# Three-Body Forces and Neutron-Neutron Effective-Range Parameters

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It is suggested that the difference which presently exists between neutron-neutron effective-range parameters extracted from the reactions $D(\pi^-,\gamma)2n$ and $D(n,2n)p$ can be explained by the same three-body force which describes $^3H$, $^3He$, and $^4He$. The effects of the three-body force are different for neutron pickup and proton knockon processes in the reaction $D(n,2n)p$.

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The neutron-neutron scattering length extracted[1] using improved theoretical treatment[2,3] from the recent measurement of the reaction $D(\pi^-,\gamma)2n$ is $a_{nn}=-18.6\pm0.48$ fm and it differs from the currently accepted value[4]: $a_{nn}=-16.3\pm0.6$ fm. Recent studies[5] of the reaction $D(n,2n)p$ reinforce[6] the value $-16.7\pm0.47$ fm for $a_{nn}$. Since differences among nuclear forces are magnified[7] in scattering lengths, it is possible that the presence of three nucleons in the final state of $D(n,2n)p$ as opposed to only two in the reaction $D(\pi^-,\gamma)2n$ can cause differences in the effective-range parameters extracted from these two reactions.

From our basic understanding of strong interactions it follows that there exists a three-body force.[8] The importance of the three-body force is confirmed by the following: (i) The best two-nucleon forces do not explain nuclear matter saturation[9] and the $^3H$ binding energy and the $^3He$ charge form factor.[10] (ii) Holes in $^3He$ and $^4He$ one-body density distributions could have their origin in a repulsive three-body force.[11] (iii) The three-body force[12] gives[13] a needed additional binding for $^3H$. (iv) A three-meson-exchange three-body force can be responsible for nuclear saturation.[14] (v) Good agreement with energies of ground and negative parity excited states in $^4He$ is obtained[15] if one adds the three-body Fujita-Miyazawa (FM) force to a standard two-body force [similar results are obtained with a partial conservation of axial-vector current (PCAC) current-algebra three-body force[16]]. (vi) This force also enhances meson-exchange-current contributions to the $^4He$ charge form factor giving almost satisfactory agreement with the data.[17]

We use the FM two-pion-exchange three-body force through $\Delta(1232)$ with a phenomenological cutoff mass and investigate its effect on $a_{nn}$ and the effective range, $r_{nn}$, extracted from the reaction $D(n,2n)p$. Information on $a_{nn}$ and $r_{nn}$ comes from neutron pickup and proton knockon processes followed by neutron-neutron final-state interaction and from neutron-neutron quasifree scattering.

The weighted average[6] of pickup data[6,18] taking into account both experimental errors and uncertainties in the theoretical analysis—exact three-body calculation using various separable two-nucleon forces ($\Delta a_{nn}^{\text{theor}}=0.3$ fm, $\Delta r_{nn}=0.5$ fm) —is

$$a_{nn}^{\text{PU}}=-16.73\pm0.47\text{ fm},\quad r_{nn}^{\text{PU}}=2.85\pm0.60\text{ fm}.$$

The proton knockon process was studied by measuring proton spectra at forward angles. Information on $a_{nn}$ from such data is subject to experimental (sensitivity to energy and angular resolution) and theoretical uncertainties (inclusions of tensor and higher partial-wave forces appreciably change the spectra) which are much larger than for correlation studies. The weighted average[6] of knockon data[19] analyzed with an exact three-body calculation with separable force and impulse approximation compared to D($p,n$)$2p$ data is

$$a_{nn}^{\rm KO} = -20.7 \pm 2\ {\rm fm}.$$

The error stems mainly from the uncertainties in the analysis.

