Monopole Pair Creation in Energetic Collisions: Is It Possible?

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It is suggested that monopole-antimonopole pair production initiated by pointlike particles (electrons, quarks) has widely different cross sections in the two cases of "pointlike" and composite monopoles, respectively. Production of 't Hooft–Polyakov monopoles is expected to be suppressed by a hugh factor, $\geqslant 10^{30}$. Furthermore, astrophysical evidence is presented suggesting that monopoles with $m_m \approx 10^4$ GeV/c^2 cannot be pointlike.

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The finite-energy topologically charged solution of the coupled Yang-Mills plus Higgs classical field equation[1,2] allows a beautiful realization of Dirac's original idea of magnetic monopoles.[3] The latter was motivated by electric-magnetic symmetry. A remarkable bonus was the charge (and magnetic charge) quantization via the relation

$$e_Q g_m = 2n\pi . \qquad (1)$$

Modern gauge-theory approaches may account for charge quantization if all particles belong to representations of a single simple underlying group.[4] Furthermore, in nonconfining gauge theories more subtle realizations of the electric-magnetic duality are possible. These realizations—of which the 't Hooft–Polyakov monopoles are the most celebrated examples—do not require the simultaneous existence (in the same phase) of pointlike electrically and pointlike magnetically charged objects.

The 't Hooft–Polyakov monopoles arise when a simple gauge group breaks spontaneously, leaving an exact U(1) subgroup. There is therefore a very nice consistency between charge quantization arising from the simple group generator, the Dirac quantization [Eq. (1)], and the fact that 't Hooft–Polyakov monopoles do require an initial simple gauge group.

The spontaneous symmetry breaking of the underlying group generates for some vector bosons ("W") masses, m_W. The monopole solution has the corresponding size scale $R_m \sim m_W^{-1}$ and its mass is

$$m_m \sim g_m^2/R_m \sim m_W/\alpha \qquad (2)$$

with $\alpha = e^2 a/4\pi$ the common gauge coupling. Low-energy weak phenomenology suggests a nonsimple SU(2)⊗U(1) group. Magnetic monopoles should then arise at a unification scale $m_{``W"} > 100$ GeV, and therefore $m_m > 10^4$ GeV.

The magnetic charges and "soft" ($k < m_X$) photon exchanges and/or emissions (ionization, bremsstrahlung, etc.) are the same for pointlike and 't Hooft–Polyakov monopoles.

Our main purpose is to show that monopole-antimonopole pair production in processes initiated via pointlike particles (electrons, quarks) has widely different cross sections in the two cases of pointlike and composite, 't Hooft–Polyakov monopoles, respectively. Specifically the latter are likely to be suppressed by a huge factor 10^{30}–10^{300}. This practically excludes $m\bar{m}$ production by future accelerators, by cosmic rays, or by near neutron stars. If "composite" monopoles—superheavy or not—are ever discovered, they should be of primordial origin.

The question, "How many bosons: W's and Higgs (φ) quanta, effectively comprise the 't Hooft–Polyakov monopoles?" transcends the pure classical solution. It is of crucial importance, however, when production of magnetic monopoles—starting with few "elementary" (φ's, W's) quanta—is concerned. Our assertion is that these numbers are both $\approx 1/\alpha$:

$$\bar{n}_W \sim 1/\alpha, \quad \bar{n}_\varphi \sim 1/\alpha. \qquad (3)$$

We assumed that λ, the φ^4 coupling, is $\sim\alpha$, in which case $n_\varphi = [4\pi/(e_Q)^3]\sqrt{\lambda} \approx 1/\alpha$ and $m_H = \lambda^{1/2}\varphi_0 \approx m_W$. We now present several arguments to justify Eq. (3).

(i) The deviation $\xi = \varphi - \varphi_0$ of the field φ from its vacuum value has a magnitude of order φ_0 over the monopole volume δv. We can approximate the potential $\lambda(\varphi^2-\varphi_0^2)^2$ near φ_0 by a harmonic-oscillator potential (remembering $m_W^2 = \lambda^2\varphi_0^2$): $v(\xi)\sim\frac{1}{2}m_H^2\xi^2$. The Higgs fields inside δv are then a collection of harmonic oscillators, each in a coherent state $|\xi\rangle$, $\xi=\varphi_0$.

