Field-Swelling Instability in Anisotropic Plasmas

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Collisionless plasmas imbedded in a magnetic field and with the electron temperature transverse to the magnetic field larger than the longitudinal temperature can be subject to an instability that produces transport of transverse electron thermal energy into the regions where the magnetic field is weakened by the perturbation while the plasma motion is decoupled from that of the magnetic field lines.

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In plasma regimes where the degree of collisionality is relatively low, the particle distribution in velocity space can depart considerably from a Maxwellian. Then plasma collective modes can be excited and, as a result, induce a substantial change in the particle distribution in velocity space. Here we consider the case where the electron distribution is characterized by an anisotropic temperature. The plasma is imbedded in a relatively weak magnetic field such that the particle pressure is of the same order as the magnetic pressure and the electron temperature anisotropy ($T_{e\perp} \neq T_{e\parallel}$) is relative to the direction of the magnetic field. We consider collective modes that can be described by moment (fluidlike) equations, and so do not rely on wave-particle resonance processes which depend on the evolution of a small portion of the particle distribution in velocity space. The plasma motion is allowed to be decoupled from the magnetic field lines. That is, the so-called "frozen-in condition" is not imposed. Thus a new kind of instability is found to be excited when $T_{e\perp} > T_{e\parallel}$. In this case the magnetic field is perturbed from its equilibrium state, and transverse electron thermal energy is transported toward the regions where the magnetic field is weakened, both by a particle flow and by an effective thermal conductivity along the magnetic field lines. The local increase in the particle transverse pressure tends to make the magnetic field "swell" further locally and the excited mode is amplified as a consequence. We point out that, as shown in a parallel paper,[1] modes of the type treated here can be important in producing so-called magnetic reconnection when a plasma is confined in a magnetic field configuration that contains a ("neutral") surface where the field vanishes. Magnetic reconnection corresponds to a change of the field topology, as in the case where magnetic "islands" are formed out of a configuration that initially has straight and parallel field lines. In fact, the formation of these "islands" would not occur if the field lines were constrained to move together with the plasma. The analysis of neutral-sheet configurations where magnetic reconnection takes place is of special interest in space physics.

We note that fluidlike instabilities driven by a plasma temperature anisotropy had been found earlier, and the one which is closest to that considered in this paper is usually referred to as the "mirror" instability.[2] However, this is derived under the assumptions that (a) the "frozen-in condition" is valid and (b) the effective thermal conductivity along the magnetic field is negligible. In particular, the threshold value of $T_{e\perp}/T_{e\parallel}$ above which this instability can be excited is 6 times that for the instability we describe in the present paper.

We write the equilibrium electron distribution as

$$f_e = \frac{n}{(2\pi T_{e\perp}/m_e)^{3/2}}\left(\frac{T_{e\perp}}{T_{e\parallel}}\right)^{1/2}\exp\left(-\frac{m_e v_\perp^2}{2T_{e\perp}} - \frac{m_e v_\parallel^2}{2T_{e\parallel}}\right),$$

and the uniform equilibrium magnetic field as $\vec{B}_0 = B_0\hat{z}$. We analyze perturbations of the form $\tilde{\vec{B}} \propto \exp(-i\omega t + ik_\perp x + ik_\parallel z)$ with $\omega < \Omega_i < \Omega_e$ and

$$v_{\text{th}i} < \omega/|k_\parallel| < v_{\text{th}e},$$

where $\Omega_e = eB_0/m_e c$, $\Omega_i = eB_0/m_i c$, $v_{\text{th}e} = (2T_{e\parallel}/m_e)^{1/2}$, and $v_{\text{th}i} = (2T_{i\parallel}/m_i)^{1/2}$. We introduce the vector and scalar potentials by $\tilde{\vec{B}} = \nabla \times (\tilde{\vec{A}})$, and $\tilde{\vec{E}} = (i\omega/c)\tilde{\vec{A}} - \nabla\tilde{\varphi}$ and consider the quasineutrality condition $\tilde{n}_i = \tilde{n}_e$ (i.e., $\nabla \cdot \tilde{\vec{A}} = 0$). We assume for simplicity that the ion temperature is negligible in comparison to the electron temperature. Thus the longitudinal (cold) ion momentum conservation equation yields

$$\tilde{u}_{i\parallel} = (ie/\omega m_i)\tilde{E}_z. \quad (1)$$

From the transverse (cold) ion momentum conservation equation we obtain, for $\omega < \Omega_i$,

$$\tilde{u}_{ix} \simeq \frac{i\omega}{B_0}\tilde{A}_y + \frac{\omega^2}{B_0\Omega_i}\tilde{A}_x - \frac{\omega}{\Omega_i}\frac{ck_\perp\tilde{\varphi}}{B_0}, \quad (2)$$

$$\tilde{u}_{iy} \simeq \frac{ick_\perp\tilde{\varphi}}{B_0} + \frac{\omega^2}{B_0\Omega_i}\tilde{A}_y - \frac{i\omega}{B_0}\tilde{A}_x. \quad (3)$$

