Do Multiquark Hadrons Exist?

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The $qq\bar{q}\bar{q}$ system has been examined by solving the four-particle Schrödinger equation variationally. The main findings are that (1) $qq\bar{q}\bar{q}$ bound states normally do not exist, (2) the cryptoexotic 0^{++} sector of this system with $K\bar{K}$ quantum numbers is probably the only exception to (1) and its bound states can be identified with the S^* and δ just below $K\bar{K}$ threshold, (3) $qq\bar{q}\bar{q}$ bound states provide a model for the weak binding and color-singlet clustering observed in nuclei, and (4) there is no indication that this system has strong resonances.

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When supplemented with ingredients from chromodynamics, nonrelativistic quark-potential models enjoy considerable success in describing mesons and baryons.[1] It is natural to try to extend these models to multiquark sectors, both in the hope of uncovering interesting new phenomena and out of a desire to understand nuclei, the only known multiquark systems. One of the principle conclusions of the work on which we report here[2] is that *apart from nuclear-physics-like states, multiquark hadrons probably do not exist.* We have drawn this conclusion after examining the $qq\bar{q}\bar{q}$ system—the simplest system which is neither strictly forbidden (like q, qq, $qq\bar{q}$, etc.) nor required (like $q\bar{q}$ and qqq) by the confinement of color—and finding, contrary to previous studies in both potential models and in the bag model, no evidence for "true" $qq\bar{q}\bar{q}$ hadrons. We do, however, find that in special circumstances a $qq\bar{q}\bar{q}$ bound state can exist with properties remarkably like that of the deuteron: It is essentially a weakly bound state of two color-singlet mesons ($q\bar{q}-q\bar{q}$) in a wave function with an extension much greater than the mesonic radius.

Even if mesonic nuclei do not exist in nature, we believe that they are an interesting quark-based model for ordinary nuclei. However, we present a series of arguments below in favor of interpreting the $J^{PC}=0^{++}$ $S^*(980)$ and $\delta(980)$ just below $K\bar{K}$ threshold as such states. Although the nature of the states is very different, we view our results in this sector as basically confirming Jaffe's proposal[3] for 0^{++} cryptoexotics based on studies of the $qq\bar{q}\bar{q}$ system in the bag model. On the other hand, as will be discussed below, we believe that the potential models offer a better understanding of not only these specific states but of the whole multiquark question.

The results we present here are based on the simplified Hamiltonian

$$H=\sum_{i=1}^{4}\left(m_i+\frac{p_i^{\,2}}{2m_i}\right)-\sum_{i<j}\left[e_0+\tfrac{1}{2}kr_{ij}^{\,2}+\frac{8\pi\alpha_s}{3m_im_j}\tilde{\delta}^3(\vec{r}_{ij})\vec{S}_i\cdot\vec{S}_j\right]\frac{\vec{\lambda}_i}{2}\cdot\frac{\vec{\lambda}_j}{2}\,, \tag{1}$$

in which m_i, $\vec{r}_i$, $\vec{p}_i$, $\vec{S}_i$, and $\frac{1}{2}\vec{\lambda}_i$ ($\vec{\lambda}_i\to-\vec{\lambda}_i^{\,*}$ for antiquarks) are the mass, position, momentum, spin, and color matrix of the quark or antiquark labeled i. [In the color hyperfine interaction in (1) we have smeared the Breit-Fermi $\delta^3(\vec{r})$ to $\tilde{\delta}^3(\vec{r})=\pi^{-3/2}\sigma^3\exp(-\sigma^2r^2)$ with $\sigma\sim O(1/m)$ as expected on physical grounds and as required for finiteness of the Schrödinger equation.] Such a Hamiltonian forbids the formation of isolated color nonsinglets while confining and giving a satisfactory description of both low-lying mesons and baryons.[1,4] On the other hand, it is missing several features of the true Hamiltonian (including an anharmonic component to the potential, $q\bar{q}$ annihilation effects, and spin-orbit effects) and we must hope for now that by taking into account the dominant effects of confinement and strong spin-spin interactions we will not be completely misled.[2]

