Interpretation of Tests of Time-Reversal Invariance

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Some of the most sensitive tests of time-reversal invariance in particle and nuclear physics involve the measurement of certain polarization quantities in elastic scattering which are thought to be exactly zero when time-reversal invariance holds. It is shown that a nonzero result for such quantities need not indicate a violation of time-reversal invariance, but can be the consequence of a dynamics which is time-reversal invariant but which violates rotation or Lorentz invariance. These two alternatives can be told apart by subsidiary measurements.

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The validity of time-reversal invariance in nuclear and particle physics has recently been under close scrutiny,[1] partly because of some tentative experimental evidence[2] suggesting a violation of this symmetry. It is likely, therefore, that experimental tests of time-reversal invariance will constitute a substantial activity in the coming years.

Such tests in nuclear or particle physics can be of several types. The conceptually simplest type simply compares[3] any experimental observable (e.g., differential cross section) is a given reaction with the same observable in the time-reversed reaction. From an experimental point of view, however, this type of a test can involve considerable uncertainties, since in most cases the time-reversed reaction is quite different from the original one, and involves different types of accelerators, different detection techniques, and different targets, and hence, the systematic errors and uncertainties in the two reactions are very different. Thus, a common calibration of the two reactions is not easy, and makes evidence derived from such reactions somewhat uncertain.

From this point of view a second type of test presents many fewer difficulties. In this type a reaction is considered which is its own time-reversed reaction[4] (that is, elastic scattering), and in it those experimental observables are measured which are supposed to vanish in a time-reversal–invariant dynamics. Not only does this type avoid the problem of intercalibration between two reactions, but it also constitutes a null experiment which allows the detection of very tiny deviations from time-reversal invariance.

The theory of such tests can be formulated independently of particular dynamic assumptions, that is, based entirely on symmetries. I will in fact do that in this note. My aim will be to demonstrate that tests of the second type depend not only on time-reversal invariance but, also, on rotation or Lorentz invariance. In other words, the apparently time-reversal–noninvariant effects that are usually listed can be a consequence either of a rotation-invariant but time-reversal–noninvariant dynamics or of a time-reversal–invariant but rotation-invariance–violating dynamics. Thus, the interpretation of such tests of the second type is not as clear-cut as was assumed previously. One can, however, carry out additional measurements which distinguish between the two possibilities. That rotation or Lorentz invariance is an ingredient in such tests was implicitly, though not explicitly,[5] recognized previously, but the similarity or dissimilarity of the ways in which the tests fail when we have a violation of time-reversal invariance or of rotation invariance, respectively, has not been discussed.

The situation here is in many ways similar to that of parity tests which was discussed recently,[6] and where a similar ambiguity exists. It is, nevertheless, advisable to carry out the reasoning for the time-reversal case separately in order to prove the ambiguity in that case also, and in order to see the similarities and differences between the two cases.

Rather than discuss the problem in terms of a general but, by necessity, abstract formalism, I will instead illustrate it on a relatively simple and well-known example, that of the elastic scattering of a spin-$\frac{1}{2}$ particle on a spin-0 particle. There is one rather inessential disadvantage of choosing this particular example. In that reaction (and in that reaction only)[7] when parity conservation holds, the additional imposition of time-reversal invariance generates no new constraints. For this reason our demonstration will be for a dynamics which is not parity conserving. It is clear that this circumstance in no way impairs the argument.

To describe the reaction matrix of the process we are considering, I will use a three-dimensional notation and span the space with a set of three unit vectors which has been used previously in polarization studies.[8] Clearly, the particular notation used does not matter from the point of view of the validity of the results, and this particular one has convenient transformation properties under time reversal.

