Thermal Fluctuations in Resonant Motion of Fluxons on a Josephson Transmission Line: Theory and Experiment

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The radiation emission from long and narrow Josephson tunnel junctions dc-current biased on zero-field steps has been ascribed to resonant motion of fluxons on the transmission line. Within this dynamic model a theoretical expression for the radiation linewidth is derived from a full statistical treatment of thermal fluctuations in the fluxon velocity. The result appears to be very general and is corroborated by experimental determination of linewidth and frequency of radiation emitted from overlap Nb-I-Pb junctions.

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Recently there has been an increasing interest in the fluxon propagation in a long and narrow overlap Josephson tunnel junction current biased on a zero-field step.[1,2] The resonant fluxon motion gives rise to an emitted radiation with frequency determined by the velocity of the fluxon and the length of the junction. The linewidth of the radiation is given by the thermal fluctuation in the fluxon velocity. From an experimental point of view the very narrow (~ 1 kHz) Josephson radiation (at 10 GHz) attracts interest for microwave applications.[3] Theoretically the interest lies in the possibility of providing specific solutions to the perturbed sine-Gordon equation which is assumed to govern the fluxon motion.[1] We derive a Langevin equation of motion which allows us to determine the linewidth of the radiation emitted from the end of the transmission line. Although the problem cannot be solved analytically we show that the theoretical quantities that determine the linewidth can be expressed in terms of the static and dynamic resistance derived from the dc I-V characteristic at the bias point. Our result for the full half-power linewidth of the mth harmonic of the fundamental frequency $\nu_0=\frac{1}{2}V_{\rm dc}/\varphi_0$ (modified Josephson relation[2]) is

$$\Delta\nu_m=m^2\frac{\pi kT}{\varphi_0^2}\frac{R_D^2}{V_{\rm dc}}I_{\rm dc}, \tag{1}$$

where $V_{\rm dc}$, $I_{\rm dc}$, and R_D are the voltage, the current, and the dynamic resistance at the bias point and φ_0 is the flux quantum $h/2e$. The result resembles published expressions for small tunnel junctions[4] and point contacts and microbridges.[5]

The quantum phase difference $\varphi(x,t)$ between the two superconductors of the Josephson junction is governed by the perturbed sine-Gordon equation[6]

$$\varphi_{xx}-\varphi_{tt}-\sin\varphi=\eta+\alpha\varphi_t+n(x,t), \tag{2}$$

where x is expressed in units of the Josephson penetration depth $\lambda_{\rm J}=(\hbar/2eJ\mu_0d)^{1/2}$ and t in units of the reciprocal maximum Josephson plasma frequency ω_p^{-1}, $\omega_p=(2eJ/\hbar C)^{1/2}$. d is the magnetic thickness of the oxide layer (sum of the thickness of the oxide layer and the London penetration depths of the two superconductors), J is the maximum Josephson current density, and C is the capacitance per unit area. The right-hand side of Eq. (2) represents the perturbations. The first term is the normalized bias current $\eta=I_{\rm dc}/JWL$, where W is the width and L the length of the junction ($W\ll\lambda_{\rm J}\ll L$). The second term is the loss term given by $\alpha\omega_p=1/rC$, where r^{-1} is the normal-state conductance per unit area. The third term represents internal thermal ($eV_{\rm dc}\ll kT$) noise connected to the loss.

A fluxon is a localized 2π kink (*from* $-\arcsin\eta$ *to* $2\pi-\arcsin\eta$) in the phase difference $\varphi(x,t)$. The rest energy of the fluxon is $E_0=8\hbar JW\lambda_{\rm J}/2e$. An equation of motion for the fluxon may be obtained by considering the momentum p (normalized to $E_0/\lambda_{\rm J}\omega_p$),

$$p=-\tfrac{1}{8}\int_{-\infty}^{\infty}\varphi_x\varphi_t\,dx. \tag{3}$$

Differentiating p with respect to time and using Eq. (2) yields[6]

$$dp/dt=-\alpha p+\tfrac{1}{4}\pi\eta+\epsilon(t), \tag{4}$$

where $\epsilon(t)$ in this Langevin type equation is the noise term stemming from $n(x,t)$.

