cally spectral width due to transit time and second-order Doppler effects. We have also demonstrated the use of laser deceleration as a highly efficient means of producing a monoenergetic atomic beam which should have important applications in spectroscopy, collision work, and other areas which use atomic beams. Slow atoms could be confined in optical traps by the sudden turning on of a trapping laser when the atoms were near the center of the trap. Alternatively, the atoms could be further decelerated and focused[7] into a continuously operating trap.[5] The atoms could also be trapped by magnetic fields, in much the same way that cold neutrons are trapped.[10] Such traps have been constructed with well depths greater than the energy of our slowest atoms.

We are continuing to work on improving the slow-atom density, which should allow the observation of still slower atoms. We are also pursuing applications of slow atoms to trapping and ultrahigh-resolution spectroscopy.

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[(a)]Permanent address: Physics Department, State University of New York, Stony Brook, Stony Brook, N.Y. 11794.

[1]T. Hänsch and A. Schawlow, Opt. Commun. 13, 68 (1975); D. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20, 637 (1975).

[2]D. Wineland *et al.*, Phys. Rev. Lett. 40, 1639 (1978); W. Neuhauser *et al.*, Phys. Rev. Lett. 41, 233 (1978).

[3]W. D. Phillips and H. Metcalf, Phys. Rev. Lett. 48, 596 (1982).

[4]Nonresonant laser deceleration has also been reported by S. V. Andreev *et al.*, in Pis'ma Zh. Eksp. Teor. Fiz. 34, 463 (1981) [JETP Lett. 34, 442 (1981)]. They reported a slight deceleration (about 10% of the initial velocity) compared to the larger, resonant deceleration (40%) reported in Ref. 3.

[5]A. Ashkin, Phys. Rev. Lett. 40, 729 (1978).

[6]The slow-atom peak is not split by the excited-state hfs because optical pumping and relative transition strengths discriminate against observing transitions to any states but $F = 3$.

[7]J. E. Bjorkholm, R. R. Freeman, A. Ashkin, and D. B. Pearson, Phys. Rev. Lett. 41, 1361 (1978).

[8]N. F. Ramsey, *Molecular Beams* (Oxford, London, 1956), pp. 404–407.

[9]A. Ashkin and J. P. Gordon, Opt. Lett. 4, 161 (1979).

[10]K.-J. Kügler, W. Paul, and U. Trinks, Phys. Lett. 72B, 422 (1978).

Double Ionization of Helium by Protons and Electrons at High Velocities

J. H. McGuire

Department of Physics, Kansas State University, Manhattan, Kansas 66506

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Double ionization of helium at high projectile velocities, v, is considered in terms of two mechanisms. In the shakeoff mechanism the ratio of double- to single-ionization cross sections, σ^{ii}/σ^{i}, is independent of v. In the two-step mechanism it is shown that $\sigma^{ii}/\sigma^{i} \sim (v^2 \ln v)^{-1}$. Combining amplitudes gives a reasonable fit to the observed velocity dependence as well as an explanation of the observed factor-of-2 differences between double ionization of helium by protons and electrons near $v \simeq 10 v_{\mathrm{Bohr}}$.

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At high velocities the physics of atomic collisions becomes relatively simple. For example, single ionization of atoms by charged particles is fairly well understood[1] in terms of the Born approximation at high projectile velocities, $v > v_{\mathrm{orbit}}$. Here the total cross sections for single ionization by protons and electrons are the same and vary as $v^{-2}\ln v$. However, double ionization at high velocities is not so well understood, even for the simplest two-electron targets. Over the past twenty years double ionization of helium has been observed by a number of independent groups[2-9] at v up to almost $40 v_B$ (where v_B is the Bohr velocity), or 20 times the electron orbit velocity in helium. There has been no satisfactory explanation of the velocity dependence of these data. And there has been no explanation of the ~50% differences between double ionization by electrons and protons.

In this Letter double ionization of helium is analyzed in terms of two mechanisms: shakeoff[10-12] (SO) and two-step[13] (TS). Both mechanisms are found to be of comparable magnitude near $v \sim 10 v_B$. Nevertheless, neither mechanism alone can explain the energy dependence observed. However, combining amplitudes for both mechanisms not only gives a reasonable energy dependence, but also indicates that cross sections for double ionization by protons and electrons differ.

