experiments in ^{48}Ca.

In summary, we have shown that the inclusion of the Δ-isobar degrees of freedom introduces a quenching of GT cross sections which depends on the momentum transfer (angle) and the spin of the excitation. This behavior could be used to distinguish experimentally between the Δ-isobar quenching effect and other possible effects like, e.g., the coupling to high-lying (40–60 MeV) 2p-2h configurations which also gives rise to a reduction of the low-lying GT strength.[18] The latter one should be independent from the momentum transfer and the angular momentum.

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Dirac Phenomenology for Deuteron Elastic Scattering

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A relativistic description of deuteron elastic scattering based on a two-particle Dirac Hamiltonian containing phenomenological nucleon-nucleus Dirac potentials is developed. The resulting effective central potential is like that obtained with a folding model based on the Schrödinger equation; the spin-orbit potential, however, is only one-half as strong. Dirac and Schrödinger fits to 80-MeV $d+{}^{58}$Ni data with potentials consistent with nucleon-nucleus phenomenology are of comparable quality.

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Considerable progress has recently been made in understanding nucleon-nucleus scattering within the framework of Dirac phenomenology.[1,2] Such an approach leads naturally to effective central and spin-orbit potentials which are combinations of very strong Dirac vector and scalar potentials and which can in principle be related to the meson-exchange picture of the fundamental nucleon-nucleon interaction.[1,3,4] Geometry differences between the effective central and spin-orbit potentials and their relation to the nuclear matter distribution are also at least qualitatively explained.[1] Much of the energy dependence in the effective potentials is automatically accounted for in the Dirac formulation.[1,2] Where feasible, the Dirac equation provides a more appropriate starting point than the Schrödinger equation for the description of a nucleon in the presence of strong potentials.

In this note we report the development of a Dirac description for deuteron-nucleus elastic scattering. We begin by writing down a two-particle Dirac equation in a Hamiltonian form similar to that used in treating the nucleon-nucleon problem[3,4]:

$$\left\{-i\hbar c\{\vec{\alpha}(1)\cdot\vec{\nabla}_1+\vec{\alpha}(2)\cdot\vec{\nabla}_2+[\beta(1)+\beta(2)]\}mc^2-(E_1+E_2)+V_D(1)+V_D(2)\right\}\Psi=0, \tag{1}$$

where

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix},\quad \beta=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

and m is the nucleon mass, E_i is the total energy of the ith nucleon in the deuteron, $V_D(i)$ is the Dirac potential for the ith nucleon, and Ψ is the two-particle Dirac spinor for the deuteron which can be written in terms of outer products of one-particle spinors.[3,4] We write

$$\Psi(i)=\begin{pmatrix}u(i)\\ w(i)\end{pmatrix}\chi^{\mathrm{P}}(i),$$

where $u(i)$ and $w(i)$ are the large and small spatial components, respectively, of the ith nucleon and $\chi^{\mathrm{P}}(i)$ is the usual Pauli two-component spinor. Then

$$\Psi(1,2)=\Psi(1)\otimes\Psi(2)=\begin{pmatrix}u(1)u(2) & u(1)w(2)\\ w(1)u(2) & w(1)w(2)\end{pmatrix}\chi^{\mathrm{P}}(1)\otimes\chi^{\mathrm{P}}(2)\to\begin{pmatrix}U(1,2) & \eta_1(1,2)\\ \eta_2(1,2) & W(1,2)\end{pmatrix}\chi^{\mathrm{P}}(1)\otimes\chi^{\mathrm{P}}(2).$$

Thus the spatial part of Ψ is a two-by-two matrix.

We now assume that the internal spatial structure of the deuteron can be ignored and that the spatial dependence of the potentials in Eq. (1) is upon the center-of-mass coordinate, $\vec{r}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$, only. Such a procedure gives the general form of the deuteron-nucleus potential in the standard Schrödinger formulation. We also adopt Dirac potentials of the form which describe nucleon-nucleus scattering, i.e.,

$$V_D(i)=\beta(i)V_s(r)+V_v(r).$$

In the present work, in contrast to other approaches,[1] we assume that the Dirac potentials are purely real. We use a Schrödinger (linear) imaginary potential. In fact, in the calculations to be discussed below, the imaginary potential is identically that used in the Schrödinger calculations.

