Simple Interpretation of Interacting-Boson-Approximation Wave Functions and Transition Rates in Deformed Nuclei

R. F. Casten and D. D. Warner
Brookhaven National Laboratory, Upton, New York 11973
(Received 23 December 1981)

It is shown that an expansion of interacting-boson-approximation wave functions for deformed nuclei in an SU(3) basis leads to a simple interpretation of intrinsic excitations and $E2$ transitions and that it is possible to express these results in terms of a single, empirical parameter. Effective $\Delta K = 0$ and 2 matrix elements can also be extracted. The former is shown to be the dominant mode of SU(3)-symmetry breaking, leading to a natural explanation for the empirical behavior of β and γ vibrations.

PACS numbers: 21.60.Cs, 21.60.Ev, 23.20.Ck

There has recently[1-3] been considerable interest and controversy surrounding the application of the interacting boson approximation (IBA) to deformed nuclei, in particular as regards the relationship between its modes of excitation and those of the familiar geometrical model. The resolution of these questions requires an improved understanding of the underlying structure and characteristics of the IBA itself. It is the purpose of this Letter to contribute towards such an understanding by presenting a simple analysis of an IBA Hamiltonian for deformed nuclei and by expressing the resulting wave functions and $E2$ matrix elements in a new basis that renders their structure, interactions, and physical interpretation particularly transparent.

The standard IBA Hamiltonian used,[1] to date, for deformed nuclei is

$$H = -\kappa Q\cdot Q - \kappa' L\cdot L + \kappa'' P\cdot P, \qquad (1)$$

where L, Q, and P are operators involving the creation and destruction of s and d bosons. (The coefficients κ, κ', and κ'' are related to the input parameters QQ, ELL, and PAIR of the computer program PHINT[4] by the relations $\kappa = -\mathrm{QQ}/4$, $\kappa' = -\mathrm{ELL}/2$, $\kappa'' = \mathrm{PAIR}$.) The SU(3) limit[5] is generated by the first two terms and consists of a series of rotational bands in degenerate groupings corresponding to representations defined by the quantum numbers (λ,μ). Note that, in general, rotational bands will enter the current discussion in three different contexts: as pure SU(3) excitations, as broken SU(3) excitations resulting from a realistic IBA calculation, and as the geometrical excitations themselves. Despite essential differences in detail, especially for the lowest $K=0$ excitation and for the multiphonon-like states,[6] the SU(3) states bear a superficial similarity to the geometrical model excitations. Thus, it is useful to define a distinguishing, yet familiar, terminology. A primed notation (g', β', γ', $\gamma\gamma'$, etc.) will refer to pure SU(3) excitations while those arising from a realistic IBA calculation will be labelled "g," "β," etc. The unadorned notation will refer to the geometrical concepts. In the SU(3) limit, the lowest four representations and the rotational bands therein[5] are $(\lambda,0)$, $K=0$ (g'); $(\lambda-4,2)$, $K=0$ (β'), 2 (γ'); $(\lambda-8,4)$, $K=0$ $(\beta\beta')$, 2 $(\beta\gamma')$, 4 $(\gamma\gamma')$; $(\lambda-6,0)$, $K=0$ $(\gamma\gamma')$. In most actual deformed nuclei, the lowest $K=0^+$ band is above the lowest $K=2^+$ band. To reproduce this, the symmetry-breaking $P\cdot P$ interaction is introduced.

Further consideration of Eq. (1) allows a remarkable simplification and insight into the behavior of the IBA. First, the operator L is diagonal and leaves wave functions unaffected. Therefore, if energies are expressed as differences between states of equal spin, the $L\cdot L$ term can be completely ignored. Second, the boson basis states on which the Hamiltonian acts are those of the SU(5) limit. Since there is no explicit boson energy (ϵ_d) term in Eq. (1), these basis states are initially degenerate. Then, it is a general quantum mechanical result that a scaling of the total interaction among them cannot affect the wave functions, which therefore can *depend on only one parameter, the ratio* κ''/κ, and not on the size of each term separately. Energy differences, such as $E_{2_2}-E_{2_1}$, however, depend on both κ''/κ and κ but, since they simply scale with the latter, the expression $(E_{2_i}-E_{2_1})/4\kappa$ depends only on the symmetry-breaking parameter κ''/κ for a specific boson number N. This dependence, shown in Fig. 1 (top), is uniquely defined. Therefore, *all results of such an IBA calculation* can be equally well determined by an *empirical energy expression.* The quantity $E_{0_2}(E_{2_2}-E_{2_1})^{-1}-1$ is convenient: It depends only on energy levels commonly known and goes to zero in the SU(3) limit (i.e.,

