Fractionally Charged Particles as Evidence for Supersymmetry

P. H. Frampton and T. W. Kephart

Institute of Field Physics, Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27514

(Received 31 March 1982)

Supersymmetric grand unified models with fractionally charged color singlets ("fractons") are considered. It is shown that the minimal supersymmetrized fracton model, which occurs in SU(7), gives a value of $\sin^2\theta(m_W)$ that is naturally lower than the minimal SU(5) supersymmetric grand unified theory prediction, and can be in agreement with experiment for fracton charge $q = e/3$.

PACS numbers: 11.30.Pb, 11.30.Ly, 12.10.En, 14.80.Pb

The reported observation of fractional electric charge,[1] if confirmed, poses a serious problem for theoretical physics. Either color is broken[2] or grand unified theories (GUT's)[3] must be enlarged[4-6] to accommodate color singlets with fractional electric charge: "fractons" (to agree with experiment at least some of the fractons must carry $\frac{1}{3}$-integral electric charge). We will discuss only the latter possibility.

With unbroken color, the minimal grand unification group beyond the Georgi-Glashow SU(5) that allows fractons is SU(7), as described in Refs. 4 and 5, where a nontrivial charge embedding is necessary. This model has two normal SU(5) families $2(\underline{5}+\underline{10}^*)_L$ and two charge-shifted conjugate families $(\underline{5}^*+\underline{10})^{\pm 1/3}$, that must be light ($\lesssim 1000$ GeV) since they only acquire mass after the breaking of the electroweak SU(2)⊗ U(1) group. The charge-shifted families contain both fractionally charged leptons and quarks with charges not in the sequence $(\frac{2}{3}+n)e$, n integer. Hence a fracton may be either a lepton or a hadron.

The symmetry in the SU(7) model of Ref. 4 is broken directly to SU(3)⊗SU(2)⊗ U(1) at the superheavy mass scale by scalars in irreducible representations (irreps) of high dimensionality, e.g., $\underline{756}$, $\underline{840}$, etc., which cannot couple directly to fermions.

In order to have agreement with the experimental value of $\sin^2\theta(m_W)$ many light Higgs doublets must be added[4] in an *ad hoc* way, since otherwise the Weinberg angle is much too small. In Ref. 5 a satisfactory Weinberg angle is obtained instead by introducing vacuum expectation values (VEV's) at a relatively low mass ($\sim 10^9$ GeV) for the irrep which fixes the charge shift (i.e., the $\underline{756}$ and/or the $\underline{840}$) of the two conjugate families. Here we shall show how supersymmetrizing the SU(7) model avoids paying either of these two prices for fractionally charged particles.

Supersymmetric grand unified theories have the advantage that supersymmetry, if unbroken, protects mass relations set up at the grand unification mass scale $m_{\rm GUT}$, by means of nonrenormalization theorems.[7] This offers the hope of a solution[8] of the long-standing hierarchy problem.[9]

If m_{ss} is the supersymmetry-breaking mass scale then we expect, and so assume, that $\alpha m_{ss} \lesssim m_W$. Our assumption allows for the possibility of generating m_W either at the tree level or radiatively at the one-loop level (the breaking of supersymmetry and of the electroweak group is not discussed here). The consequences of allowing m_{ss} to range all the way up to $m_{\rm GUT}$ has been discussed.[10]

If we require $\alpha m_{ss} \lesssim m_W$, then the predictions for $\sin^2\theta(m_W)$ appear to be somewhat too high in the minimal SU(5) supersymmetric GUT[11] (see, however, Ref. 10). Including fractons necessarily increases $\mathrm{Tr}Q^2$ at $m_{\rm GUT}$, hence reducing $\sin^2\theta(m_{\rm GUT})$, while also changing its evolution. We will find that a minimal SU(7) fracton supersymmetric GUT with the minimum two light Higgs doublets and no intermediate mass scales (neither gauge- nor supersymmetry-breaking mass scales) can give a value for $\sin^2\theta(m_W)$ in agreement with experiment.

Let us summarize the properties that we assume for SU(N) fracton supersymmetric GUT's[12]: (a) light fermions (i.e., those in complex irreps) in totally antisymmetric ($\underline{1}^k$) irreps only (this disallows color exotics); (b) light scalars in ($\underline{1}^k$) irreps only; (c) anomaly freedom; (d) family unification[13,14]; (e) the standard charge assignment and embedding in minimal SU(5) [i.e., $\underline{N} \to \underline{5}+(N-5)\underline{1}$]; (f) pure vector couplings for SU(3)⊗ $\mathrm{U_{em}}$(1); and (g) color singlets with fractional electric charge.