Equation (1) then becomes, if one assumes $E_1=E_2=E$,

$$\left\{\frac{-i\hbar c}{2}[\vec{\alpha}(1)+\vec{\alpha}(2)]\cdot\nabla_r+[\beta(1)+\beta(2)][mc^2+V_s(r)]-2[E-V_v(r)]\right\}\Psi(\vec{r})=0.$$

After insertion of the explicit form of Ψ and performance of some algebra, uncoupled equations for $U+W$ and $U-W$ can be obtained.[3,4] After dropping all terms containing four $\vec{\sigma}\cdot\vec{\nabla}$ operations and adding the equations for $U+W$ and $U-W$, we obtain the following:

$$\begin{aligned}\frac{-\hbar^2c^2}{4E}&\left\{\nabla^2-\frac{\mathfrak{M}'}{\mathfrak{M}}\hat{r}\cdot\vec{\nabla}+\frac{1}{r}\frac{\mathcal{E}'}{\mathcal{E}}\vec{\sigma}_d\cdot\vec{\mathrm{L}}+\frac{1}{r}\left(\frac{\mathcal{E}'}{\mathcal{E}}-\frac{\mathfrak{M}'}{\mathfrak{M}}\right)+\frac{\mathfrak{M}}{2}\left[\frac{1}{\mathfrak{M}}\left(\frac{\mathcal{E}'}{\mathcal{E}}-\frac{\mathfrak{M}'}{\mathfrak{M}}\right)\right]'\right\}U-\left\{\frac{\mathcal{E}^2-\mathfrak{M}^2c^4}{E}\right\}U\\ &=\frac{-\hbar^2c^2}{8E}\left\{\frac{\mathcal{E}'}{\mathcal{E}}-\frac{\mathfrak{M}'}{\mathfrak{M}}(\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\vec{\nabla}+\vec{\sigma}_2\cdot\hat{r}\vec{\sigma}_1\cdot\vec{\nabla})+\mathfrak{M}\left[\frac{1}{\mathfrak{M}}\left(\frac{\mathcal{E}'}{\mathcal{E}}-\frac{\mathfrak{M}'}{\mathfrak{M}}\right)\right]'\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}\right.\\ &\qquad\left.+\frac{1}{r}\left(\frac{\mathcal{E}'}{\mathcal{E}}-\frac{\mathfrak{M}'}{\mathfrak{M}}\right)(\vec{\sigma}_1\cdot\vec{\sigma}_2-\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r})\right\}W,\end{aligned} \tag{2}$$

where $\mathcal{E}=E-V_v$, $\mathfrak{M}c^2=mc^2+V_s$, $\vec{\sigma}_d=(\vec{\sigma}_1+\vec{\sigma}_2)/2$, and primes indicate differentiation with respect to r. Note that in defining $\vec{\sigma}_d$ we have specified that the two nucleons are coupled to spin 1 and that this is the only element of the structure of the deuteron which is addressed explicitly. All terms multiplying W in Eq. (2) are roughly of the same magnitude as the smallest terms multiplying U. Since W itself is the "lower-lower" component of the scattering wave function and is smaller than U by roughly a factor of $[(E-mc^2)/(E+mc^2)]^2$, we are well justified in setting the right-hand side of Eq. (2) to zero. We then have

$$\left\{\frac{-\hbar^2c^2}{2E_d}\nabla^2-\frac{\hbar^2c^2}{2E_d r}\left[\frac{\mathcal{E}'}{\mathcal{E}}\vec{\sigma}_d\cdot\vec{\mathrm{L}}-\frac{\mathfrak{M}'}{\mathfrak{M}}\vec{\mathrm{r}}\cdot\vec{\nabla}\right]+\frac{\hbar^2c^2}{2E_d}\mathfrak{M}\left[\left(\frac{1}{r\mathfrak{M}}+\frac{d}{2dr}\frac{1}{\mathcal{E}}\right)\frac{d}{dr}\left(\frac{\mathcal{E}}{\mathfrak{M}}\right)\right]\right.$$
$$\left.+2V_v+\frac{m_dc^2}{E_d}2V_s+\frac{(2V_s)^2-(2V_v)^2}{2E_d}-\frac{E_d^2-(m_dc^2)^2}{2E_d}\right\}U=0, \tag{3}$$

where $E_d=2E$ and $m_d=2m$. This equation is written with no Coulomb potential and in the no-recoil limit appropriate for heavy target nuclei. The actual calculations, however, add a Coulomb term to the vector potential and use center-of-momentum energies and a reduced total energy for E_d to treat recoil in a standard approximate manner.[5] Note that retention of terms dropped in obtaining Eq. (3) would lead to weak tensor interactions of various kinds.

