Nuclear Scaling in Inelastic Electron Scattering from d, ^{3}He, and ^{4}He

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Data for electron scattering from d, ^{3}He, and ^{4}He in the region between the elastic and quasielastic peaks are found to be unified by a nuclear scaling function. A simple model is used to understand the general features of the data, to understand the approach to scaling with increasing momentum transfer, and to extract the deuteron wave function from the deuteron data.

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In recent years it has been recognized[1] that cross sections for quasielastic scattering from nuclei at a variety of kinematical conditions can be unified by a nuclear scaling function $F(y)$. In the simplest picture, and with use of the impulse approximation, the variable y is the smallest possible Fermi momentum that can give a final-state electron of the observed energy and angle. The function $F(y)$ describes the shape of the quasielastic peak and is related to an integral over the single-particle wave function.

The scaling prediction has been verified for $|y| < 0.3$ GeV/c for calcium[2] with incident energies of 0.2 to 0.5 GeV and angles of 60° to 160°, and for a much larger range of y with ^{3}He data at 8° and much higher energies.[3] In this Letter we use new data for deuterium which exhibit remarkable y scaling, and compare the results to those from ^{3}He and ^{4}He. We use a simple model to understand the approach to scaling and extract the deuteron wave function.

The relation between experimental inclusive electron cross sections $d^2\sigma/d\Omega\, dE'$ and the corresponding scaling-function value that we use is

$$F(y) = (d^2\sigma/d\Omega\, dE')(dE'/dy)[Z\sigma_p(E,\theta) + (A-Z)\sigma_n(E,\theta)]^{-1}, \quad (1)$$

where y is the solution of

$$E + M_A = E' + (M + q^2 + 2qy + y^2)^{1/2} + (M_{A-1}{}^2 + y^2)^{1/2}, \quad (2)$$

where $\sigma_p(E,\theta)$ and $\sigma_n(E,\theta)$ are the on-shell proton and neutron elastic cross sections evaluated for stationary nucleons, E and E' are the incident and final electron energies, and q is the absolute value of the three-momentum transfer. The variables A and Z are the nucleon and atomic numbers, and M_A, M, and M_{A-1} are the masses of the target, ejected nucleon, and spectator nucleons.

There now exist data for electron scattering from deuterium from two experiments,[4,5] at scattering angles 8° and 10°, respectively, with incident energies ranging from 6 to 21 GeV. There are ten data sets in Ref. 4, with average four-momentum transfer (Q^2) values from 0.8 to 6.0 (GeV/c)^2, all of which span a final electron energy range very close to the deuteron elastic peak (the threshold inelastic region). These data probe the region of the largest possible negative-y values for a given Q^2 and angle. The data of Ref. 5 come in five sets with average Q^2 values from 2.5 to 10 (GeV/c)^2 and concentrate on the region around the quasielastic peak.

In Fig. 1 we show $F(y)$ for deuterium obtained by use of Eq. (1) with on-shell elastic cross sections σ_p and σ_n taken from a fit to the world's data.[10] There was a substantial contribution to the high-Q^2 data from pion production (up to 60% at the highest Q^2 for $y < 0$). This was subtracted by means of smeared inelastic electron-proton data normalized to the deuteron data in the large positive-y region. In order to minimize effects from final-state interactions, a few data points very close to the deuteron elastic peak for which $p^* < 0.2$ GeV/c were not used, where p^* is the momentum of the final-state nucleons in their center of mass. It can be seen that the remaining data cluster in a very narrow band.