The weighted average[6] of neutron-neutron quasi-free scattering data,[20] again with consideration of experimental and theoretical ($\Delta r_{nn}^{\rm theor} = 0.08$ fm) uncertainties, is

$$r_{nn}^{\rm QFS} = 2.68 \pm 0.16\ {\rm fm}.$$

These results should be compared with the weighted average[6] of the D($\pi^-,\gamma$)$2n$ data[1,21]:

$$a_{nn} = -18.45 \pm 0.46\ {\rm fm},\quad r_{nn} = 2.83 \pm 0.16\ {\rm fm}, \quad (1)$$

yielding

$$\begin{aligned}\Delta a_{nn}^{\rm PU} &= a_{nn}^{\rm PU} - a_{nn} = +1.72 \pm 0.66\ {\rm fm},\\ \Delta r_{nn}^{\rm PU} &= r_{nn}^{\rm PU} - r_{nn} = +0.02 \pm 0.62\ {\rm fm},\\ \Delta a_{nn}^{\rm KO} &= a_{nn}^{\rm KO} - a_{nn} = -2.25 \pm 2.05\ {\rm fm},\\ \Delta r_{nn}^{\rm QFS} &= r_{nn}^{\rm QFS} - r_{nn} = -0.15 \pm 0.22\ {\rm fm}.\end{aligned} \quad (2)$$

The neutron-neutron system is in a $^1S_0$ state and the only part of the FM three-body force $W_i(\vec{r}_{ki}, \vec{r}_{ij})$ having a nonzero expectation value is the central force $W_3^c$, where we have labeled the proton as particle 3 and two neutrons by 1 and 2:

$$W_3^c = \tfrac{2}{3} C_p [1 + (3\cos^2\theta - 1) T_\mu(r_{31}) T_\mu(r_{23})] Y_\mu(r_{31}) Y_\mu(r_{23}) \equiv V_{123},$$

where $C_p = 0.45$ MeV,[15] $\theta$ is the angle between $\vec{r}_{31}$ and $\vec{r}_{23}$, $Y_\mu(r) = e^{-\mu r}/(\mu r)$, and $T_\mu(r) = 1 + 3/(\mu r) + 3/(\mu r)^2$. This is in contrast to the $^3$H, $^4$He, and nuclear-matter binding-energy calculations where the tensor part of FM plays a main role.[15]

The contribution of the three-body force depends[22] on the pionic form factor. We use

$$H(Q^2) = (\Lambda^2 - \mu^2)/(\Lambda^2 + Q^2),$$

where $\mu$ is a pion mass, $Q$ is the exchange pion momentum, and $\Lambda$ is a cutoff mass. It modifies the three-body force as $Y_\mu \to Y_\mu - (\Lambda/\mu)^3 Y_\Lambda$ and $T_\mu Y_\mu \to T_\mu Y_\mu - (\Lambda/\mu)^3 T_\Lambda Y_\Lambda$, and smaller $\Lambda$ gives longer-range modification. The value $(\Lambda/\mu)^2 = 10$ reproduces the Goldberger-Treiman discrepancy, while $(\Lambda/\mu)^2 = 50$ is in agreement with the one-boson-exchange model.[23]

The transition matrix element is

$$T_{fi} = \langle \varphi_f^- | (V_{23} + V_{31}) | \Psi_i^+ \rangle, \quad (3)$$

$$\Psi_i^+ = \{1 + (E - H + i\epsilon)^{-1}(V_{12} + V_{31})\}(2\pi)^{-3/2} \exp(i\vec{k}_i \cdot \vec{\rho}_1)\varphi_d(\vec{r}_{23}), \quad (3a)$$

$$\varphi_f^- = (2\pi)^{-3} \exp(i\vec{k}_f \cdot \vec{\rho})\{1 + (e_f^\dagger)^{-1} V_{12}\} e^{i\vec{q}\cdot\vec{r}}, \quad (3b)$$