The average number of quanta in a coherent state $|\xi\rangle$ is $\bar{n}_\xi=\frac{1}{2}k\xi^2/\hbar\omega$ with $H_{\rm osc}=\frac{1}{2}k\xi^2+\frac{1}{2}\mu\dot{\xi}^2$, $\omega=(k/\mu)^{1/2}$. The analog field-theoretic expression for the Hamiltonian density $\mathcal{H}$ is $\mathcal{H}=\frac{1}{2}m_H^2\varphi^2+\frac{1}{2}\dot{\varphi}^2$, i.e., formally $k\to m_H^2$, $\mu\to 1$, $\omega\to m_H$. Hence the Higgs-quantum density in the monopole is $dn/dV=m_H\varphi_0^2$. The total number of quanta in the monopole is obtained by multiplying by $\delta v=\frac{4}{3}\pi R_m^3$:

$$\bar{n}_\varphi=\frac{4}{3}\pi R_m^3 m_H\varphi_0^2$$

$$=\frac{4}{3}\pi\frac{m_H}{e_Q\varphi_0}\frac{1}{e_Q^2}\sim\frac{\lambda^{1/2}4\pi}{e_Q^3}\sim\frac{1}{\alpha}, \qquad (4)$$

where $m_H\sim m_W\sim e_Q\varphi_0$ was used in the last step.

Also since the W vector fields have, inside the monopole, magnitude

$$A_\mu\sim\frac{g_m}{R_m}\sim\frac{1}{e_Q}\frac{1}{R}\sim\frac{m_W}{e_Q}\equiv\varphi_0,$$

or alternatively $eA_\mu\sim g_\mu\sim 1/R_m$, we expect the monopole to "contain" $1/\alpha$ gauge quanta.

(ii) We try to understand the monopole stability and mass, viewing it as a collection of $\bar{n}$ quanta in a sphere of radius $R_m\sim m_W^{-1}$. The kinetic energy is $K=n[R_m^{-2}+m_W^2]^{1/2}$ and the potential energy $U\sim\binom{n}{2}(e_Q^2/R_m)\sim n^2\alpha m_W$. Demanding $U\sim K$ fixes $\bar{n}\sim 1/\alpha$. Furthermore we find $m_m\sim K\sim U\sim m_W/\alpha$.

(iii) The large magnetic charge $g_m\approx(1/\alpha)e_Q$ can be naturally interpreted as the collective coupling of the $1/\alpha$ bosons in the monopoles to a soft photon.

(iv) If $\bar{n}\sim 1/\alpha$ then the quantum (loop) $O(\alpha)$ corrections of the monopole classical solution are, indeed, as expected, proportional to $1/\bar{n}_{\rm quanta}$.

If the monopole is an elementary, pointlike object, the $m\bar{m}$ production cross section starting with pointlike, electrically charged objects (say e^+e^- collisions) is given by the simple perturbation (Feynman) diagram [Fig. 1(a)]:

$$\sigma^{(0)}(e^+e^-\to m\bar{m})\approx\frac{4\pi}{3}\frac{\alpha_e\alpha_m}{Q^2}\sim\frac{1}{Q^2} \qquad (5)$$

e.g.,

$$\sigma^0(e^+e^-\to m\bar{m};\ Q^2\approx 4m_\mu^2\approx(10^4\ \text{GeV})^2)=10^{-35}\ \text{cm}^2$$

exceeds then μ pair production (at the same energy) by $(1/\alpha)^2$! The strong magnetic charge implies that the pure $m\bar{m}$ production is extremely rare. However, just as in the case of $\bar{q}q$ production in QCD, this still allows Eq. (5) to predict the inclusive $e^+e^-\to m\bar{m}+\gamma$'s $+W_\mu$'s + Higgs + ... cross section.

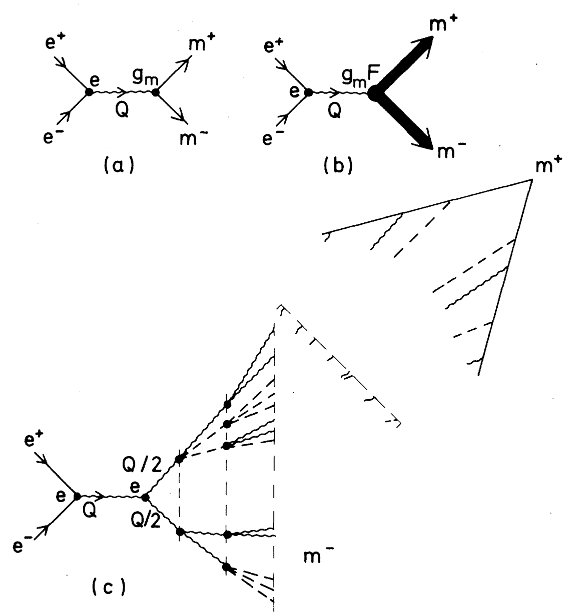

FIG. 1. $m\bar{m}$ pair production with the monopole viewed as (a) pointlike particles with coupling g_m to photons, (b) extended objects with form factor F, and (c) coherent states of many W's and φ's. The latter production mechanism requires many stages indicated by the broken vertical lines in the figure. Only when enough ($\sim 1/\alpha$) quanta are thus perturbatively generated can they "collapse" into the monopoles. The gap at the upper right-hand corner in the figure represents our inability to describe this stage appropriately.