Also shown in Fig. 1 are data from an experiment[6] for ^{3}He and ^{4}He at 8°. There are threshold inelastic data sets with average Q^2 values from 0.8 to 5.0 (GeV/c)^2, with fourteen sets for ^{3}He and seven sets for ^{4}He. There are six quasielastic sets for ^{3}He, with Q^2 from 0.15 to 4.0 (GeV/c)^2. A cut on p^* was made, and the subtraction of real pion production was performed, in the same

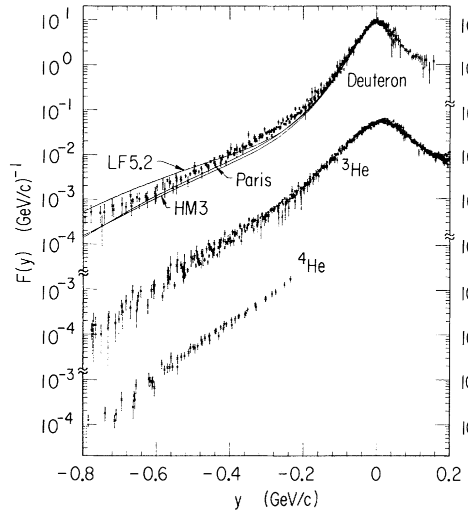

FIG. 1. Experimental values for $F(y)$ obtained from recent data for d, ^{3}He, and ^{4}He (Refs. 4, 5, and 6). The data have been radiatively corrected and pion-production contributions have been subtracted. The solid lines are calculations of $F(y)$ for deuterium using Eq. (5) with three different deuteron wave functions: Paris (Ref. 7), Lomon-Feshbach with 5.2% d state (Ref. 8), and Holinde-Machleidt (Ref. 9).

manner as for deuterium. It can be seen that again $F(y)$ for ^{3}He and ^{4}He groups into narrow bands. Notice that the ^{3}He peak is wider than the d peak, reflecting the larger average Fermi momentum and stronger two-body correlations. At large negative y, the $F(y)$ values for all three elements are about the same order of magnitude, but the slope increases slightly with A.

In order to understand the general features of y scaling, we derive a simple formula for $F(y)$ for deuterium, starting with the nonrelativistic model of McGee as modified by Durand.[11] We have kept only the terms proportional to u^2 and w^2, where u and w are the s-state and d-state parts of the deuteron wave function. The inclusion of other terms, such as those proportional to uw or u', makes less than a 6% difference over the kinematic range of our data, but may be more important at other energies and angles. We then obtain

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{M^2 p^* E(\sigma_p+\sigma_n)}{8\pi E^*}\int \psi^2(k)d\Omega^* \qquad (3)$$

where $\psi^2 = u^2+w^2$, k is the laboratory momentum of the spectator nucleon in the impulse approximation, and E^* and $d\Omega^*$ are the center-of-mass energy and solid angle of the final-state nucleons. Using Eq. (1), we find

$$F(y) = \frac{M^2E}{2qE'}\frac{dE'}{dy}\int_{|y|}^{|y+q|}\frac{\psi^2(k)k\,dk}{(k^2+M^2)^{1/2}}\,. \qquad (4)$$

It should be noted that the range of integration

$$|q+y| - |y| = (E+2M-E')p^*/E^*$$

is proportional to p^*. Since p^* grows, at fixed y, with Q^2, and ψ^2 decreases quickly as a function of k, the upper limit of the integral can be replaced with infinity at large Q^2. Taking the limit $Q^2\to\infty$ gives

$$F(y) \to \frac{\frac{1}{2}M^2(1+y/E_y)}{2m-y-E_y}\int_{|y|}^{\infty}\frac{\psi^2(k)k\,dk}{(k^2+M^2)^{1/2}}\,, \qquad (5)$$

where $E_y = (y^2+M^2)^{1/2}$. This equation now depends only on y.

Similar expressions have been derived for elements other than deuterium,[12] where the high-Q^2 limits take the same form as Eq. (5). At finite Q^2 the wave function must be replaced by a spectral function $S(k,E_{\rm rel})$, where $E_{\rm rel}$ is the relative energy of the spectator nucleons, and both k and $E_{\rm rel}$ must be integrated over.