where $\vec{k}_i$ and $\vec{k}_f$ are initial-state neutron and final-state proton momenta, respectively, $\varphi_d$ is a deuteron wave function, $\vec{r} = \vec{r}_1 - \vec{r}_2$, $\vec{\rho} = \vec{r}_3 - (\vec{r}_1 + \vec{r}_2)/2$, $\vec{\rho}_1 = \vec{r}_1 - (\vec{r}_2 + \vec{r}_3)/2$, $\vec{q}$ is a neutron-neutron relative momentum, and $e_f = \hbar^2 q^2/M - T_r - V_{12} + i\epsilon$. The effective-range parameters extracted from the pickup and knockon processes depend on the shape[24] of the spectra, which are determined by the final-state interaction. Thus, we do not consider the effect of the three-body force in the initial state[25] and introduce the three-body force in Eq. (3) only as a final-state interaction replacing $V_{12}$ in Eq. (3b) by $V_{12} + V_{123}$. This changes $T_{fi}$ into $T_{fi} + \Delta T_{fi}$, with

$$\Delta T_{fi} = \langle \varphi_f^- | V_{123} e_f^{-1}(V_{31} + V_{23}) | \Psi_i^+ \rangle = \langle \varphi_f^- | V_{123}(1 + e_f^{-1} V_{12}) e_0^{-1}(V_{31} + V_{23}) | \Psi_i^+ \rangle, \quad (4)$$

where $e_0 = \hbar^2 q^2/M - T_r + i\epsilon$ and we have approximated $e_f^\dagger - V_{123}$ by $e_f^\dagger$.

We consider knockon and pickup reactions in the final-state interaction condition, where two neutrons are in a $q \sim 0$, $S$-wave final state. Then

$$\langle \vec{r} | \frac{1}{e_0} | \vec{r}' \rangle = \frac{-e^{iq|\vec{r}-\vec{r}'|}}{4\pi|\vec{r} - \vec{r}'|} \to \frac{1}{4\pi|\vec{r} - \vec{r}'|}$$

and $\{1 + e_f^{-1} V_{12}\} e^{iqr} \to$ an $S$-wave scattering function of $r$ as $q \to 0$.[26]

We define an effective two-body potential $\Delta V_{12}$ so that, when it is added to $V_{12}$, it reproduces $T_{fi} + \Delta T_{fi}$:

$$\Delta V_{12}(r) = \frac{\int d^2\hat{r} \int d^3\rho \exp(-i\vec{k}_f \cdot \vec{\rho}) V_{123}(\vec{r}, \vec{\rho}) \int d^3 r' |\vec{r} - \vec{r}'|^{-1} \mathcal{V}(\vec{r}', \vec{\rho}) \Psi_i^+(\vec{r}', \vec{\rho})}{\int d^2\hat{r} \int d^3\rho \exp(-i\vec{k}_f \cdot \vec{\rho}) \int d^3 r' |\vec{r} - \vec{r}'|^{-1} \mathcal{V}(\vec{r}', \vec{\rho}) \Psi_i^+(\vec{r}', \vec{\rho})}, \quad (5)$$

TABLE I. Calculated values $\Delta a_{nn} = a_{nn}^{\mathrm{RSC}+\Delta V_{12}} - a_{nn}^{\mathrm{RSC}}$ and $\Delta r_{nn} = r^{\mathrm{RSC}+\Delta V_{12}} - r_{nn}^{\mathrm{RSC}}$ for pickup and knockon processes and effective three-body force $\Delta V_{12}$ with various $(\Lambda/\mu)^2$.

| | Pickup | | Knockon | |
|---|---|---|---|---|
| $(\Lambda/\mu)^2$ | $\Delta a_{nn}$ (fm) | $\Delta r_{nn}$ (fm) | $\Delta a_{nn}$ (fm) | $\Delta r_{nn}$ (fm) |
| 10 | −0.18 | −0.01 | 0.29 | 0.01 |
| 25 | +0.64 | 0.0 | −0.09 | 0.01 |
| 50 | +1.71 | +0.03 | −0.45 | 0.0 |
| 75 | +2.37 | +0.04 | −0.68 | 0.0 |
| 100 | +2.83 | +0.06 | −0.84 | −0.01 |
| 150 | +3.44 | +0.07 | −1.09 | −0.01 |

where $v$ is either $V_{31}$ or $V_{23}$. In the knockon case one uses $V_{31}$ and $\vec{k}_i$ is parallel to $\vec{k}_f$. In the pickup case one has $V_{23}$ and $\vec{k}_f$ is antiparallel to $\vec{k}_i$.