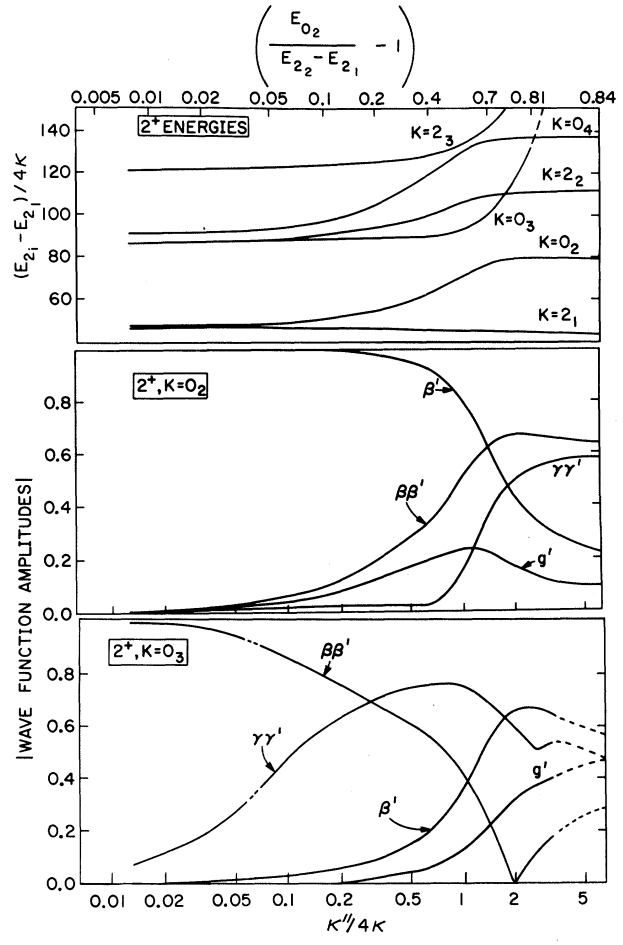

FIG. 1. Energies and major SU(3) wave function amplitudes (absolute values) for 2^+ states in the lowest band vs magnitude of SU(3) symmetry breaking. $N = 16$. Note that $\kappa''/4\kappa$ is used as the symmetry-breaking parameter because of its close relation to PAIR/QQ as is evident from the definitions below Eq. (1).

as $\kappa'' \to 0$). In Fig. 1 this equivalent parameter is indicated along the top.

As mentioned, the IBA wave functions are normally expressed[4] in an SU(5) basis. As this is a poor basis for deformed nuclei the wave functions are extremely complex. Furthermore, none of the wave-function or matrix-element components, corresponding as they do to states or transitions in a vibrational nucleus, has a clear interpretation in a deformed basis. An enormous simplification occurs, however, if the wave functions are reexpanded in the SU(3) basis itself. Not only will there now be fewer sizable components, but each will have a much closer correspondence to a deformed description and a simple behavior as a function of the SU(3) symmetry breaking.