Shown in Fig. 1 are the predictions for the asymptotic $F(y)$ for deuterium from Eq. (5) with three different commonly used nonrelativistic deuteron wave functions.[7-9] The predictions agree very well with the data near the quasielastic peak, but are somewhat low around $y=-0.3$ GeV/c. This could be due to a lack of high momentum components in the wave functions used (see below), or to corrections to our simple model from relativistic effects or off-shell form factors. Other processes that could account for the discrepancies, such as final-state interactions or meson exchange currents, would not in general be expected to show scaling in y, but could still be of some importance. It is interesting to note that a calculation[3] for ^{3}He using the best available Faddeev wave functions falls below the data in a manner similar to our deuteron calculation. This discrepancy is attributed to a lack of high-momentum components in the ^{3}He wave functions.

Although on a logarithmic scale the grouping of the experimental $F(y)$ data is impressive, systematic differences between different data sets in a given y region exist. This could be expected since much of the data was taken at Q^2 values for which Eq. (4) has not yet reached the asymptotic limit of Eq. (5). The gradual increase of the ratio of Eq. (4) to Eq. (5) towards 1.0 is depicted by the curves in Fig. 2, where it can be seen that the onset of scaling occurs at higher Q^2 as $|y|$ is increased. In the evaluation of Eq. (4) for ^{3}He and ^{4}He the kinematics were changed, but no integral over the relative energy was performed. In addition, the same wave function[7] was used as for deuterium. Since the effects of the wave function largely cancel out in taking the ratio of Eq. (4) to Eq. (5), this approximation should be reasonably valid. The solid points in Fig. 2 represent the average of the experimental data points in the y intervals indicated. Each group has been normalized with a single multiplicative constant to each of the theoretical curves. It can be seen that the shape of the experimentally observed approach to scaling is consistent with that expected from Eqs. (4) and (5).

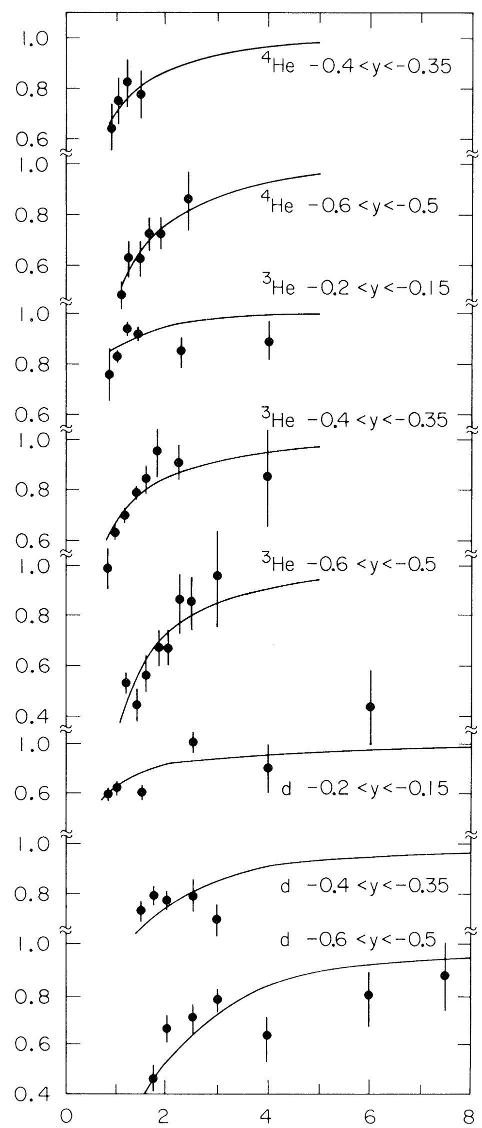

FIG. 2. Comparison of experimental and theoretical approach to scaling. The solid lines are the ratio of Eq. (4) to Eq. (5), using the Paris wave function (Ref. 7). The points are the average experimental data in the indicated y bins, normalized to solid lines with a single multiplicative constant for each set.