There will be various coefficients ("amplitudes") appearing in the discussion. In order to exhibit explicitly the time-reversal properties of such coefficients, they will be written in two parts, the first invariant under time reversal, the second changing sign under such transformation. Thus, we write

$$C \equiv C^T + C^\perp. \tag{1}$$

If rotation invariance holds, the M matrix for the reaction $\frac{1}{2}+0 \rightarrow \frac{1}{2}+0$ will have four terms, and thus can be written as

$$M^{(R)} = (a_0^T + a_0^\perp) + (a_1^T + a_1^\perp)\vec{\sigma}\cdot\hat{l} + (a_2^T + a_2^\perp)\vec{\sigma}\cdot\hat{m} + (a_3^T + a_3^\perp)\vec{\sigma}\cdot\hat{n}, \tag{2}$$

where

$$\hat{l} = \frac{\vec{q}_2 - \vec{q}_1}{|\vec{q}_2 - \vec{q}_1|}, \quad \hat{m} \equiv \frac{\vec{q}_2 \times \vec{q}_1}{|\vec{q}_2 \times \vec{q}_1|}, \quad \hat{n} \equiv \hat{l} \times \hat{m}, \tag{3}$$

with $\vec{q}_1$ denoting the initial center-of-mass momentum (say, of particle A) and $\vec{q}_2$ the final center-of-mass momentum (say of particle C in the reaction $A+B \rightarrow C+D$). Note that under time reversal $\vec{\sigma}$, $\hat{l}$, $\hat{m}$, and $\hat{n}$ transform into $-\vec{\sigma}$, $+\hat{l}$, $-\hat{m}$, and $-\hat{n}$, respectively. The components of $\vec{\sigma}$ are the Pauli spin matrices. Furthermore, if rotation invariance holds, the a_i's can depend only on the rank-zero tensors that can be formed from the vectors that describe the kinematics. In this case these vectors are $\vec{q}_1$ and $\vec{q}_2$, and so these rank-zero tensors are q_1^2, q_2^2, and $\vec{q}_1 \cdot \vec{q}_2$. Since $q_1^2 = q_2^2$, and under time reversal $\vec{q}_1 \rightarrow -\vec{q}_2$, all three rank-zero tensors are in fact time-reversal invariant and thus in (2) we have

$$a_0^\perp = a_1^\perp = a_2^\perp = a_3^\perp = 0. \tag{4}$$

If, keeping rotation invariance, we now impose time-reversal invariance also, then the second term must vanish, and we are left with three instead of four independent nonzero terms in the M matrix:

$$M^{(R,T)} = a_0^T + a_2^T\vec{\sigma}\cdot\hat{m} + a_3^T\vec{\sigma}\cdot\hat{n}. \tag{5}$$

It is this reduction from four to three terms in the M matrix that generates all the tests of time-reversal invariance that can be devised for this reaction. If there is no such reduction, no experimental observable vanishes and, hence, there are no tests. This remark will be important in what follows.

Let us assume now that rotation invariance does not hold and thus there is some preferred direction in space. For the sake of simplicity, but thereby not reducing the generality of the argument, I will assume that this anisotropy of space can be described in terms of a vector (or pseudovector), $\hat{U}$, which points in the preferred direction. In order to keep the discussion general, we will write $\hat{U}$ in the most general form,

$$\hat{U} = (\alpha^T + \alpha^\perp)\hat{l} + (\beta^T + \beta^\perp)\hat{m} + (\gamma^T + \gamma^\perp)\hat{n}. \tag{6}$$

In the presence of such a preferred direction, the M matrix will be modified in three ways. First, the M matrix need no longer be a rank-zero tensor, but could contain vector, tensor, etc. terms. I will *not* discuss this particular aspect of the change in M, simply because for my argument it will be sufficient to consider the other two. The second way in which M will be different is the dependence of the a_i's on rank-zero tensors, since now, in addition to the two $\vec{q}$'s, we have $\hat{U}$ also at our disposal to form rank-zero tensors. Finally, the third way of modifying M is to add new terms in the M matrix since now we can form such terms not only out of the $\vec{q}$'s and $\vec{\sigma}$, but also out of $\hat{U}$. My argument will utilize these last two ways of modifying M.