The (approximate) Dirac equation can readily be compared with the standard Schrödinger equation for deuteron scattering. We note an extra $\vec{\mathrm{r}}\cdot\vec{\nabla}$ or Darwin term which has little effect on elastic scattering[1] and an unpleasant but small term involving derivatives of $\mathcal{E}/\mathfrak{M}$. In examining Eq. (3) it is useful to compare it with the corresponding (exact) Dirac equation for nucleon-nucleus scattering,

$$\left\{\frac{-\hbar^2c^2}{2E_p}\nabla^2-\frac{\hbar^2c^2}{2E_p r}\frac{\mathfrak{M}_p'}{\mathfrak{M}_p}[\vec{\sigma}_p\cdot\vec{\mathrm{L}}-\vec{\mathrm{r}}\cdot\vec{\nabla}]+V_v+\frac{m_pc^2}{E_p}V_s+\frac{V_s^2-V_v^2}{2E_p}-\frac{E_p^2-m_pc^2}{2E_p}\right\}u=0, \tag{4}$$

where u is the large component of the nucleon spinor and $\mathfrak{M}_pc^2=E_p+m_pc^2+V_s-V_v$. It is seen that, except for the "unpleasant" term mentioned above, the central potential for the deuteron is twice that for the proton, in keeping with standard folding-model concepts.

Beyond this, we begin to see significant differences between the Schrödinger and Dirac results. The proton and deuteron spin-orbit potentials, which in the Dirac formulation are purely relativistic effects resulting from an r-dependent coupling between large and small components, do not have the same relationship as in the nonrelativistic picture. Nonrelativistically, the deuteron spin-orbit potential is approximately the sum of the proton and neutron spin-orbit potentials, i.e.,

$$V_{so}^d(\vec{\mathrm{L}}_d,\vec{\sigma}_d,r)\simeq\frac{1}{r}\frac{d}{dr}V_{so}^p(r)(\vec{\mathrm{L}}_p\cdot\vec{\sigma}_p+\vec{\mathrm{L}}_n\cdot\vec{\sigma}_n)$$
$$\simeq\frac{1}{r}\frac{d}{dr}V_{so}^p(r)\vec{\mathrm{L}}_d\cdot\vec{\sigma}_d. \tag{5}$$

Relativistically[1] we have

$$V_{so}^p(\vec{\mathrm{L}}_p,\vec{\sigma}_p,r)=\frac{-\hbar^2c^2}{2E}\frac{1}{r}\frac{\mathfrak{M}_p'}{\mathfrak{M}_p}\vec{\mathrm{L}}_p\cdot\vec{\sigma}_p \tag{6a}$$

$$V_{so}^d(\vec{\mathrm{L}}_d,\vec{\sigma}_d,r)=\frac{-\hbar^2c^2}{2E_d}\frac{1}{r}\frac{\mathcal{E}'}{\mathcal{E}}\vec{\mathrm{L}}_d\cdot\vec{\sigma}_d. \tag{6b}$$

Since $V_s\simeq-V_v$, we have $\mathfrak{M}_p'/\mathfrak{M}_p\simeq\mathcal{E}'/\mathcal{E}$. Consequently we find

$$V_{so}^p/V_{so}^d\simeq 2\vec{\mathrm{L}}_p\cdot\vec{\sigma}_p/\vec{\mathrm{L}}_d\cdot\vec{\sigma}_d$$

versus the ratio of unity obtained above by nonrelativistic folding arguments. This is perhaps a surprising result, especially since most phenomenological deuteron Schrödinger potentials have spin-orbit potentials approximately equal to the proton potentials.[6] However, it is also well known that spin observables in deuteron elastic-scattering calculations at energies below ~100 MeV result from a complicated interplay of the spin-orbit and central potentials. Therefore it is to some extent an open question as to how well determined the ratio of proton to deuteron spin-orbit strengths is phenomenologically. Higher-energy measurements, where spin observables are likely to depend more straightforwardly on the spin-dependent potentials, would be of obvious value in this regard. It is also possible that the ratio of 2 discussed above is somewhat misleading because the functional forms of the spin-orbit potentials are appreciably different in the two formulations. In any case, we have attempted to fit the highest-energy complete set of deuteron elastic scattering data available, 79-MeV $d+{}^{58}$Ni data of Stephenson *et al.*,[7] using Eq. (4). For convenience, we have used the phenomenological imaginary Schrödinger parameters determined in Ref. 7. Thus, as discussed above, only the real potentials have been treated as Dirac potentials. The resulting fit—which was obtained after limited searching—is compared with the Schrödinger best fit in Fig. 1. The quality of the fits is comparable. The phenomenological vector and scalar strengths for the Dirac potential are roughly double those obtained by us for nucleon-nucleus scattering in the energy range of 40 MeV. Thus the phenomenological Dirac potential is consistent with our folding arguments, the apparent factor

