\rm tot}^{1/2} - \rho_{\rm BH}/\rho_{\rm space}(\dot{M}/M)\,. \tag{10}$$

Equations (7)–(10) allow one in principle to evolve an initial state with BH's of mean mass $M_0$ and mass density $\rho_{\rm BH}{}^0$. In practice, they must be solved numerically and I shall describe the quantitative results in a future paper. However, the general qualitative features are easily described. Depending on the initial parameters there may be an adiabatic period where $T_{\rm space} = T_{\rm BH}$ and both are increasing. However, it is easy to show that once $4\rho_{\rm space} \geq \rho_{\rm BH}$ black holes must go out of equilibrium. (In fact they will often go out of equilibrium before this, depending on the relative magnitudes of $\dot{M}$ and $\dot{R}$.) After this point the BH's, at a mass $M_C$, increase in temperature and evaporate on a time scale of order[13] $\tau \approx 10^{-18} M_C{}^3$ sec, while the temperature of space reaches a maximum and then decreases.

The initial values $M_0$ and $\rho_{\rm BH}{}^0$ are constrained by a variety of requirements. First, for a given $M^0$, $\rho_{\rm BH}{}^0$ must be less than the value given by dense packing of BH's, $\rho_{\rm crit}$ (in practice $\rho_{\rm BH}{}^0 \ll \rho_{\rm crit}$ in order for our approximations to be valid), and greater than a minimum value below which BH states would no longer be favored in the initial tunneling bubble formation. This dual requirement then can be shown to imply $M_0 \gtrsim (10\text{–}100) \times M_{\rm planck}$ ($kT_0{}^{\rm BH} \lesssim 10^{17}$ GeV).

There are also limits on primordial BH density for $M_0 \geq 10^9$ kg in order not to affect big-bang nucleosynthesis,[14] and so we will take our initial mass constraint as $10^{-6}$ kg $< M_0 < 10^9$ kg.

This range can be restricted further by considering baryon and monopole production by black holes. It has been shown that unless $CP$ is not microscopically conserved, black holes may produce a net baryon number only via superheavy $X$-boson production.[15] The advantage of such production in the present scenario is that if $kT_s < kT_{\rm BH} \lesssim 10^{14}$ GeV, all $X$ bosons produced subsequently by black-hole evaporation will be out of equilibrium (inverse decays are suppressed) and will decay producing net baryon number. Hence mass limits on the $X$ boson needed in the standard model in order to get departure from thermal equilibrium are unnecessary. Noting that $X$ bosons will only be radiated after $kT_{\rm BH} > M_X$ ($M_{\rm hole} \geq M_i$) one can estimate the number of such particles produced per black hole:

$$N_X \approx M_i/1.5k\langle T\rangle N^* = 5\times10^8/N^*\,, \tag{11}$$

where $\langle T\rangle$ is the average temperature at which the BH radiates after reaching mass $M_i$, and $N^*$ is the number of species of particles being radiated.

If the expansion of the universe is adiabatic after the $X$-particle decay products thermalize then

$$(n_B/n_\gamma)_{\rm present} = 7\epsilon(N_X \rho_{\rm BH}{}^0/M_0)[S_0 + \{M_X N^* N_X \rho_{\rm BH}{}^i/3kT_i M_i\}]^{-1}\,,$$

where $\rho_{BH}{}^0$ and $M_0$, and $S_0$ are the initial values of BH mass density and mass and the total entropy density, respectively, $T_i$ and $\rho_{BH}{}^i$ are the values of the temperature of space and BH mass density at the time the $X$ bosons are emitted ($M_{BH} = M_i$), and $\epsilon$ is the net baryon number per $X$-$\bar{X}$ pair decay. Note that the term in curly brackets arises because black-hole radiation is out of equilibrium and its subsequent decay can produce significant entropy. (This term has been neglected by other authors[14] but it need not be small.) Because of this term $n_B/n_\gamma$ can be at most equivalent to that of the standard scenario. Hence, here, in order to agree with the observed $n_B/n_\gamma$, we can avoid problems with tuning the $X$ boson, but the potential entropy production by $X$-boson decays constrains the size of the temperature difference between the BH's and radiation at the time $X$ bosons are radiated, which constrains $M_0 < 10^{-2}$ kg.

On the other hand, monopole production via phase transitions provides a severe constraint on grand unified theories.[15] In this scenario monopole production may occur via two different mechanisms. They may be produced in the initial transition, or produced via subsequent black-hole evaporation. The latter effect may be exponentially suppressed by semiclassical effects.[16] Similarly, production in the initial transition can be washed out by entropy generation via black-hole evaporation. Numerical estimates of both these effects will be given in a future paper.

Thus the early universe may have been much cooler than naive extrapolations would imply. In this scheme, during the period in which big-bang expansion dynamics apply, space need never have exceeded the critical temperature for the restoration of grand unified symmetry. If so (modulo various numerical computations now underway), it seems possible in principle to allow baryosynthesis, while suppressing monopole production. Also being considered are such questions as the possibility of producing remnant inhomogeneities on the scale of galaxies; refinements to include an initial mass distribution of BH's; and a discussion of the horizon, flatness, and cosmological-constant problems in the context of the present model. While such investigations, in the absence of a fundamental theory, provide only circumstantial evidence for the existence of a gravitational transition, they illustrate the possibility that finite-temperature gravitational effects may significantly alter our models of the early universe, as well as our understanding of quantum gravity.

I would like to thank Roscoe Giles for immeasurable aid, A. Guth for insightful discussions, and S. Glashow, D. Gross, N. Isgur, J. Preskill, and J. Primack for useful comments. This work was supported in part by the U. S. Department of Energy under Contract No. DE-AC02-76ER03069.

(a)Present address: Department of Physics, Lyman Laboratories, Harvard University, Cambridge, Mass. 02138.

[1]See M. Yoshimura, in *Proceedings of the Fourth Kyoto Summer Institute on Grand Unified Theories* (World Science, Singapore, 1981), and references therein.

[2]For example, see A. Guth, Phys. Rev. D 23, 347 (1981); A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); A. D. Linde, Phys. Lett. 103B, p. 389.

[3]L. Krauss, Gravitation Research Foundation Essay 1981, Massachusetts Institute of Technology Report No. MIT CTP 986 (unpublished).

[4]S. Weinberg, in *General Relativity: An Einstein Centenary Survey* (Cambridge Univ. Press, Cambridge, 1979).

[5]For example, see L. Smollin, to be published.

[6]See D. Gross, M. Perry, and L. Yaffe, Phys. Rev. D 25, 330 (1982).

[7]S. W. Hawking and G. Ellis, *The Large Scale Structure of Space-Time* (Cambridge Univ. Press, Cambridge, 1973).

[8]See S. W. Hawking, in Ref. 4.

[9]See G. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2738 (1977), and references therein.

[10]S. W. Hawking, Phys. Rev. D 13, 191 (1976).

[11]For example, see B. J. Carr, Astrophys. J. 201, 1 (1975).

[12]S. Coleman, Phys. Rev. D 15, 2929 (1977).

[13]D. Page, Phys. Rev. D 13, 198 (1976).

[14]See references in M. Turner, Phys. Lett. 89B, 155 (1979).

[15]D. Toussaint, S. B. Truman, F. Wilczek, and A. Zee, Phys. Rev. D 19, 1036 (1976).

[16]J. Preskill, Phys. Rev. Lett. 43, 1365 (1979).