This being the case, it is plausible to use Eq. (4) to explore the discrepancy described earlier between the experimental and calculated $F(y)$ values for deuterium, the only case for which we calculate absolute cross sections. If we make the assumption that the discrepancy is not due to relativistic corrections, off-shell form factors, or other reaction mechanisms, but solely to a deficiency in high-momentum components, we can use Eq. (4) to find a wave function that would better describe the experimental $F(y)$ values. This is of particular interest since there seems to be a similar lack of high-momentum components in ^{3}He. Using Eq. (4) and neglecting some small terms proportional to $F(y)$, we find

$$\psi^2(k) = (2E_y E' q / E M^2 y)(dy/dE') dF(y)/dy, \qquad (6)$$

where $k = |y|$. The wave-function values obtained from Eq. (6) are shown in Fig. 3. They were obtained by differentiation of all pairs of adjacent $F(y)$ points in each data set. Values at different Q^2 in a given k region were found to be consistent with each other, and were therefore averaged together to obtain smaller error bars. No systematic errors are included. Also shown in Fig. 3 are three commonly used potential models and data from two $(e,e'p)$ experiments.[13,14] It can be

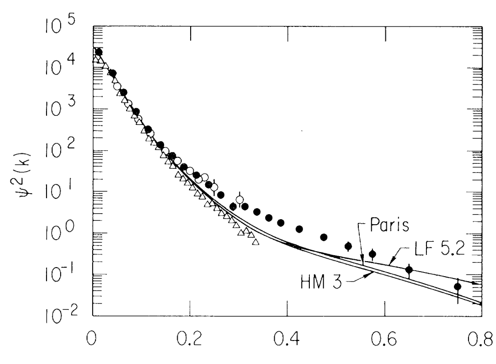

FIG. 3. Experimental deuteron wave functions obtained from Eq. (6) (solid circles) compared to three wave-function models labeled as in Fig. 1. Also shown are results from two $d(e, e'p)$ experiments: Berheim *et al.* (Ref. 13, triangles) and Agranovich *et al.* (Ref. 14, open circles).

seen that the results of our analysis contain more high-momentum components in the 0.2- to 0.5-GeV/c region than any of the models, and disagree with one $(e,e'p)$ experiment, but agree with the other.

As with the $(e,e'p)$ experiments, a full study of all the processes that could complicate the interpretation of the inclusive electron measurements in terms of the plane-wave impulse approximation and nonrelativistic wave functions must be undertaken before any definite conclusions can be drawn. The result remains, however, that any theory must be able to explain the impressive experimental observation that nucleon scaling unifies inclusive electron-scattering data for the three lightest nuclei over a large kinematic region and for y values that imply very short distances within the nucleus.

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Role of Mean Free Paths of Product Particles in High-Energy Nucleus-Nucleus Collisions

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The role of mean free paths of product particles in high-energy nuclear collisions has been studied. In inclusive energy spectra the observed slope difference among p, π, and K^+ can be interpreted as due to the difference in mean free paths of these particles, suggesting that particles with longer mean free paths probe most sensitively the early, highly excited, hot phase of the collision. With use of the data of pp and $\pi\pi$ interferometries further discussions on the space-time evolution of the system are developed.

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One of the main goals in the research of high-energy nucleus-nucleus collisions is to probe experimentally the highly excited, compressed, hot phase of nuclear matter. Obviously, a nuclear collision is time dependent, and, as demonstrated in cascade calculations,[1] the time interval during which the system is at this hot phase would be of the order of $(2-3)\times 10^{-23}$ s. Unfortunately, particles detected by actual experiments, such as p, π, K^+, etc., will record the entire (time-integrated) history of the collision. Thus, an important question to ask here is whether a particular type of these particles can most sensitively probe this hot phase. In this Letter I first discuss this question using the data of inclusive energy spectra of p, π, and K^+.