The implication of property (f) with the charge operator given by

$$Q=\mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},0,-1,q_1,q_2,\ldots,q_p,q_{p+1},\ldots,q_{N-5}) \qquad (1)$$

and arranged with $q_i \neq 0$ for $i=1,2,\ldots,p$ and $q_i=0$ for $i>p$ is that the fermion representation $\{F\}$ must reduce to a "spinor" form at SU($5+p$).[15] By this we mean to the spinor σ of SO($10+2p$) which has the reduction

$$\sigma \rightarrow 1^1+1^3+1^5+\ldots \qquad (2)$$

under

$$\mathrm{SO}(10+2p)\rightarrow \mathrm{SU}(5+p). \qquad (3)$$

Furthermore, p must be even; otherwise the number of families is zero. Other choices for $\{F\}$ always lead to unpaired charged massless Weyl fields.

Let us systematically search for an SU(N) model with properties (a)–(g). For simplicity we will consider only models with the breaking

$$\mathrm{SU}(N)\rightarrow \mathrm{SU}(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1). \qquad (4)$$

It has been pointed out that phenomenology may necessitate an extra $\tilde{\mathrm{U}}(1)$ for models that are supersymmetric down to the weak-breaking mass scale[16,17] (but see also Ellis, Nanopoulos, and Rudaz[18]), and so we could have included that possibility here.

One more remark is in order at this point. Although SU(5) supersymmetric GUT's are in agreement with experiment for the ratio m_b/m_τ and with the lower limit on the proton lifetime, the value of $\sin^2\theta$ calculated at m_W ($0.23 \leq \sin^2\theta \leq 0.26$) is somewhat higher than the measured value: $\sin^2\theta(\mathrm{expt})\cong 0.215$.[10,18] The reason for this higher value of $\sin^2\theta$ is that the gauge coupling runs more slowly since the gauge-field contribution to the β function is reduced by a factor of $\frac{9}{11}$,[19-21] because of the fermionic partners of the gauge fields. Thus the calculated value of $\sin^2\theta$ does not drop quickly enough to reach the experimental value. The addition of light Higgs doublets gives a positive contribution to $\sin^2\theta$, which only makes the problem worse. We will see below how this situation can be improved by including fractons.

SU(7).—The nonsupersymmetric case has already been thoroughly investigated.[4,5] Thus let us consider an SU(7) supersymmetric GUT. In fact, let us consider a generalization of the nonsupersymmetric model with

$$Q=\mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1,q,-q), \qquad (5)$$

where q is real and nonzero but otherwise unspecified. Next we assume that the breaking (4) is possible (this will be justified when we fix q; see below). In order to fix q we make the further requirement: (h) The predicted value of $\sin^2\theta$ must fall within the experimentally accepted range. To satisfy (h) we now carry out the simple one-loop renormalization-group analysis.

In the notation of Georgi, Quinn, and Weinberg (GQW)[22] we find by straightforward manipulation of the one-loop renormalization-group equations that

$$\sin^2\theta(m_W)=\frac{1}{1+C^2}-\left[\frac{1}{1+C^2}-\frac{\alpha}{\alpha_s}\right]\frac{C^2(b_1-b_2)}{C^2b_1+b_2-(1+C^2)b_3}. \qquad (6)$$

The b_i are given by

$$16\pi^2 b_3=-9+N_f, \qquad (7a)$$

$$16\pi^2 b_2=-6+N_f+\tfrac{1}{2}N_\mathrm{H}, \qquad (7b)$$

$$16\pi^2 b_1=N_f+\tfrac{1}{2}C^{-2}N_\mathrm{H}, \qquad (7c)$$

where N_f is the number of light quark flavors and N_H is the number of light Higgs doublets (N_H must be an even number in supersymmetric GUT's). As an aside, we observe that for the classification $\mathrm{SU}(7)\rightarrow\mathrm{SU}_\mathrm{GG}(5)$, the $\underline{7}$ in Eq. (5) gives $\underline{7}\rightarrow\underline{5}+\underline{1}^{+q}+\underline{1}^{-q}$. We have assumed that the two charge-shifted singlets are light; in fact, they must be massless at tree level when SU(7) breaks via (4) and so they must stay massless until supersymmetry breaks at $\sim m_W$, because they are protected by the nonrenormalization theorems.[7] If these singlets were heavy, as, say, in the conventional SU(7) model, then they would modify b_1 (and b_1 only) by replacing N_f with N_f-2/q^2.

Using the above values for the b_i we find

$$\sin^2\theta(m_W)=A+B\,\alpha(m_W)/\alpha_s(m_W), \qquad (8)$$

with

$$A=(3+\tfrac{1}{2}N_\mathrm{H})/(18+36q^2+N_\mathrm{H}), \qquad (9)$$

$$B=[10-\tfrac{1}{3}N_\mathrm{H}+4q^2(6-\tfrac{1}{2}N_\mathrm{H})]/(18+N_\mathrm{H}+36q^2). \qquad (10)$$

Here we used $2\,\mathrm{Tr}Q^2=1+C^2=\frac{8}{3}+4q^2$.