With neglect of the thermal noise [$n(x,t)$ and hence $\epsilon(t)$], Eq. (2) has stationary 2π-kink solutions $\varphi^u(x,t)$ moving with the normalized velocity u. No analytic expression for $\varphi^u(x,t)$ has been given but the momentum is [Eq. (4)] *exactly* given by

$$p(u) = \pi\eta/4\alpha = u\gamma(u) = \tfrac{1}{8}u \int_{-\infty}^{\infty} (\varphi_x{}^u)^2 dx. \qquad (5)$$

In the last step we have used Eq. (3) together with $\varphi_t{}^u = -u\varphi_x{}^u$. Equation (5) defines the function $\gamma(u)$ which for *small* perturbations is the Lorentz factor $(1-u^2)^{-1/2}$.

The noise term $n(x,t)$ in Eq. (2) introduces fluctuations in the momentum and accordingly in the velocity of the fluxon. We assume a Gaussian white noise with average $\langle n(x,t)\rangle = 0$ and autocorrelation function[7]

$$\langle n(x,t)n(x',t')\rangle = (16\alpha kT/E_0)\delta(x-x')\delta(t-t'), \qquad (6)$$

the δ's being Dirac delta functions. The noise term in Eq. (4) $\epsilon(t)$ is [$\langle\epsilon(t)\rangle = 0$]

$$\epsilon^u(t) = \tfrac{1}{8}\int_{-\infty}^{\infty} \varphi_x{}^u(x,t)n(x,t)\,dx \qquad (7)$$

with the autocorrelation function $R_\epsilon(t-t')$ [from Eq. (6)],[8]

$$R_\epsilon(t-t') = \langle\epsilon(t)\epsilon(t')\rangle = (2\alpha kT/E_0)\gamma(u)\delta(t-t'), \qquad (8)$$

where $\gamma(u)$ is defined by Eq. (5). The constant in Eq. (6) is determined by applying the fluctuation dissipation theorem for small velocities (Brownian motion).[9] The appearance of the factor $\gamma(u)$ in Eq. (8) reflects the Lorentz contraction of the fluxon. The resulting fluctuations are small since kT/E_0 is typically 10^{-4} to 10^{-5}.

The power spectrum for $\epsilon(t)$ is

$$S_\epsilon(\omega) = \int_{-\infty}^{\infty} dt\, e^{-i\omega\tau} R_\epsilon(t) = (2\alpha kT/E_0)\gamma(u). \qquad (9)$$

The standard theory of stochastic processes[10] yields the power spectrum for $\Delta p = p - p_0$,

$$S_{\Delta p}(\omega) = \frac{2\alpha kT}{E_0}\frac{\gamma(u_0)}{\omega^2+\alpha^2}, \qquad (10)$$

where the subscript zero refers to the bias point, u_0, η_0, and p_0 being related by the power balance Eq. (5).

Since the noise term is small, the fluctuation in the velocity $\Delta u = u - u_0$ has the power spectrum

$$S_{\Delta u}(\omega) = (\partial u_0/\partial p_0)^2 S_{\Delta p}(\omega). \qquad (11)$$

We only consider the radiation emitted at one end of the junction. When a fluxon is reflected as an antifluxon or vise versa, a voltage pulse appears there and is repeated every time the fluxon/antifluxon has traveled a distance $2l$ ($l = L/\lambda_J$). The instantaneous frequency of the fluxon motion $\nu = u/2l$ fluctuates around the bias value $\nu_0 = u_0/2l$. The power spectrum of the deviation $\Delta\nu = \nu - \nu_0$ is given by

$$S_{\Delta\nu}(\omega) = (2l)^{-2} S_{\Delta u}(\omega). \qquad (12)$$

Collecting the results of Eqs. (9)–(12) we obtain

$$S_{\Delta\nu}(\omega) = \left(\frac{1}{2l}\right)^2\left(\frac{\partial u_0}{\partial p_0}\right)^2 \gamma(u_0)\frac{kT}{E_0}\frac{2\alpha}{\omega^2+\alpha^2} = \frac{\Delta f}{2\pi}\frac{\alpha^2}{\omega^2+\alpha^2}, \qquad (13)$$

defining a frequency Δf. From standard frequency modulation noise theory[11] it follows that the full half-power linewidth of the radiation $\Delta\nu_1$ is equal to Δf if $\pi\Delta f \ll \alpha$, which here is the case. Expressed in unnormalized quantities[8]

$$\Delta\nu_1 = \left(\frac{2\pi}{2l}\right)^2\left(\frac{\partial u_0}{\partial p_0}\right)^2 \frac{p_0}{u_0}\frac{2}{\alpha}\frac{kT}{E_0}\frac{\omega_p}{2\pi}. \qquad (14)$$