Our main observation is that the large degree of compositeness of the 't Hooft–Polyakov monopoles invalidates Eq. (5). The "pointlike" value (5) saturates at all Q^2 the unitarity bound for total e^+e^- pointlike S-wave cross section and is inconsistent with the asymptotic freedom of the original weakly coupled simple gauge theory. The inclusive $m\bar{m}$ production is given by $\sigma^{\text{'t H.-P.}}(e^+e^-\to m\bar{m})\approx F^2\sigma^0$ with the compositeness factor $F\sim e^{-2/\alpha}$.

To estimate F we note that many steps [see Figs. 1(b) and 1(c)] are required to bridge the gap between the initial few-quanta (1γ, W^+W^-, or $\varphi^+\varphi^-$, ...) state and the final m, $\bar{m}$ state with $\sim 2/\alpha$ W's and Higgs quanta. In fact we have to go to $(2/\alpha)$th order in perturbation. It is generally believed that the actual formation of the nonperturbative "classical" monopoles cannot be addressed in perturbation theory.[5] However, this concerns only the very last stage in Fig. 1(c). All previous steps involve virtual momenta

$$Q_j^2 \gg m_W^2 \quad (Q_0 \sim 2m_m,\ Q_1 \sim Q_0/4,\ Q_l \sim Q_0/2^l,\ \text{and } \bar{n} = 2^l)$$

and would be amenable to perturbative treatment even if the theory considered was QCD-like let alone for the case of electromagnetic weak. We therefore suggest that the perturbative estimate amplitude $e_Q^{(\mathrm{order})}$ applies, i.e.:

$$F \approx e_Q^{2/\alpha} \sim e^{-2/\alpha}, \quad |F|^2 \approx e^{-4/\alpha} \approx 10^{-250} \tag{6}$$

[we use the numerical coincidence $e_Q \equiv (4\pi\alpha)^{1/2} \sim e^{-1}$].

Instead of going to high orders we could attempt to utilize the fact that the monopole is a coherent state with average number of quanta $\bar{n} = \alpha^{-1}$. It has a Poisson probability $\bar{n}^K e^{-\bar{n}}/K!$ of having K quanta. If we couple to the monopole via its K-quanta component, the probability of its production is

$$[(\alpha\bar{n})^K/K!]e^{-\bar{n}} \sim (1/K!)e^{-\bar{n}} \sim (1/K!)e^{-1/\alpha}.$$

The joint probability of $m\bar{m}$ production is

$$P_{(K,L)} = [(\alpha\bar{n})^K/K!][(\alpha\bar{n})^L/L!]e^{-2\bar{n}}.$$

Even if we sum over all $\binom{K+L}{L}$ partitionings[6] of the quanta into the upper and lower vertices we have a suppression factor

$$\sum_{\substack{L\ K \\ K+L\leq 2\bar{n}}}\sum \binom{K+L}{L} P(K,L) = [2(\alpha\bar{n})]^{2\bar{n}} e^{-2\bar{n}} \approx (e/2)^{-2\bar{n}} \approx (e/2)^{-2/\alpha} \approx 10^{-50}.$$

The small probability for generating $m\bar{m}$ starting (essentially) with the perturbative vacuum is reminiscent of the similar suppression by a tunneling factor $\exp(-8\pi^2/g^2)$ of instanton and monopole-antimonopole configuration in the Euclidean Yang-Mills action.[7]

On a more pedestrian level we can interpret Eq. (6) as a form-factor suppression. QR for the $m\bar{m}$ production is $QR \geq 4m_m R_m \sim 4/\alpha$ and[8]

$$F \approx e^{-QR}.$$

Accelerator searches for monopoles are energy limited. Cosmic rays extend to about 10^{12} GeV and they have been used to obtain the upper limits for the cross section of monopole creation in nucleon-nucleon interactions. The best limits are those obtained from lunar samples,[9] namely

$$\sigma(NN \to m + x) \leq 10^{-44}[m_m/(1\ \text{GeV})]^{3.4}\ \text{cm}^2.$$

For $m_m = 10^4$ this bound, 10^{-30} cm^2, is bigger than the pointlike cross section [Eq. (5)].