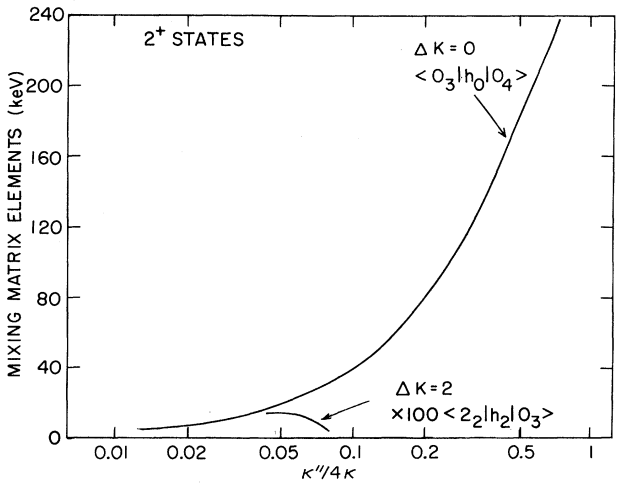

FIG. 2. $\Delta K = 0$ and 2 interaction matrix elements between SU(3) states vs magnitude of symmetry breaking. $N = 16$.

These concepts are illustrated for two 2^+ states in Fig. 1 for $N = 16$. [For other N, the wave functions are very little altered while the energies roughly scale with N (to $\approx 10\%$).] The simplicity and systematic behavior of the wave functions are evident. Thus, for example, the "β" band, initially situated between the g' and $\beta\beta'$ bands, contains substantial amplitudes for both and, for typical deformed nuclei ($\kappa''/4\kappa \approx 0.3 - 1.5$), can hardly be considered a pure SU(3) excitation. The next excited, or $(0^+)_3$ band, has $\beta\beta'$ character in SU(3) but, as the $P \cdot P$ interaction increases, the $2_{\beta\beta'}$ level strongly mixes with the nearby $2_{\gamma\gamma'}$ level. For a $\kappa''/4\kappa$ value of ≈ 1 (e.g., ^{168}Er), the 2^+ $K = (0^+)_3$ state is already largely $\gamma\gamma'$ in character. The "g" and "γ" bands (not shown), however, are much purer: The principal effect of symmetry breaking is to induce small admixtures of β' and $\beta\gamma'$ components, respectively.

The mixings just discussed correspond, in effect, to $\Delta K = 0$ interactions. For $\kappa''/4\kappa \leq 0.5$, Fig. 1 shows that the 2^+ state of the $(0^+)_3$ band has essentially only two dominant amplitudes, $\beta\beta'$ and $\gamma\gamma'$. Thus, given the initial SU(3) and admixed energies, and the two-state mixing amplitudes, it is trivial to deduce the approximate $\Delta K = 0$ matrix element. Figure 2 shows that this is indeed a very large interaction, ≈ 100–200 keV for typical deformed nuclei. Similar analyses show that the β'-g' and γ'-$\beta\gamma'$ $\Delta K = 0$ matrix elements are similar in size and that all these matrix elements depend little on spin as well (for $I < 6$). Thus, a nearly structure-independent $\Delta K = 0$ mixing be-

tween certain pairs of bands is, in fact, the dominant mode of SU(3) symmetry breaking, since $\Delta K = 2$ mixing is virtually absent. The latter is only noticeable in principle in Fig. 1 for $2_{\beta\beta'}$-$2_{\beta\gamma'}$ mixing since these levels are initially degenerate. They admix strongly only over the range of $\kappa''/4\kappa$ (too narrow to display in Fig. 1) corresponding to the dashed gap (at $\kappa''/4\kappa \approx 0.066$) in the plotted $K = (0^+)_3$ amplitudes. By a two-state mixing analysis similar to the above, a $\Delta K = 2$ matrix element of ≈ 0.1 keV is found for this range of $\kappa''/4\kappa$ (see Fig. 2).

