Continuity of the Chemical Potential across an Oscillating Superleak Transducer

In a recent Letter,[1] Liu and Stern have modeled the operation of a second-sound transducer (an oscillating porous membrane) by assuming that the normal fluid velocity, v_n, is equal to the membrane velocity, v_M, at the position of the membrane and that the chemical potential is continuous across the (very thin) porous membrane. They further assumed that the oscillatory part of the chemical potential, $\delta\mu$, is zero at the membrane position and showed that the relative amplitude of first sound ($A_1 \equiv \delta\rho/\rho$) is very small compared with that of second sound ($A_2 \equiv \delta\sigma/\sigma$); they found, in fact, that $A_1/A_2 = \rho_n c_2^2/\rho_s c_1^2$, which is generally orders of magnitude smaller than a previous result[2] based on the boundary condition $v_s = 0$ at the membrane. The purpose of the present Comment is to investigate the implications of relaxing the requirement that $\delta\mu = 0$ at the membrane but retaining the requirement that μ be continuous across it. The relevant result is $A_1/A_2 = \rho_n c_2^3/\rho_s c_1^3$ which is generally another order of magnitude smaller still than the Liu and Stern result. In this Comment, I will use Liu and Stern's notation.

The geometry is an oscillating membrane of negligible thickness, l, occupying the plane $x=0$ whose pores are small enough to clamp the normal fluid. There is a rigid backing plate at $x = -L$. The membrane oscillates with a velocity $v_M e^{-i\omega t}$, thus generating first- and second-sound waves of amplitudes A_1 and A_2 propagating to the right and to the left. These latter are reflected back towards the right at $x = -L$. The boundary conditions determine all six unknown amplitudes. At any position $\delta\mu$, v_n, and v_s are linearly related[1] to $\delta\rho$ and $\delta\sigma$.

If one assumes that this process is adiabatic, the boundary conditions at $x=-L$ are $\rho_s v_s + \rho_n v_n |_{x=-L} = 0$, and $\rho\sigma v_n |_{x=-L} = 0$ (i.e., $v_s = v_n = 0$). The density and entropy variations in the region $-L < x < 0$ are, therefore, of the form $\delta\rho/\rho = A_1'[\exp(-iq_1 x) + \exp(2iq_1 L)\exp(iq_1 x)]$ and $\delta\sigma/\sigma = A_2'[\exp(-iq_2 x) + \exp(2iq_2 L)\exp(iq_2 x)]$. The four unknown amplitudes A_1, A_1', A_2, and A_2' are determined from the four boundary conditions at $x=0$: $v_n(0^-) = v_n(0^+) = v_M$, $\delta\mu(0^-) = \delta\mu(0^+)$, $\rho_s v_s(0^-) + \rho_n v_n(0^-) = \rho_s v_s(0^+) + \rho_n v_n(0^+)$. The resulting solutions are particularly simple in two limiting cases:

(A) In the limiting case $L \to \infty$, the waves reflected from the back wall never return (under the assumption that q_1, q_2 have small imaginary parts) and the solutions (for $x > 0$) are identical in all respects to those derived by Liu and Stern.

(B) The opposite limit, appropriate to most experiments ($L \ll q_2^{-1}, q_1^{-1}$), is radically different. The determinant of the 4×4 matrix goes to zero as L^1; thus the unknown amplitudes can be expanded in a power series in L beginning with L^{-1}, i.e., $A_1' = \alpha_1' L^{-1} + \beta_1' + O(L^1)$, etc. By collecting like powers of L, one can solve for the α's and β's explicitly. The resulting amplitudes for first and second sound are

$$A_1 = v_M (c_2^2/c_1)[c_2^2 + (\rho_s/\rho_n)c_1^2]^{-1},$$

$$A_2 = v_M (c_1^2/c_2)[c_1^2 + (\rho_n/\rho_s)c_2^2]^{-1},$$

$$A_1/A_2 = (\rho_n/\rho_s)(c_2/c_1)^3.$$

Thus, the ratio of first- to second-sound amplitudes is down by another factor of c_2/c_1 compared to the Liu-Stern result, in this limit. As a corollary, the quantities δP, δT, $\delta\rho$, and $\delta\sigma$ are all nonzero and independent of position in the region $-L < x < 0$. To order L^{-1} these values are, coincidentally, the same as those calculated by Liu and Stern (what they call ΔP, ΔT, etc.). Therefore, to order L^{-1}, $\delta\mu = 0$ in this region; to order L^0, $\delta\mu(0^-)$ has the same (nonzero) value as $\delta\mu(0^+)$.

The Liu-Stern analysis of the nuisance effects is unaffected. Specifically, the normal-fluid slip due to Poiseuille flow through the pores is

$$Q = N\rho_n \pi R^4 \{(\rho_n/\rho)[\delta P(0^-) - \delta P(0^+)] + \rho_s \sigma[\delta T(0^-) - \delta T(0^+)]\}/8\eta_n l.$$

For small L, the quantity in curly brackets is equal to $\delta P(0^-)$, Q is the same as that given by Liu and Stern, and the condition for suppression of Poiseuille flow [their Eq. (8)] is unchanged.

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[1] M. Liu and M. R. Stern, Phys. Rev. Lett. 48, 1842 (1982).

[2] R. A. Sherlock and D. O. Edwards, Rev. Sci. Instrum. 41, 1603 (1970).