Let us first consider briefly the SO mechanism.[10-12] This corresponds to single ionization followed by final-state rearrangement. The rearrangement, or shakeoff, is due to a change in electronic screening of the nucleus when the first electron is removed. Hence the initial (screened) bound state of the second electron, $\varphi_2{}^i$, is not orthogonal to the final (unscreened) continuum state, $\varphi_2{}^f$, i.e., $\langle\varphi_2{}^f|\varphi_2{}^i\rangle \neq 0$. Thus (retaining the notion of distinguishable electrons for the moment) the probability amplitude for double ionization by shakeoff is given[10-12] by

$$a_{SO} = i\int_0^\infty \exp[i(E_f - E_i)t]\langle\varphi_1{}^f|V|\varphi_1{}^i\rangle dt\,\langle\varphi_2{}^f|\varphi_2{}^i\rangle = -Z_p c_1. \tag{1}$$

In Eq. (1) it has been noted that $a_{SO} \sim -Z_p$ (where Z_p is the charge of the projectile) since a_{SO} is proportional to V, the interaction potential between the projectile and the target electron (the minus sign corresponding to the sign of V).

From Eq. (1) it is evident that the double-ionization cross section, σ^{ii}, is proportional to the single-ionization cross section, $\sigma_1{}^i$, and for shakeoff,

$$R = \sigma^{ii}/\sigma^i = |\langle\varphi_2{}^f|\varphi_2{}^i\rangle|^2, \tag{2}$$

which is independent of the projectile velocity, v. Some calculations[10-12] corresponding to Eq. (2) have been reported. However, it has been found[12,14] that R is sensitive to electron-electron correlation in both the initial- and final-state wave functions. Although no definitive calculation of R in the SO mechanism exists, a rough estimate is $R \sim 5\times10^{-3}$.

Now consider the TS mechanism,[13] where both electrons are ejected via interaction with the projectile during a single collision. Here, if the probability of ionizing electron 1 (2) is P_1 (P_2), then the probability for ionizing both electrons is P_1P_2. The probability amplitudes are also multiplicative so that the probability amplitude for double ionization in the TS mechanism at high velocities, where first-order pertubation theory is applicable, is given by

$$a_{TS} = a_1 a_2 = \{i\int_0^\infty \exp[i(E_f - E_i)t]\langle\varphi_1{}^f|V|\varphi_1{}^i\rangle dt\}\{i\int_0^\infty \exp[i(E_f - E_i)t]\langle\varphi_2{}^f|V|\varphi_2{}^i\rangle dt\} = Z_p{}^2 c_2. \tag{3}$$

Here it has been noted that $a_{TS} \sim V^2 \sim Z_p{}^2$. Like c_1 in Eq. (1), c_2 is a complex number whose phase and exact magnitude depend on detailed calculation.

The v dependence of σ^{ii} in the TS mechanism, which has not been previously given, is now determined. For two-electron atoms where both electrons are counted and $|a_1|^2 = |a_2|^2 = |a|^2$ in Eq. (3), one has

$$\sigma^i = 2\pi\int_0^\infty (|a_1|^2 + |a_2|^2) b\,db = 4\pi\int_0^\infty |a|^2 b\,db \tag{4a}$$

and

$$\sigma^{ii} = 2\pi\int_0^\infty |a|^4 b\,db, \tag{4b}$$

where b is the impact parameter. Now the probability amplitude, $a(b)$, is related[15] to the scattering amplitude $f(q)$, by

$$a = (2\pi v)^{-1} i\int_0^\infty J_0(q_\perp b) f(q) q_\perp dq_\perp, \tag{5}$$

where q is the magnitude of the momentum transfer and $q_\perp$ is the magnitude of the component of $\vec{q}$ perpendicular to the beam direction. Here J_0 is a zeroth-order cylindrical Bessel function.

It is well known[1,16,17] that the ln$v$ behavior in σ^i at high velocity, v, comes from small q (or large b). Furthermore, for dipole-allowed transitions[16] near $q=0$, $f(q) = cZ_p q^{-1}$, where c is a constant. Hence, with $q^2 = q_0{}^2 + q^2$ one has[18]

$$a = \frac{icZ_p}{2\pi v}\int_0^\infty \frac{J_0(q_\perp b) q_\perp dq_\perp}{[q_0{}^2 + q_\perp{}^2]^{1/2}} = \frac{\bar{c}Z_p}{v}\frac{q_0}{[q_0 b]^{1/2}} K_{1/2}(q_0 b) = C\frac{Z_p}{v}\frac{\exp(-q_0 b)}{Z_T b}, \tag{6}$$

where Z_T is the nuclear charge of the target and C is a constant.