The model quantities $p_0/u_0 = \gamma(u_0)$ and $\partial p_0/\partial u_0$, which in general cannot be expressed analytically, can be related to the static and dynamic resistance of the junction at the bias point. The average voltage across the junction is $V_{dc} = \hbar\omega_p \times \overline{\varphi}_t/2e$, where the time average of φ_t is $\overline{\varphi}_t = 2 \times 2\pi u_0/2l$ because of a phase shift of $2\times 2\pi$ when a fluxon is reflected as an antifluxon (modified Josephson relation). Within the model the static resistance is given by

$$R_S = \frac{V_{dc}}{I_{dc}} = \frac{\pi^2}{2l}R\frac{1}{\gamma(u_0)} \qquad (15a)$$

and the dynamic resistance by

$$R_D = \frac{\partial V_{dc}}{\partial I_{dc}} = \frac{\pi^2}{2l}R\frac{\partial u_0}{\partial p_0}, \qquad (15b)$$

where $R = r/WL$ is the normal-state resistance of the junction. With these resistances the linewidth Eq. (14) can be written

$$\Delta\nu_1 = \frac{\pi kT}{\varphi_0{}^2}\frac{R_D{}^2}{R_S}. \qquad (16)$$

It is interesting to note that our expression Eq. (16) is identical to the result of Ref. 4 and that of Ref. 5 (except for a factor of 4 due to the modified Josephson relation) despite the quite different nature of the three models. Intuitively the origin of the forms is simple. A current noise spectrum $\sim 2kT/R_S$ is via the dynamic resistance converted into a voltage noise spectrum $\sim R_D^2 2kT/R_S$ which via the (modified) Josephson relation is transformed into a frequency noise spectrum $\sim R_D^2 2kT/R_S(2\varphi_0)^2$. The last step using FM noise theory, in order to reach Eq. (16) [or (1)], is *not* trivial. The absence, however, of model parameters in Eq. (16) indicates that the result as inferred from the dc I-V characteristic is more general than the derivation.

The experimental determination of emitted power, linewidths, and frequencies was performed with a sensitive (overall single-sideband noise figure 5.2 dB), broadband (8–11 GHz) microwave receiver. Standard phase locking of the local oscillators allowed us to measure absolute frequency and linewidth to better than ~ 500 Hz. A series of microwave rejection filters and isolators, RF filtering, and passive electrical and magnetic shielding prevented unwanted signals and fields from interfering with the measurements. The samples were Nb-I-Pb tunnel junctions in an overlap goemetry and the first harmonic of the radiation remained within the frequency of the receiver when biased at the first zero-field step. The microwave coupling was facilitated by using a 50-Ω stripline and a very thin and narrow (~ 0.5 mm) silver stub mounted very close (~ 12–15 μm) to one end of the junction. The detected power increased approximately in proportion to the inverse square of this distance and was only weakly frequency dependent. The power varied with the bias conditions, and the maximum received power with this preliminary coupling configuration amounted to 5×10^{-14} W.

The corresponding values of linewidth $\Delta\nu_1$ and frequency ν_0 were recorded with the junction dc-current biased on the first zero-field step maintaining constant temperature. The temperature had to be stabilized to within ~ 10 μK. Extreme stability of the bias parameters was crucial. Typical frequency tunings were ~ 2 MHz/μA (dynamic resistance ~ 10 mΩ) and ~ 0.1 MHz/mK. A small external magnetic field could be applied and its strength was chosen so that the critical supercurrent attained its maximum value, experimentally corresponding to an extremum in the frequency ν_0 (zero magnetic tuning).[8] In this situation both first zero-field steps ($N = \pm 1$) were identical within experimental accuracy.