These formulas have a suggestive structure. First, note that for $N_{\rm H}=q=0$ we obtain $A=\frac{1}{6}$, $B=\frac{5}{9}$, exactly as in the GQW analysis[22] for minimal Georgi-Glashow SU(5)[3]; as is well known, this gives a very successful prediction for $\sin^2\theta(m_W)$. In minimal supersymmetric GUT's, where $N_{\rm H}\geq 2$ and even, the value of A is raised (for $N_{\rm H}=2$) from $\frac{1}{6}$ to $\frac{1}{5}$: This is the root of the problem with fitting the Weinberg angle. Increasing $N_{\rm H}$ to ≥ 4 makes matters worse.

Remarkably, putting $N_{\rm H}=2$ in Eq. (9), we can restore A to its "canonical" GQW value $A=\frac{1}{6}$ by setting $q=\frac{1}{3}$! The second term in Eq. (8) has coefficient B reduced by only $\frac{2}{27}$ and since it multiplies the small ratio α/α_s this reduces $\sin^2\theta(m_W)$ only by 0.005 hence maintaining excellent agreement. The price of extra Higgs doublets or new intermediate mass scales need not be paid.

By fixing q at $\frac{1}{3}$ we are now able to specify the Higgs representations that do the superheavy breaking.[23] To be specific, we choose a $\underline{756}$ H and an adjoint A along with their complex conjugates with VEV's $H_{77}{}^{[1236]}$ (see Ref. 4) for the $\underline{756}$ and $\langle A\rangle \propto \lambda_{15}$. Apart from $N_{\rm H}=2$ light doublets (coming from, for example, the adjoint A), all these particles are superheavy and do not affect the renormalization-group analysis.

Taking $N_{\rm H}=2$ and using

$$8\pi[C^2b_1+b_2-(1+C^2)b_3]\ln\left(\frac{M_{\rm GUT}}{m_w}\right)=\frac{1}{\alpha(m_w)}-\frac{1+C^2}{\alpha_s(m_w)}$$

we find $M_{\rm GUT}$ in the range $(8.8\times10^{12},\ 4.2\times10^{14}$ GeV) and $\sin^2\theta(m_W)$ in the range (0.204, 0.186) for α_s within the phenomenologically acceptable bounds (0.10, 0.19 GeV). This can be compared with the range (0.210, 0.189) for $\sin^2\theta(m_W)$ for the one-loop predictions of the nonsupersymmetric minimal SU(5) GUT for α_s in the same range as above.

This SU(7) model predicts (for dimension-six operators) a measurable proton lifetime similar to that of minimal SU(5). As pointed out in Refs. 18 and 24, there may also be dimension-five operators which, if not eliminated by discrete symmetries, predict $p\to K\bar{\nu}_\tau$, etc. decay modes whose partial rates are, however, supressed by high powers of small unknown Yukawa coupling constants.

We stress that we have made only the most naive one-loop estimates here. Because of the large number of superheavy thresholds, the new low-energy thresholds coming from the charge-shifted families and their scalar partners, etc.,[18] we expect the errors in the one-loop values of m_W and $\sin^2\theta$ to be large enough to permit complete agreement with experiment. (A full-blown two-loop calculation including the effects of Yukawa couplings[25] would be necessary to rule out this model.)

By requiring asymptotic freedom for α_s we find that the only other possibilities for fracton supersymmetric GUT's (or GUT's) are the trivial embeddings of the above SU(7) model in SU(N) [i.e., all $q_i=0$ in Eq. (3) for $i\geq 3$] and in O(14). Note that it is impossible to arrange Q in E_6 to include fractons.

We summarize the state of fracton supersymmetric GUT's as follows:

(1) The nontrivial embedding of SU(5) allows for agreement with the experimental value of $\sin^2\theta$, without intermediate mass scales or a proliferation of light Higgs doublets. Here, we have decided to take at face value the quoted angle $\sin^2\theta(m_W)=0.215\pm0.012$ and hence found a prediction of $\sin^2\theta(m_W)=0.245\pm0.015$ to be unacceptably high. Of course, the phenomenological analysis may change again as it has previously. If it settles on the present value, however, we now see the first justification for introducing particles with charge $0<q<1$, and in particular with $q=\frac{1}{3}e$. In that sense, the confirmation of the observations of fractional charge[1] could provide indirect evidence of supersymmetry.

(2) At least one fracton will be absolutely stable and hence could explain the observations of Ref. 1.