Then the ion-mass conservation equation gives

$$\frac{\tilde{n}_i}{n} \simeq \frac{ik_\perp}{B_0}\tilde{A}_y + \frac{iek_\parallel}{m_i\omega^2}\tilde{E}_z, \quad (4)$$

for $\tilde{u}_{ix} \approx i\omega\tilde{A}_y/B_0$ that corresponds to $|k_\parallel/k_\perp| > |\omega/\Omega_i|$. In order to proceed to evaluate $\tilde{n}_e$ we write the electron pressure tensor as

$$\overleftrightarrow{\mathrm{P}}_e = p_{e\perp}\overleftrightarrow{\mathrm{I}} + (p_{e\parallel} - p_{e\perp})\vec{b}\vec{b},$$

where $\overleftrightarrow{\mathrm{I}}$ is the unit dyadic, $\vec{b} = \vec{B}/B$, and the p_e's are the scalar components. The relevant linearized longitudinal electron momentum conservation equation is

$$0 = -en\tilde{E}_z - ik_\parallel\tilde{p}_{e\parallel} + ik_\parallel(p_{e\parallel} - p_{e\perp})\tilde{B}_z/B_0.$$

Thus

$$\tilde{p}_{e\parallel} = T_{e\parallel}\tilde{n}_e = ien\tilde{E}_z/k_\parallel + ik_\perp(p_{e\parallel} - p_{e\perp})\tilde{A}_y/B_0,$$

as $\tilde{T}_{e\parallel} = 0$ given that $\omega/k_\parallel < v_{the}$, and

$$\frac{\tilde{n}_e}{n} = \frac{ie}{k_\parallel T_{e\parallel}}\tilde{E}_z + \frac{ik_\perp}{B_0}\left(1 - \frac{T_{e\perp}}{T_{e\parallel}}\right)\tilde{A}_y. \quad (5)$$

Then the quasineutrality condition gives

$$\left(1 - \frac{\omega_s^2}{\omega^2}\right)\frac{e\tilde{E}_z}{k_\parallel T_{e\parallel}} = \frac{T_{e\perp}}{T_{e\parallel}}\frac{k_\perp}{B_0}\tilde{A}_y, \quad (6)$$

where $\omega_s^2 = k_\parallel^2 T_{e\parallel}/m_i$. From the transverse electron-momentum conservation equation we find

$$\tilde{u}_{ex} = \frac{i\omega}{B_0}\tilde{A}_y - \frac{ck^2}{eB_0^2}(T_{e\parallel} - T_{e\perp})\tilde{A}_x. \quad (7)$$

Then, from Ampere's law

$$\tilde{J}_x = (ck^2/4\pi)\tilde{A}_x$$

we obtain

$$\left[1 - \frac{\omega^2}{k^2 v_A^2} - \tfrac{1}{2}(\beta_\parallel - \beta_\perp)\right]\tilde{A}_x = -\frac{\omega^2}{k^2 v_A^2}\left(\frac{ck_\perp\tilde{\varphi}}{\omega}\right), \quad (8)$$

where $k^2 = k_\parallel^2 + k_\perp^2$, $v_A^2 = B_0^2/(4\pi n m_i)$, $\beta_\parallel = 8\pi p_{e\parallel}/B_0^2$, and $\beta_\perp = 8\pi p_{e\perp}/B_0^2$. Next, we consider

$$\tilde{J}_y = (ck^2/4\pi)\tilde{A}_y, \quad (9)$$

and derive $\tilde{J}_y$ from the total momentum conservation equation

$$-i\omega n m_i\tilde{u}_{ix} = -ik_\perp\tilde{p}_{e\perp} - ik_\parallel(p_{e\parallel} - p_{e\perp})\tilde{B}_x/B_0 + c^{-1}\tilde{J}_y B_0. \quad (10)$$