Turning now to $E2$ matrix elements, in IBA-1 the $E2$ operator[5] is $T(E2) = [(s + \tilde{d} + d^+ s)^{(2)} + (R/\sqrt{5})(d^+\tilde{d})^{(2)}]$. In the strict SU(3) limit $R = -\frac{1}{2}\sqrt{35} = -2.958$ and the $E2$ selection rule is $\Delta(\lambda, \mu) = 0$. However, it has been shown[6] that one *must* utilize a smaller R value to account for nonzero interrepresentation (e.g., $\beta' \to g'$ or $\gamma' \to g'$) transition strengths in actual deformed nuclei. It has also been shown,[6] though, that a narrow range of R values approximately fits all rare-earth deformed nuclei in which the "β" band lies above the "γ" band. Since this feature characterizes the nuclei treated in this Letter, this range may be approximated by its mean, $R = -0.85$, which may be used to calculate, for a given boson number, a set of *elementary SU(3) transitions*. Then, the simplicity of the IBA wave functions in the SU(3) basis can be exploited to simplify enormously the structure of the $M(E2)$ values by expressing them as a sum of terms, each consisting of the strength of a given standard SU(3) transition times the amplitudes for the appropriate initial and final SU(3) states in the calculated IBA wave functions. Indeed, instead of the hundreds of components comprising an $M(E2)$ value in the SU(5) basis, most $M(E2)$ values can now be closely approximated by a coherent combination of < 5 contributions, each corresponding to a single SU(3) transition (e.g., $\gamma' \to g'$ or $\beta\beta' \to \beta'$) with a simple interpretation in a deformed context.

Figure 3 gives two examples, for deexcitations of the lowest $K = 0^+$ band, and Fig. 4 shows the composition of several transitions for the specific $\kappa''/4\kappa$ value of 0.94 used[1] for ^{168}Er. Close to the SU(3) limit, the transitions in Fig. 3 closely resemble the SU(3) $\beta' \to \gamma'$ and $\beta' \to g'$ transitions, but, as the symmetry breaking grows, the wave functions become more complex, and so does the composition of the $E2$ transitions. Thus, for the transitions to the nearly pure "γ" band (Fig. 3, bottom) the growing importance of g' and $\beta\beta'$ amplitudes in the $(0^+)_2$ band is reflected in the growing $g' \to \gamma'$ and $\beta\beta' \to \gamma'$ amplitudes. The transition (Fig. 3, top) to the ground band naturally consists primarily of the $\beta' \to g'$ amplitude near the SU(3) limit. However, because the β' and g' bands mix via the strong $\Delta K = 0$ interaction, the diagonal components $g' \to g'$ and $\beta' \to \beta'$ quickly dominate all others because of the large quadrupole moments

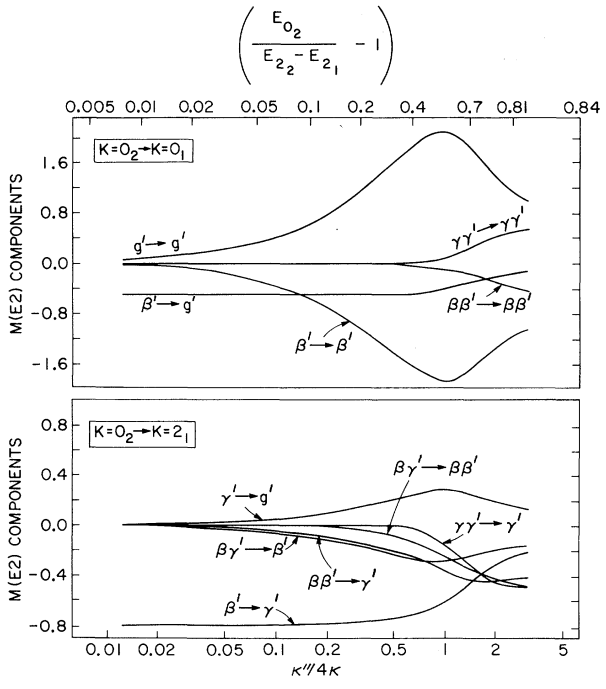

FIG. 3. Major SU(3) components of two $2^+ \to 2^+$ transitions vs symmetry breaking. $N = 16$.