Using Eq. (6) in Eq. (4a) one may obtain the high-energy behavior of the cross section for single ionization. Contributions from small b [which have not been properly included in Eq. (6)] may be ignored since $bP(b)$ goes to zero at small b. Thus, with $q_0 \sim Z_T^2/v$, $\bar{b} = Z_T b$, and $\bar{q}_0 = q_0/Z_T$, one has[19]

$$\sigma^i = C\int_{b_{\min}\sim Z_T^{-1}}^{\infty} \left| \frac{Z_p}{v} \frac{\exp(-q_0 b)}{Z_T b} \right|^2 b\,db = C Z_T^{-2}\left(\frac{Z_p}{v}\right)^2 \int_{\bar{b}\sim 1}^{\infty} \exp(-2\bar{q}_0\bar{b})\frac{d\bar{b}}{\bar{b}}$$

$$\xrightarrow[v\to\infty]{} C Z_T^{-2}\left(\frac{Z_p}{v}\right)^2 \int_{\bar{b}\sim 1}^{\sim 1/q_0} \exp(-2\bar{q}_0 b)\frac{d\bar{b}}{b}$$

$$\to A Z_T^{-2}\left(\frac{Z_p}{v}\right)^2 \ln(q_0/Z_T) = A\left(\frac{Z_T Z_p}{v}\right)^2 Z_T^{-4}\ln(Z_T/v), \tag{7}$$

where A is the first Bethe constant.[1]

The high-velocity dependence of the double-ionization cross section may be similarly[20] obtained with Eq. (6) in Eq. (4b), namely

$$\sigma^{ii} = C\left(\frac{Z_p}{v}\right)^4 \int_{b_{\min}\sim Z_T^{-1}}^{\infty} \left|\frac{\exp(-q_0 b)}{Z_T b}\right|^4 b\,db = C\left(\frac{Z_p}{v}\right)^4 \int_{\bar{b}\sim 1}^{\infty} \left|\frac{\exp(-\bar{q}_0\bar{b})}{\bar{b}}\right|^4 \frac{\bar{b}\,d\bar{b}}{Z_T^2}$$

$$\xrightarrow[v\to\infty]{} \left(\frac{Z_p}{v}\right)^4 Z_T^{-2}\left\{A + \frac{B}{Z_T} q_0\right\} \xrightarrow[v\to\infty]{} \left(\frac{Z_p Z_T}{v}\right)^4 Z_T^{-6} A, \tag{8}$$

where $q_0 \sim Z_T^2/v$, and A and B are constants. It is noted that $P^2 = |a|^4$ peaks at smaller b than P. Correspondingly, the logarithmic divergence in σ^i as $q_0 \to 0$ does not occur in σ^{ii} since $\int b^{-1} db$ does not occur in Eq. (8).

In order to estimate $R = \sigma^{ii}/\sigma^i$ with the TS mechanism, tabulated[21] values of the ionization probability for protons on helium were used. At $v = 6v_B$, this probability is about 10^{-2}, so that from Eqs. (4a) and (4b) one finds $R \sim 5\times 10^{-3}$.

Now consider Fig. 1, where the data were taken under single-collision conditions. In the SO mechanism R is independent of energy, and in the TS mechanism R varies as $(v^2 \ln v)^{-1}$, as sketched in Fig. 1. It is apparent that neither mechanism alone fits the velocity dependence observed. However, at the lower velocities the data are consistent with $(v^2 \ln v^2)^{-1}$, and at the higher velocities the data are consistent with a constant energy dependence (with a value lower than 5×10^{-3}). Combining the SO and TS mechanisms gives a plausible fit through the data.