In particular, the fact that, in forming rank-zero tensors on which the a_i's can depend, we now have $\hat{U}$ available also means that these tensors now include not only q_1^2, q_2^2, and $\vec{q}_1 \cdot \vec{q}_2$ (all three of which, as we saw, are time-reversal invariant), but also tensors like $\vec{q}_1 \cdot \hat{U}$, $\vec{q}_2 \cdot \hat{U}$, $\vec{q}_1 \cdot \vec{q}_2 \times \hat{U}$, etc., the transformation properties of which are mixed under time reversal. Thus, with $\hat{U}$ present the a_i's now will have both a time-reversal–invariant and a time-reversal–changing part.

The fact that additional terms in the M matrix are now possible has the following consequences. We can write these new terms in the following form:

$$M' = M^{(R)} + (b_1^T + b_1^\perp)\vec{\sigma}\cdot\hat{U} + (b_2^T + b_2^\perp)\vec{\sigma}\cdot\hat{l}\times\hat{U} + (b_3^T + b_3^\perp)\vec{\sigma}\cdot\hat{m}\times\hat{U} + (b_4^T + b_4^\perp)\vec{\sigma}\cdot\hat{n}\times\hat{U}. \tag{7}$$

Inserting U from (6) we can write, using $\hat{l}\times\hat{m}=\hat{n}$, $\hat{m}\times\hat{n}=\hat{l}$, $\hat{n}\times\hat{l}=\hat{m}$,

$$\begin{aligned}
\vec{\sigma}\cdot\hat{U} &= (\alpha^T+\alpha^\perp)\vec{\sigma}\cdot\hat{l} + (\beta^T+\beta^\perp)\vec{\sigma}\cdot\hat{m} + (\gamma^T+\gamma^\perp)\vec{\sigma}\cdot\hat{n},\\
\vec{\sigma}\cdot\hat{l}\times\hat{U} &= -(\gamma^T+\gamma^\perp)\vec{\sigma}\cdot\hat{m} + (\beta^T+\beta^\perp)\vec{\sigma}\cdot\hat{n},\\
\vec{\sigma}\cdot\hat{m}\times\hat{U} &= (\gamma^T+\gamma^\perp)\vec{\sigma}\cdot\hat{l} - (\alpha^T+\alpha^\perp)\vec{\sigma}\cdot\hat{n},\\
\vec{\sigma}\cdot\hat{n}\times\hat{U} &= -(\beta^T+\beta^\perp)\vec{\sigma}\cdot\hat{l} + (\alpha^T+\alpha^\perp)\vec{\sigma}\cdot\hat{m}.
\end{aligned} \tag{8}$$

Combining (7) and (8) we obtain

$$\begin{aligned}
M' &= (a_0^T+a_0^\perp) + [(a_1^T+a_1^\perp) + (b_1^T+b_1^\perp)(\alpha^T+\alpha^\perp) + (b_3^T+b_3^\perp)(\gamma^T+\gamma^\perp) - (b_4^T+b_4^\perp)(\beta^T+\beta^\perp)]\vec{\sigma}\cdot\hat{l}\\
&\quad + [(a_2^T+a_2^\perp) + (b_1^T+b_1^\perp)(\beta^T+\beta^\perp) - (b_2^T+b_2^\perp)(\gamma^T+\gamma^\perp) + (b_4^T+b_4^\perp)(\alpha^T+\alpha^\perp)]\vec{\sigma}\cdot\hat{m}\\
&\quad + [(a_3^T+a_3^\perp) + (b_1^T+b_1^\perp)(\gamma^T+\gamma^\perp) + (b_2^T+b_2^\perp)(\beta^T+\beta^\perp) - (b_3^T+b_3^\perp)(\alpha^T+\alpha^\perp)]\vec{\sigma}\cdot\hat{n}\\
&\equiv (a_0^T+a_0^\perp) + [(a_1^T+a_1^\perp) + (\omega_1^T+\omega_1^\perp)]\vec{\sigma}\cdot\hat{l} + [(a_2^T+a_2^\perp) + (\omega_2^T+\omega_2^\perp)]\vec{\sigma}\cdot\hat{m}\\
&\quad + [(a_3^T+a_3^\perp) + (\omega_3^T+\omega_3^\perp)]\vec{\sigma}\cdot\hat{n}.
\end{aligned} \tag{9}$$