Combining Eqs. (2), (9), and (10) we obtain

$$\left[1 - \frac{\omega^2}{k^2 v_A^2} - \frac{k_\parallel^2}{2k^2}(\beta_\parallel - \beta_\perp)\right]\tilde{A}_y = i\frac{4\pi k_\perp}{k^2 B_0}\tilde{p}_{e\perp}. \quad (11)$$

We derive $\tilde{p}_{e\perp}$ from the linearized equation of state

$$\partial\tilde{p}_{e\perp}/\partial t + 2p_{e\perp}\nabla\cdot\tilde{\vec{u}}_e - p_{e\perp}\nabla_\parallel\tilde{u}_{e\parallel} = -\nabla\cdot(\hat{z}\tilde{q}_{e\parallel}{}^\perp), \quad (12)$$

where $\nabla_\parallel \equiv (\vec{B}_0\cdot\nabla)/B_0$ and

$$\tilde{q}_{e\parallel}{}^\perp \equiv \tfrac{1}{2}m_e\int d^3v\, v_\perp^2(v_\parallel\tilde{f}_e - \tilde{u}_{e\parallel}f_e), \quad (13)$$

$\tilde{f}_e$ being the perturbed electron distribution.

Since we have not found an easy way to derive $\tilde{q}_{e\parallel}{}^\perp$ from a "fluid" approach we calculate it directly from Eq. (13) after solving the perturbed linearized Vlasov equation. Introducing the polar coordinates θ, $v_\parallel$, and $v_\perp$ in velocity space we obtain the moment

$$\int_0^{2\pi} d\theta\,\tilde{f}_e = \frac{2\pi ei}{T_{e\perp}}\left\{\frac{\tilde{E}_z}{k_\parallel} - \frac{\omega - (1-\alpha_e)k_\parallel v_\parallel}{\omega - k_\parallel v_\parallel}\left[\frac{\tilde{E}_z}{k_\parallel} - \frac{v_\perp}{2c}\frac{k_\perp v_\perp}{\Omega_e}\tilde{A}_y\right] - \frac{v_\parallel}{2c}\frac{k_\perp v_\perp^2}{\Omega_e^2}(1-\alpha_e)\tilde{B}_y\right\}f_e \quad (14)$$

that is relevant to the evaluation of $\tilde{u}_{e\parallel}$ and $\tilde{q}_{e\parallel}{}^\perp$. Here $\alpha_e = T_{e\perp}/T_{e\parallel}$ and we find

$$\tilde{q}_{e\parallel}{}^\perp = -inT_{e\perp}\frac{\omega}{k_\parallel}\frac{T_{e\perp}}{T_{e\parallel}}\frac{k_\perp\tilde{A}_y}{B_0} - inT_{e\perp}(T_{e\parallel} - T_{e\perp})\frac{ck_\perp\tilde{B}_y}{eB_0^2}. \quad (15)$$

Noting that $2p_{e\perp}\nabla\cdot\tilde{\vec{u}}_e - p_{e\perp}\nabla_\parallel\tilde{u}_{e\parallel} = i\omega p_{e\perp}(\tilde{n}_e/n) + ik_\perp p_{e\perp}\tilde{u}_{ex}$, we obtain, using Eqs. (5), (7), and (15) in (12),

$$\frac{\tilde{p}_{e\perp}}{p_{e\perp}} = \frac{ie}{k_\parallel T_{e\parallel}}\tilde{E}_z + 2i\left(1 - \frac{T_{e\perp}}{T_{e\parallel}}\right)\frac{k_\perp\tilde{A}_y}{B_0}, \quad (16)$$

and, since $\tilde{p}_{e\perp}=T_{e\perp}\tilde{n}_e+n\tilde{T}_{e\perp}$,

$$\tilde{T}_{e\perp}/T_{e\perp}=(1-T_{e\perp}/T_{e\parallel})\tilde{B}_z/B_0. \tag{17}$$

Then combining Eqs. (6), (11), and (16) we finally arrive at the dispersion relation

$$\left(\frac{\omega^2}{\omega_s^2}-1\right)\left[1-\frac{\omega^2}{k^2v_A^2}+\frac{k_\perp^2}{2k^2}\frac{T_{e\perp}}{T_{e\parallel}}(2\beta_\parallel-\beta_\perp)-\frac{k_\parallel^2}{2k^2}(\beta_\parallel-\beta_\perp)\right]+\frac{\beta_\perp}{2}\frac{k_\perp^2}{k^2}\frac{T_{e\perp}}{T_{e\parallel}}=0. \tag{18}$$