However, the SO and TS mechanisms do not correspond to distinguishable processes. Consequently it is appropriate to add the amplitudes, i.e.,

$$|a|^2 = |a_{\rm SO} + a_{\rm TS}|^2 = |-Z_p c_1 + Z_p^2 c_2|^2. \tag{9}$$

Now it is evident for protons ($Z_p = +1$) that $|a|^2 = |c_1 - c_2|^2$, while for electrons ($Z_p = -1$), $|a|^2 = |c_1 + c_2|^2$. In other words, a difference between the cross sections for protons and electrons is apparent due to interference between amplitudes

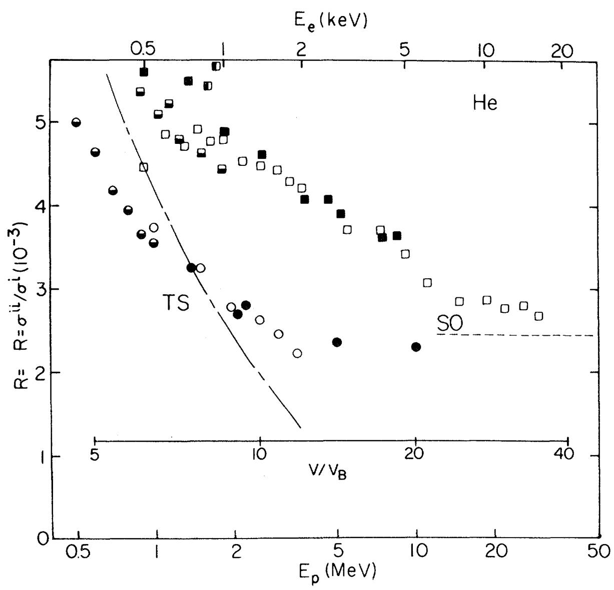

FIG. 1. Ratio, R, of double- to single-ionization cross sections in helium vs projectile velocity (in units of $v_B = 2.2\times 10^9$ cm/sec). The proton data are due to Haugen *et al.* (Ref. 2), closed circles; Wexler (Ref. 4), open circles; and Puckett and Martin (Ref. 3), half-open circles. The electronic data are due to Schram, Boerboom, and Kistemaker (Ref. 5), open squares; Nagy, Skutlartz, and Schmidt (Ref. 9), closed squares; Adamczyk *et al.* (Ref. 6), squares divided into horizontal halves; and Harrison (Ref. 7), squares divided into vertical halves. The curve TS denotes the $(v^2 \ln v)^{-1}$ velocity dependence of the two-step mechanism, and curve SO represents the constant velocity dependence of the shakeoff mechanism. Amplitudes for these mechanisms interfere.

of the SO and TS mechanisms. The coefficients c_1 and c_2 are complex numbers whose phases and exact magnitudes are not given here. Nevertheless, it is clear that the magnitudes of both mechanisms are comparable, and that the relative sign is opposite for protons and electrons.

In order to obtain total cross sections one intergrates $|a|^2$ over the momentum of each ejected electron, $\vec{k}_1$ and $\vec{k}_2$, as well as the momentum transfer, q (or equivalently the impact parameter, b), of the projectile. Differences in the differential cross sections for ionization by protons and electrons can be no smaller than differences in the total integrated cross section. In particular, the greatest differences in the differential cross sections are expected in between regions where the SO and TS mechanisms are dominant. This suggests further experimental observations of interference between the SO and TS mechanisms at $v \simeq 10v_B$.

At large impact parameters (corresponding to small q or small-angle scattering) the SO mechanism, proportional to σ^i, is expected to dominate over the TS mechanism which, as noted, peaks at smaller impact parameters, $b \sim Z_T^{-1}$. Hence in helium one may expect to find the largest interference at scattering angles corresponding to impact parameters between $\sim \frac{1}{2}a_0$ and several a_0 (where $a_0 = 5.3 \times 10^{-9}$ cm is the Bohr radius). Of course the corresponding scattering angles for protons and electrons are quite different.

In terms of the momenta of the ejected electrons, most electrons are ejected with a momentum $k \sim \frac{1}{2}Z_T$ for direct ionization by projectile impact. Hence in the TS mechanism both electrons are most often ejected from the target with $k_1 \simeq k_2 \simeq \frac{1}{2}Z_T$. In the SO amplitude, the overlap of the continuum wave function $\varphi_{k_2}{}^f$ with the initial-state wave function $\varphi_1{}^i$ will be greatest when k_2 is small, since $\varphi_{k_2}{}^f$ oscillates slowly[22] for small k_2. Hence the SO mechanism is expected to dominate when $k_1 \sim \frac{1}{2}Z_T$ and $k_2 < \frac{1}{2}Z_T$. Consequently, the interference is expected to be maximum when one electron is ejected with momentum $\sim \frac{1}{2}Z_T$, and the other electron is emitted with momentum less than $\frac{1}{2}Z_T$.