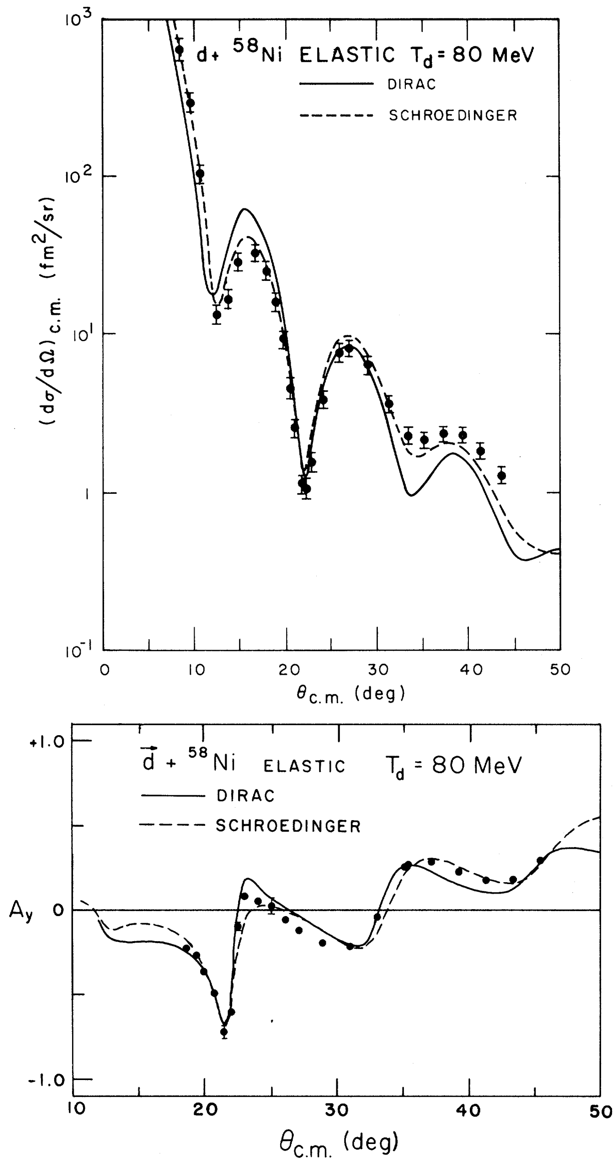

FIG. 1. Dirac and Schrödinger fits to the cross section and vector analyzing power data of Stephenson *et al.* (Ref. 7).

of 2 reduction in spin-orbit strength notwithstanding.

Another interesting feature is that the deuteron spin-orbit potential is determined exclusively by the vector potential through the factor $\mathcal{E}'/\mathcal{E} = -V_v'/(E-V_v)$ while the Darwin term depends only on the scalar potential through $\mathfrak{M}'/\mathfrak{M} = V_s'/(mc^2+V_s)$. This is to be contrasted with the nucleon spin-orbit and Darwin potentials which are both determined by $\mathfrak{M}_p'/\mathfrak{M}_p = (-V_v'+V_s')/(E+mc^2-V_v+V_s)$ which contains both vector and scalar potentials. These various dependences may facilitate experimental determination of the vector and scalar potentials by themselves rather than in combination with one another. The form of the deuteron spin-orbit potential also has interesting implications for the scattering of antideuterons. As is well known, when antiparticles interact with nuclear vector and scalar potentials the signs of the vector potentials are reversed. Thus antiprotons should see a very deep central potential (~1 GeV) and a negligible spin-orbit potential. On the basis of these notions and Eq. (3), antideuterons should see a correspondingly deep central potential, while the spin-orbit potential, rather than becoming small, merely changes sign relative to the deuteron case. The deuteron potentials developed here are also crucial in Dirac calculations for the (p,d) reaction.[8]

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