Let us now impose time-reversal invariance on (9). We get

$$\begin{aligned}
M' &= a_0^T + [a_1^\perp + (b_1^T\alpha^\perp + b_1^\perp\alpha^T) + (b_3^T\gamma^\perp + b_3^\perp\gamma^T) - (b_4^T\beta^\perp + b_4^\perp\beta^T)]\vec{\sigma}\cdot\hat{l}\\
&\quad + [a_2^T + (b_1^T\beta^T + b_1^\perp\beta^\perp) - (b_2^T\gamma^T + b_2^\perp\gamma^\perp) + (b_4^T\alpha^T + b_4^\perp\alpha^\perp)]\vec{\sigma}\cdot\hat{m}\\
&\quad + [a_3^T + (b_1^T\gamma^T + b_1^\perp\gamma^\perp) + (b_2^T\beta^T + b_2^\perp\beta^\perp) - (b_3^T\alpha^T + b_3^\perp\alpha^\perp)]\vec{\sigma}\cdot\hat{n}.
\end{aligned} \tag{10}$$

We note that even in the presence of time-reversal invariance, we continue to have four terms in the M matrix and hence, as mentioned earlier, no experimental observables will vanish identically. In particular, those experimental observables that would vanish if we had $M^{(R,T)}$ for the M matrix will now be nonzero, *not* because our dynamics is time-reversal changing, but because it is rotation-invariance violating.

We can see that this effect is quite independent of the time-reversal behavior of $\hat{U}$. For example, even if we constrain $\hat{U}$ to be entirely time-reversal invariant, that is, we choose

$$\hat{U} = \alpha^T\hat{l} + \beta^\perp\hat{m} + \gamma^\perp\hat{n} \quad (\alpha^\perp = \beta^T = \gamma^T = 0), \tag{11}$$

we will continue to have four terms in $M'^{(T)}$, for two reasons: (a) because the a_i's continue to have both a time-reversal–invariant and a time-reversal–changing part, and (b) because the ω_i's also continue to have both kinds of parts. As a result, both the a_i's and the ω_i's continue to make a contribution to each of the four terms. The same holds if we make $\hat{U}$ entirely time-reversal changing.

The effect, therefore, does *not* arise because of the specific time-reversal properties of $\hat{U}$. It arises *because of the mere existence of* $\hat{U}$, which bestows on this four-particle reaction properties "normally" associated with more-than-four-particle reactions, and we know[9] in the latter case that tests of symmetries cannot be carried out.

There are ways, however, to tell the effects of rotation noninvariance and the effects of time-reversal noninvariance apart. For example, if the preferred direction is a macroscopic one due to cosmological circumstances pointing in a given direction in the universe, then carefully averaging the measurements over a day or over a year, during which the earth rotates around that preferred direction, eliminates the effect due to rotation noninvariance. To be sure, the existing experimental data were probably also taken over an extended period of time, for example, during parts of many days, but it is unlikely that running schedules of accelerators and other factors would have automatically assured that the data taking was in fact carefully averaged over the various orientations of the earth in space.

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Experimental Tests of Higher-Order Quantum Electrodynamics at Small Distances

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A direct test is presented of higher-order QED (α^4) at large momentum transfers (up to ~ 100 GeV^2). These tests were carried out with the MARK-J detector at PETRA by comparing the measured cross section for the process $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ with the prediction of QED for 12 GeV $\leq \sqrt{s} \leq$ 36.7 GeV. The cross sections and the various kinematic distributions agree with QED.

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There have been many tests of QED processes over the last two decades, and they can be grouped into two classes[1]:

(a) Precision tests of QED such as the anomalous magnetic moment ($g-2$) of muons and electrons. These processes test higher-order QED at small momentum transfers.

(b) High-energy processes such as photoproduction of lepton pairs, Bhabha scattering, and muon-pair production from e^+e^- colliding beams. These processes test first-order QED at large momentum transfers and small distances.

We present new results from an experiment which is a combination of both classes and which