Finally, it is pointed out that there may also be differences in the double ionization of helium by positrons and electrons, with an equivalence between positrons and proton cross sections in this velocity regime.

In summary, the observed energy dependence of the cross section for the double ionization of helium near $v \simeq 10v_B$ and observed differences in this cross section for ionization by proton and electron impact have been explained in terms of a combination of amplitudes for a shakeoff mechanism and a two-step mechanism. This explanation suggests that effects due to the interference of these two mechanisms may be better defined by further studies of differential cross sections, as well as studies of single and double ionization by positron impact, leading to an understanding of simple two-step scattering mechanisms at high velocities.

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[1]M. Inokuti, Rev. Mod. Phys. 43, 297 (1971).

[2]H. K. Haugen, L. H. Andersen, P. Hvelplund, and H. Knudsen, Phys. Rev. A, to be published.

[3]L. J. Puckett and D. W. Martin, Phys. Rev. A 1, 1432 (1970).

[4]S. Wexler, J. Chem. Phys. 41, 1714 (1964), and 44, 2221 (1966).

[5]B. L. Schram, Physica (Utrecht) 32, 197 (1966); B. L. Schram, A. J. H. Boerboom, and J. Kistemaker, Physica (Utrecht) 32, 185 (1966).

[6]B. Adamczyk, A. J. H. Boerboom, B. L. Schram, and J. Kistemaker, J. Chem. Phys. 44, 4640 (1966).

[7]H. Harrison, Ph.D. thesis, Catholic University, Washington, D. C., 1966 (unpublished).

[8]V. Schmidt, N. Sander, and H. Kuntzemüller, Phys. Rev. A 13, 1743 (1976).

[9]P. Nagy, A. Skutlartz, and V. Schmidt, J. Phys. B 13, 1249 (1980).

[10]M. H. Mittleman, Phys. Rev. Lett. 16, 498, 779 (1966). The value for the SO cross section used in Fig. 1 is lower than Mittleman's estimate. Correlation in the final state could give high velocity limits for single ionization by electrons (and protons) that differ from the photoionization limit.

[11]F. W. Byron and C. J. Jochain, Phys. Rev. Lett. 16, 1139 (1966), and Phys. Rev. 164, 1 (1967).

[12]T. Åberg, in *Proceedings of the International Conference on Inner Shell Ionization Phenomena and Future Applications, Atlanta, Georgia, 1972*, edited by R. W. Fink, S. T. Manson, J. M. Palms, and R. V. Rao, CONF-720404 (U. S. Atomic Energy Commission, Oak Ridge, Tenn., 1973), p. 1509; C. A. Quarles, *ibid.*, p. 1617.

[13]J. H. McGuire and L. Weaver, Phys. Rev. A 16, 41 (1977), and references therein.

[14]M. R. C. McDowell and J. P. Coleman, *Introduction to the Theory of Ion-Atom Collisions* (North-Holland, Amsterdam, 1970), pp. 337–339.

[15]R. McCarroll and A. Salin, C.R. Acad. Sci. 263, 329 (1966); Ref. 14, appendix 4.1.

[16]Ref. 14, Chap. 7.

[17]Cf. Fig. 18 of Ref. 1.

[18]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1965), Eq. (6.565-4); M. A. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (National Bureau of Standards, Washington, D. C., 1964), Eq. (10.2.17).

[19]Gradshteyn and Ryzhik, Ref. 18, Eq. (3.315-4).

[20]The high-velocity limit for the cross section for ionizing n electrons also may be similarly determined, namely $\sigma^n \sim Z_T^{-2}(Z_p/Z_T)^{2n}(Z_T/v)^{2n}$ for $n > 1$. The classical and quantum cross sections have the same high-energy dependence for $n > 1$.

[21]J. M. Hansteen, O. M. Johnsen, and L. Kocback, At. Data Nucl. Data Tables 15, 305 (1975).

[22]M. Day, private communication.