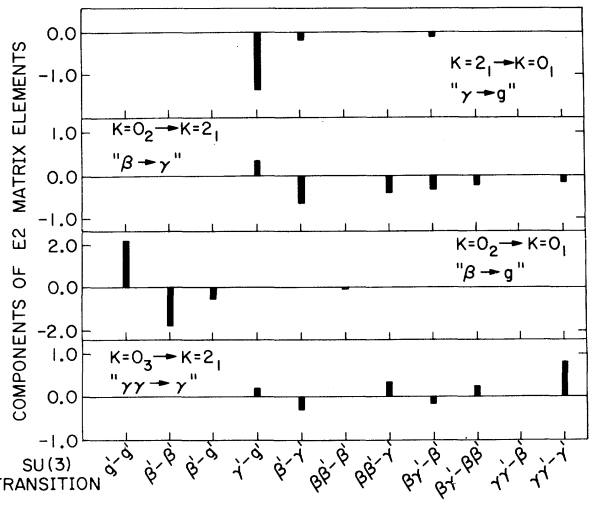

FIG. 4. SU(3) components of four $E2$ $2^+ \to 2^+$ transitions for $\kappa''/4\kappa = 0.94$ [^{168}Er (Ref. 1)].

involved. Therefore, for typical deformed nuclei (see also Fig. 4), the net "β" $\rightarrow$ "g" $E2$ transition strength arises from both a direct $\beta' \rightarrow g'$ matrix *element and a difference in the quadrupole moments of the β' and g' SU(3) states.* It is striking that this latter mechanism is analogous to that suggested in a microscopic interpretation[7] of the geometrical β band.

As a final example, Fig. 4 (bottom) vividly shows that while the $\gamma\gamma' \rightarrow \gamma'$ SU(3) transition dominates the $2^+[K=(0^+)_3] \rightarrow 2^+[K=(2^+)_1]$ decay, the total $M(E2)$ depends critically on the detailed coherence. Thus, it is simplistic to consider the transition as simply the decay of a $\gamma\gamma'$ vibration even though the net strength does happen to approximate that of this elementary SU(3) transition.

It is of interest in concluding to draw an analogy with the conventional geometrical description of deformed nuclei where the elementary excitation modes are γ and β vibrations which are normally identified directly with the lowest $K=0^+$ and 2^+ excitations in each nucleus. Thus, in general, these "elementary" modes can in fact differ substantially from nucleus to nucleus. In the IBA, as treated here, the elementary modes are the basic SU(3) excitations and, as such, have similar structure throughout the region. In the geometrical description, perturbations to the leading-order predictions for β and γ vibrations are parametrized[7] in terms of $\Delta K=0$ or 2 matrix elements both of which are deduced to be small (tenths of a kiloelectronvolt). In the IBA, $\Delta K=2$ mixing between SU(3) states is also small but the $\Delta K=0$ mixing is orders of magnitude larger. Thus, the lowest $K=2^+$ band remains a rather pure SU(3) excitation (γ') across the deformed region, displaying a constancy in structure reflected in the empirical data. For the lowest $K=0^+$ excitation, however, there are large empirical variations which arise naturally in the IBA via the high sensitivity of the $\Delta K=0$ matrix element to the value of $\kappa''/4\kappa$. The phenomenological geometrical approach effectively disguises such variations by redefining the β vibration in each nucleus. As such, it cannot easily, for example, reproduce such features as the observed[1] strong $\Delta K=2$, "β" $\rightarrow$ "γ" $E2$ transitions which arise in the IBA as remnants of the SU(3) symmetry.

This research was performed under Contract No. DE-AC02-76CH00016 with the U. S. Department of Energy.

[1]D. D. Warner, R. F. Casten, and W. F. Davidson, Phys. Rev. Lett. 45, 1761 (1980), and Phys. Rev. C 24, 1713 (1981).

[2]A. Bohr and B. R. Mottelson, Phys. Scr. 22, 468 (1980), and 25, 28 (1982).

[3]F. K. McGowan, Phys. Rev. C 24, 1803 (1981); P. VanIsacker *et al.*, to be published.

[4]Code PHINT written by O. Scholten.

[5]A. Arima and F. Iachello, Ann. Phys. (N.Y.) 111, 201 (1978).

[6]D. D. Warner and R. F. Casten, Phys. Rev. C, to be published.

[7]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.