It seems fairly certain that the observed magnetic field of neutron stars is the relic from the time of star formation, i.e., has persisted essentially unchanged for 10^6 years. However, in the magnetosphere of pulsars, there may exist "external gaps" populated with e^+ and e^-, each with $E = 10^{14}$–10^{15} eV. Thus, magnetic monopoles with masses up to 10^5 GeV/c^2 could be produced. One of the monopoles will be accelerated towards the surface of the neutron star, where it will be thermalized. It seems plausible that such monopoles will be trapped in the dense ($d = 10^5$–10^6 g/cm^2) layers of the neutron-star crust. These should lead to local changes of the magnetic field at the polar caps, and the pulsar should stop radiating. A very stringent bound for the cross section of monopole creation is obtained by requiring that the total number of monopoles produced during the pulsar's lifetime is smaller than that required to change the field.[10] It is

$$\sigma(ee \to m\bar{m}) \leq 10^{-38}\text{–}10^{-39}\ \text{cm}^2.$$

Furthermore, even without appealing to external gaps, limits of the order 10^{-34} cm^2 are obtained by considering the possibility of secondary monopole creation in the process $m + n \to m + (m + \bar{m}) + \ldots$. For the case of a "composite" monopole, this process requires a $1/\alpha$ higher threshold and hence a longer acceleration length for the monopoles. This is because only the elementary constituents of the monopole, e.g., the W's and the φ's take part in the collision with the quarks and the center-of-mass energy of that collision [which is $(2m_W E)^{1/2} = (2m_m \alpha E)^{1/2}$ rather than $(2m_m E)^{1/2}$] has to exceed the $m\bar{m}$ threshold. It appears that all by itself this effect quenches the $m\bar{m}$ cascade. Thus, the extremely small $m\bar{m}$ cross section may preclude any observable effects due to composite monopoles even if we utilize such sensitive, large, "counters" with long integration times like magnetic white dwarfs and neutron stars. However, it is amusing to note that pointlike monopoles are on the verge of being excluded by use of the astrophysical data alone.

In principle one can conceive of other more exotic mechanisms of nonprimordial monopole pair creations.

(a) In sufficiently intense magnetic fields $m\bar{m}$ pairs could be created via vacuum polarization[11] or tunneling effects. Since formally only many and very soft photons participate in this semiclassical process, one could imagine that this production will not be suppressed by compositeness. The expression of the production rate is

$$dN_{m\bar{m}}/dV\,dt \approx g_m{}^2 B^2 \exp(-\pi m_m{}^2/g_m B).$$

Thus sizeable $m\bar{m}$ production requires truly huge magnetic fields (which give rise to a potential difference m_m over a length $1/m_m$, the Compton wavelength of the monopole). Such fields are not available in known astrophysical objects.

(b) $m\bar{m}$ pairs could be produced by Hawking's black-hole evaporation for sufficiently small black holes. If $m_{\rm bh} \approx 10^{-3}$ g, then its effective surface temperature exceeds 10^{17} GeV and it could conceivably also radiate $m\bar{m}$ pairs. Such small mass black holes tend to evaporate in $t \sim 10^{-36}$ sec, which is just a bit longer than $t_{\rm Planck}$, and the corresponding monopoles should be considered primordial. (If a massive black hole decays, it will tend to yield many more photons, quarks, electrons, etc., than monopoles; the latter may arise only from the last 10^{-3} g evaporating.)

All this still leaves us with one efficient source of 't Hooft–Polyakov monopoles: the phase transition in which the simple group is broken in the early universe.[12,13]

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[6]In doing this we are extremely conservative. In fact the $W_\mu + \varphi$ quanta which make up the m and $\bar{m}$ are very specific, since they have total topological charges of $g \sim 2\pi/e$.

[7]If $\alpha \sim 1$ the distinction between perturbative and nonperturbative (electric and magnetic) phases blurs and, as suggested by 't Hooft and Mandelstam, the ensuing condensation of $m\bar{m}$ pairs in the vacuum yields color electric confinement. Our argument above suggests that since $\bar{n} \sim 1$ in this case the perturbative approach may, paradoxically, be reliable for detecting the onset of this phenomenon.

[8]A simple power law, $1/(Q^2)^{n-1}$, applies for systems composed of n constituents (the Brodsky-Farrar counting rules) but the high compositeness of the monopole leads to exponentiation, $(1+RQ/\bar{n})^{-\bar{n}} = e^{-QR}$. From this point of view $m\bar{m}$ production should be as rare as that of ^{56}Fe + $^{56}\overline{\rm Fe}$.

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