We calculate the neutron-neutron effective-range parameters by adding $\Delta V_{12}$ to a two-body Reid soft-core potential and approximating an exact $\Psi_i^+$ by a distored wave. The results,

$$\Delta a_{nn}^{\mathrm{theor}} = a_{nn}^{\mathrm{RSC}+\Delta V_{12}} - a_{nn}^{\mathrm{RSC}}$$

and

$$\Delta r_{nn}^{\mathrm{theor}} = r_{nn}^{\mathrm{RSC}+\Delta V_{12}} - r_{nn}^{\mathrm{RSC}},$$

for pickup and knockon processes at 14-MeV incident energy are given in Table I. Some data used in Eq. (2) are obtained at 11,[19] 18,[18] 17–27,[5] and 49.6 MeV.[19] Test calculations done at 10 and 20 MeV suggest rather negligible energy dependence of the calculated $\Delta a_{nn}$ and $\Delta r_{nn}$.

Comparing the calculations in Table I and the data in Eq. (2) one concludes that the effects of the three-body force are different for pickup and knockon processes and that for $(\Lambda/\mu)^2 \sim 25-100$ the calculation agrees with the data. The same $(\Lambda/\mu)^2$ values give[15] good results for $^3$H and $^4$He. The data tend to exclude values $(\Lambda/\mu)^2 \leq 10$. We conclude that the FM three-body force can explain why the neutron-neutron effective-range parameters extracted from the reaction D$(n,p)2n$ differ from those obtained from the reaction D$(\pi^-,\gamma)2n$.[27]

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[24]For kinematically complete measurement of a pickup process the exact three-body model gives (Ref. 18) the same value for $a_{nn}$ as a final-state interaction model of Watson and Migdal. The initial-state interaction changes the overall normalization.

[25]We compare $a_{nn}$ and $r_{nn}$ extracted from the reactions $d\pi^-$ and $dn$ and in both processes there is a three-hadron force in the initial state. We argue that in both cases these three-hadron forces do not change the shapes of the spectra and thus do not change the extracted $a_{nn}$ and $r_{nn}$. Since our knowledge of the three-hadron force in the $d\pi^-$ state is still not quite adequate, it seems to us that the comparison between data in our Eqs. (1) and (2) and those from the reaction $D(\mu^-, 2n)\nu_\mu$ represents the best way to test the validity of the conjecture that the three-nucleon force in the final state is the explanation of the results in Eq. (2) and that the initial-state three-hadron force effects do not influence $a_{nn}$ and $r_{nn}$. Since the value of $r_{nn}^{\rm QFS}$ does depend on the absolute cross section and the initial-state interaction, we postpone the analysis of $\Delta r_{nn}^{\rm QFS}$ for a later paper.

[26]If one reads Eq. (4) from right to left it follows that after the interaction $V_{31}+V_{23}$ the stage of the $n$ + D reaction is essentially the final state in which two neutrons are in a $q \sim 0$, $L=0$ state (the admixture of higher partial waves is negligible). The operator $1+e_f^{-1}V_{12}$ operates on this single state and the nonlocality (dependence on $r_{12}$ and $r_{12}'$) reduces to a function of $r_{12}$. Thus, the effect of the operator cancels out between the numerator and the denominator of the equation, (5), defining $\Delta V_{12}$.

[27]M. J. Moravcsik (private communication) has suggested that the explanation of differences between effective-range parameters extracted from $d\pi^-$ and $dn$ reactions could be independent of a specific model (i.e., FM) and indeed it could stem from the general features of the longest-range three-nucleon force.