It should be mentioned that the above dispersion relation can be obtained from the linearized Vlasov-Maxwell equations by evaluating $\tilde{n}_e$ from Eq. (14) and $\tilde{u}_{ey}$ from

$$\int_0^{2\pi}d\theta\sin\theta\tilde{f}_e=\frac{\pi e i}{T_{e\perp}}[\omega-(1-\alpha_e)k_\parallel v_\parallel]\left\{\frac{k_\perp v_\perp}{\Omega_e}\frac{\tilde{\varphi}}{\omega-k_\parallel v_\parallel}-\frac{k_\perp v_\perp}{\Omega_e}\left(1-\frac{k_\perp^2}{k_\parallel^2}\frac{k_\parallel v_\parallel}{\omega-k_\parallel v_\parallel}\right)\frac{\tilde{A}_x}{ck_\perp}+i\frac{v_\perp}{c}\left(\frac{\omega-k_\parallel v_\parallel}{\Omega_e^2}-\frac{k_\perp^2v_\perp^2}{2\Omega_e^2}\frac{1}{\omega-k_\parallel v_\parallel}\right)\tilde{A}_y\right\}f_e.$$

We first consider $k_\parallel^2/k^2\ll 2/\beta_\parallel$. Then one of the roots of Eq. (18) corresponds to the slow magnetosonic mode with $\omega^2\sim\omega_s^2$, and is given by

$$\frac{\omega^2}{\omega_s^2}\left[1+\frac{\beta_\perp}{2}\left(2-\frac{T_{e\perp}}{T_{e\parallel}}\right)\right]\simeq 1+\beta_\perp\left(1-\frac{T_{e\perp}}{T_{e\parallel}}\right). \tag{19}$$

This becomes unstable if

$$1<(T_{e\perp}/T_{e\parallel})[\beta_\perp/(1+\beta_\perp)]<2.$$

The other root, corresponding to the fast magnetosonic mode with $\omega^2\sim k^2v_A^2$, is given by

$$1-\frac{\omega^2}{k^2v_A^2}+\frac{\beta_\perp}{2}\left(2-\frac{T_{e\perp}}{T_{e\parallel}}\right)\simeq 0,$$

and becomes unstable if

$$(T_{e\perp}/T_{e\parallel})[\beta_\perp/(1+\beta_\perp)]>2.$$

This we call the field-swelling instability.

In the other limit, $k_\parallel\gg k_\perp$, we have

$$\omega^2=k_\parallel^2v_A^2[1-\tfrac{1}{2}(\beta_\parallel-\beta_\perp)],$$

the instability criterion being

$$\beta_\parallel-\beta_\perp>2,$$

and this is the so-called "firehose" instability.[2] In this case, $\tilde{E}_z=0$, $\tilde{B}_z=0$ implying $\tilde{n}_e=\tilde{n}_i=0$ and $\tilde{T}_{e\perp}=0$.

Another well-known instability driven by an electron temperature anisotropy is the Weibel mode.[3] This differs from the one we present here in that it develops only in zero or weak magnetic fields, for which $\omega>\Omega_e$, it does not involve the ion population, it does not produce density perturbations, and it is characterized by $\vec{k}\cdot\vec{E}=0$. Another related kind of instability reported in the literature[4] involves resonant interaction of the mode having $\omega=k_\parallel v_A$ and $k_\parallel\gg k_\perp$ with the tail of the ion distribution in velocity space. However, in this case the driving temperature anisotropy is that of the ion population.

Finally we refer to Eq. (19) and note that for $T_{e\perp}=T_{e\parallel}$ and $\beta>1$ we have $\omega^2\approx\omega_s^2(2/\beta)=k_\parallel^2v_A^2$. Thus $\omega\to 0$ as $B_0\to 0$, and we may argue that this is the limit from which the familiar "tearing" mode, that is driven by the (spatial) gradient of the current density, emerges in a plane "neutral" sheet configuration. In fact, the influence of an electron temperature anisotropy, adding the effects of the Weibel instability to the theory of the tearing mode, was analyzed by Coppi and Rosenbluth.[5] In Ref. 1 it is pointed out that, for $T_{e\perp}\neq T_{e\parallel}$, the root of Eq. (19) $\omega^2\simeq\omega_s^2(1-T_{e\perp}/T_{e\parallel})/[1-T_{e\perp}/(2T_{e\parallel})]$ does not vanish as $B_0\to 0$ and a new kind of "fast" magnetic reconnection process associated with this mode can take place in a neutral sheet configuration.

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