(3) Astrophysics provides phenomenological constraints on the production of fractons in the big bang, on subsequent annihilation processes, and on the present observed upper bound on their number density in matter. These topics are discussed in detail by Goldberg.[26]

(4) If, for instance, the stable fractons were heavy $q=-\frac{1}{3}$ leptons, then they could bind to the protons of hydrogen atoms and produce a shift $E_n\to\frac{4}{9}E_n$ in the spectral lines. Hence fractons may show up, with a distinctive signature, in the infrared spectra of Population II (i.e., metal-poor) stars. Concentrations of one part in 10^8 are about the level of the detection sensitivity of present astronomical infrared detectors.[27,28]

(5) The masses of fractons can be expected to be on the order of the mass scale of the breaking of the electroweak group and/or perhaps are as small as 50–100 GeV. Thus they may soon show

up in $\bar{p}p$ experiments at CERN or in e^+e^- at LEP.

It is a pleasure to thank B. Carney, H. Goldberg, and M. T. Vaughn for useful discussions. This work was supported in part under Contract No. DE-AS05-79ER10448 by the U. S. Department of Energy.

[1]G. S. LaRue, J. D. Phillips, and W. M. Fairbank, Phys. Rev. Lett. 46, 967 (1981), and references therein.

[2]A. de Rújula, R. C. Giles, and R. L. Jaffe, Phys. Rev. D 17, 285 (1978); R. Slansky, T. Goldman, and G. L. Shaw, Phys. Rev. Lett. 47, 887 (1981).

[3]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974). For a review see P. Langacker, Phys. Rep. 72, 185 (1981).

[4]H. Goldberg, T. W. Kephart, and M. T. Vaughn, Phys Phys. Rev. Lett. 47, 1429 (1981).

[5]L.-F. Li and F. Wilczek, Phys. Lett. 107B, 64 (1981).

[6]O. V. Kancheli and Dzh. L. Chkaruli, Pis'ma Zh. Eksp. Teor. Fiz. 33, 667 (1981) [JETP Lett. 33, 654 (1981)].

[7]J. Wess and B. Zumino, Phys. Lett. 49B, 52 (1974); J. Iliopoulos and B. Zumino, Nucl. Phys. B76, 310 (1974); S. Ferrara, J. Iliopoulos, and B. Zumino, Nucl. Phys. B77, 413 (1974); M. Grisaru, M. Rocek, and W. Siegel, Nucl. Phys. B158, 429 (1979); L. Girardello and M. Grisaru, Nucl. Phys. B194, 65 (1982).

[8]For a discussion see E. Witten, Nucl. Phys. B188, 513 (1981). For a list of references see L. Hall and I. Hinchliffe, Lawrence Berkeley Laboratory Report No. LBL-14020, 1982 (to be published).

[9]E. Gildener and S. Weinberg, Phys. Rev. D 13, 3333 (1976); S. Weinberg, Phys. Lett. 82B, 387 (1979).

[10]W. Marciano and G. Senjanovic, Phys. Rev. D 25, 3092 (1982). This paper contains references to the most recent phenomenological of $\sin^2\theta$.

[11]M. B. Einhorn and D. R. T. Jones, Nucl. Phys. B196, 475 (1982).

[12]For a general discussion of SU(N) supersymmetric GUT's see P. H. Frampton and T. W. Kephart, Nucl. Phys. B (to be published), and Phys. Rev. Lett. 48, 1237 (1982).

[13]P. H. Frampton, Phys. Lett. 88B, 299 (1979).

[14]H. Georgi, Nucl. Phys. B156, 126 (1979).

[15]See Appendix A of the first paper in Ref. 12.

[16]S. Weinberg, Phys. Rev. D 26, 287 (1982).

[17]P. Fayet, Nucl. Phys. B187, 184 (1981), and references therein.

[18]J. Ellis, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B202, 43 (1982).

[19]S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D 24, 1681 (1981).

[20]S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981).

[21]N. Sakai, Z. Phys. C 11, 153 (1981).

[22]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[23]It has been argued on the basis of the observation in Ref. 1 that the observed fractons have electric charge $e/3$; see J. P. Schiffer, Phys. Rev. Lett. 48, 213 (1982). Since these fractons appear to be mobile they may well be leptons. For a detailed discussion of fractional charge in matter see K. S. Lackner and G. Zweig, California Institute of Technology Report No. CALT 68-865, 1981 (to be published).

[24]S. Dimopoulos, S. Raby, and F. Wilczek, University of California, Santa Barbara Report No. NSF-ITP-82-08 (to be published).

[25]M. Fischler and J. Oliensis, to be published; M. Machacek and M. T. Vaughn, to be published. See also M. T. Vaughn, in *Proceedings of the Third Workshop on Grand Unification*, edited by P. H. Frampton, S. L. Glashow, and H. Van Dam (Birkhäuser, Basel, Switzerland, to be published).

[26]H. Goldberg, Phys. Rev. Lett. 48, 1518 (1982).

[27]These shifted spectral lines are presently being searched for by C. Sneden.

[28]B. Carney, private communication.