Comment on "Resolution of the Einstein-Podolsky-Rosen and Bell Paradoxes"

Pitowsky[1] makes use of a remarkable function $s_0(x)$ that assumes the values $\pm\frac{1}{2}$ on the surface $S^{(2)}$ of the unit sphere, and which he argues has the following property: Let $R_1, R_2, \ldots$ be a random[2] sequence of rotations. For any pair of points x,y in $S^{(2)}$ let $N_n(x,y)$ be the number of rotations R_i with $i \leq n$ for which $s_0(R_i(x))$ differs from[3] $s_0(R_i(y))$. Then for large n the ratio $N_n(x,y)/n$ approaches a limiting value $g(x,y)$ given by $\sin^2(\theta_{xy}/2)$, where θ_{xy} is the angular distance between x and y. Bell's theorem would appear to rule out a function with these properties, but Pitowsky argues that s_0 fails to satisfy certain conditions of Lebesgue measurability without which, he implies, the theorem cannot be proved.

It is possible, however, to reach Bell's conclusion in a way that quite clearly requires neither the measurability of s_0 on the sphere, nor the measurability in O(3) of sets of rotations such as those with $s_0(R(x)) \neq s_0(R(y))$, nor any other properties of s_0 beyond those I have specified above. Here is the argument:

For any three points x, y, and z, and any rotation R, it is impossible for all three of the numbers $s_0(R(x))$, $s_0(R(y))$, and $s_0(R(z))$ to be different, since s_0 assumes only two values. Consequently $N_n(x,y) + N_n(y,z) + N_n(z,x)$ cannot exceed $2n$. Thus if the limits exist, $g(x,y) + g(y,z) + g(z,x)$ cannot exceed 2, and hence at least one of the three g's cannot exceed $\frac{2}{3}$. But if we take, for example, x, y, and z to be 120° apart in the equatorial plane, then all three g's are given by $\sin^2(60°) = \frac{3}{4}$. There can therefore be no function s_0 with the properties specified above, regardless of how the sequence of rotations is chosen.

My argument does not necessarily imply the falsehood of the extraordinary existence theorem which Pitowsky quotes at the start of his argument, but if the theorem is indeed true then for some values of x, y, and z, any denumerable sequence of rotations "random" enough to behave as Pitowsky requires for the pairs xy and yz, will necessarily be perversely nonrandom for the pair xz.

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[1] Itamar Pitowsky, Phys. Rev. Lett. 48, 1299 (1982).

[2] My point is independent of the precise sense in which the sequence is random.

[3] Pitowsky phrases his argument in terms of the number of rotations for which $s_0(R(x)) = s_0(R(y)) = \frac{1}{2}$. My way of putting it is easily shown to be equivalent.