The shape of the dc I-V curve was inferred from an absolute counting of the frequency $\nu_0(I_{dc})$ relying on the modified Josephson relation and hence only the absolute and relative calibration of the current axis limits the accuracy. The detected radiation was displayed on a spectrum analyzer and recorded. The full half-power linewidth $\Delta\nu_1(I_{dc})$ was determined by repeating the trace with a 3.0-dB attenuator (300 K) inserted in the waveguide between the junction and the receiver and correcting for the corresponding change in the background noise level.

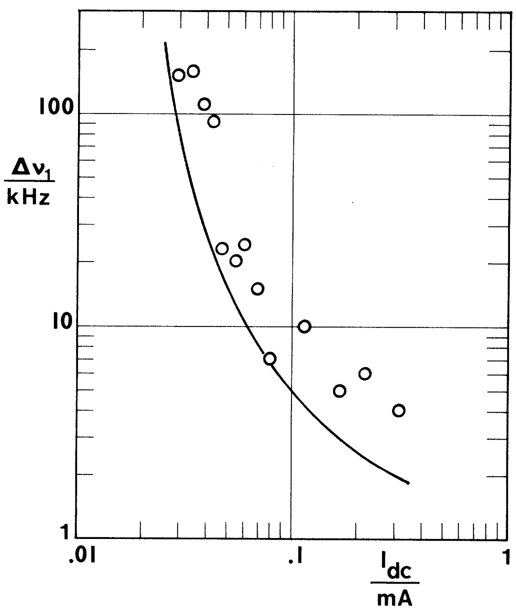

FIG. 1. The full half-power linewidth of 9.6–9.9 GHz radiation emitted from an overlap Nb-I-Pb Josephson junction biased at the first zero-field step vs the bias current. The solid curve is the theoretical result Eq. (16) and the points are experimentally determined linewidths at 4.1 K. $L = 490\ \mu$m, $W = 70\ \mu$m, and $L/\lambda_J = 1.5$.

Figure 1 shows experimentally determined linewidths versus bias current I_{dc} (points) at 4.1 K together with the theoretical result Eq. (16) (solid curve). The resistances entering Eq. (16) are determined from the measured frequencies $\nu_0(I_{dc})$. Preliminary measurements on junctions with different lengths ($l \simeq 2-12$) and at 2.1 K also showed good agreement with theory. At 2.1 K the experimental voltage resolution exposed step regions with fine structure as well as hysteresis.[8]

We emphasize that the theoretical result Eq. (16) may be expressed in terms of measurable quantities. In that way we have avoided the determination of $\gamma(u_0)$ and $\partial p_0/\partial u_0$. The experimental form of the zero-field step is not fitted by the small perturbation form of $\gamma(u_0)$ [Eq. (15)].

In conclusion, we have ascribed the radiation emitted from a long and narrow Josephson tunnel junction biased on the first zero-field step to resonant motion of a fluxon; and the linewidth of the radiation to thermal fluctuations in the fluxon velocity. Second-order effects such as plasmon-fluxon interaction, fluctuation in the fluxon profile, and geometrical irregularities have not been included because of the smallness of the fluctuations. We have found excellent agreement between the theory and experiments even on short junctions, suggesting that the theoretical result Eq. (16) with the absence of model parameters is more general than is its derivation and applicable also on short junctions. The discrepancy (less than a factor of 2) is probably due to external noise entering via bias current or temperature fluctuations.

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Role of Spin-Nonconserving Forces in the Critical Dynamics of Fe at the Curie Point

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Modern, high-resolution neutron-scattering techniques, in particular neutron spin echo, were applied to reinvestigate the critical dynamics of iron at temperatures above the Curie point. It was found that previously claimed evidence for a crossover was in error, and that the dynamical scaling hypothesis unexpectedly breaks down at small wave numbers. This breakdown is shown to be due to spin-nonconserving forces, which are tentatively identified with the thermodynamic random fields.

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The nature of the transition between the ferromagnetic and the paramagnetic states at the Curie point of the most common ferromagnet, iron, is still an object of controversy. The problem has attracted attention since decades ago and subsequent investigations raised as many questions as they answered.

In magnetism research neutron scattering plays a pivotal role by offering the only way of direct, model-independent measurement of the static and dynamic spin-spin correlation functions. However, in many cases insufficient resolution makes the neutron scattering data limited to a too narrow band, as was the case with the previous investigations of the dynamics of critical fluctuations in iron. Other microscopic experiments