The usual discussion of the $qq\bar{q}\bar{q}$ sector[5] begins by noting that a color singlet can be obtained in this system in two ways, so that the color state is in general a superposition of $|\underline{3}^*_{12}\underline{3}_{34}\rangle$ and $|\underline{6}_{12}\underline{6}^*_{34}\rangle$ (we take 1 and 2 to be quarks, 3 and 4 to be antiquarks; the notation C_{ij} means that particles i and j are in the color state C). These two sectors [usually called T- (for "true") and M- (for "mock") baryonium, respectively] have heretofore been treated either in isolation or else with mixing between them treated perturbatively. Since in terms of the variables $\vec{x}=\frac{1}{2}(\vec{r}_1+\vec{r}_3-\vec{r}_2-\vec{r}_4)$, $\vec{y}=\frac{1}{2}(\vec{r}_1+\vec{r}_4-\vec{r}_2-\vec{r}_3)$, and $\vec{\lambda}=(\vec{r}_1+\vec{r}_2-\vec{r}_3-\vec{r}_4)$ we have (discarding the center-of-mass variables and looking at the equal-mass case with no hyperfine interactions for simplicity)

$$H - H_{\text{hyp}} = \frac{1}{2m}(p_x^{\,2} + p_y^{\,2} + p_\lambda^{\,2}) + \frac{8e_0}{3} + \tfrac{1}{2}k \begin{bmatrix} 2x^2 + 2y^2 + \frac{4}{3}\lambda^2 & -\sqrt{2}(x^2 - y^2) \\ -\sqrt{2}(x^2 - y^2) & x^2 + y^2 + \frac{10}{3}\lambda^2 \end{bmatrix} \tag{2}$$

in the $(|\underline{3}^*{}_{12}\underline{3}_{34}\rangle, |\underline{6}_{12}\underline{6}^*{}_{34}\rangle)$ basis, if we neglect mixing we have confinement in all three relative coordinates in each color subspace and two towers of excited baryonia spectra. Of course, if one treats the mixing term perturbatively, these discrete spectra will persist; we have found, on the contrary, that the mixing in a potential model like ours is actually so strong that it can completely destroy the discrete spectra, leaving only a continuum of free mesons in this (equal mass, $\alpha_s = 0$) limit. In retrospect this is not surprising: The same Hamiltonian, if it is to be at all relevant, must describe free mesons which could only arise in the $\underline{33}$-$\underline{66}^*$ picture if the mixing term is nonperturbative. In some versions of the bag model the neglect of mixing is justifiable, but this merely avoids the issue: The central question of multiquark stability has merely been hidden in *ad hoc* assumptions about the unknown properties of the bag surface.[6]

We have rigorously examined this issue in the Hamiltonian (1) with equal masses by solving for the ground state of this system variationally.[2] As trial state vectors we use

$$|\psi_\pm\rangle = \cos(30^\circ + \theta_\pm)|(AA)_\pm\rangle + \sin(30^\circ + \theta_\pm)|(SS)_\pm\rangle, \tag{3}$$

where the normalized state vectors $|(AA)_\pm\rangle$ and $|(SS)_\pm\rangle$ are

$$|(SS)_\pm\rangle = (N_{SS\pm}/\sqrt{2})[\psi_{SS\pm}(x,y,\lambda)|\underline{1}_{13}\underline{1}_{24}\rangle \mp \psi_{SS\pm}(y,x,\lambda)|\underline{1}_{14}\underline{1}_{23}\rangle]|S_{12}S_{34}\rangle, \tag{4}$$

$$|(AA)_\pm\rangle = (N_{AA\pm}/\sqrt{2})[\psi_{AA\pm}(x,y,\lambda)|\underline{1}_{13}\underline{1}_{24}\rangle \pm \psi_{AA\pm}(y,x,\lambda)|\underline{1}_{14}\underline{1}_{23}\rangle]|\vec{A}_{12}\cdot\vec{A}_{34}\rangle, \tag{5}$$

where the spin states $|S_{12}S_{34}\rangle$ and $|\vec{A}_{12}\cdot\vec{A}_{23}\rangle$ have the quarks and antiquarks in spin 0 and spin 1, respectively, with total spin 0, and $\pm$ denotes symmetry or antisymmetry of the color-space-spin wave function under interchange of the quarks or antiquarks. These state vectors respect the important symmetries of the Hamiltonian (1) with equal masses and in particular as $x \to \infty$ they give free pseudoscalar and vector mesons in both the (13)(24) and (14)(23) color-singlet channels. In the limit $\alpha_s = 0$, (1) also separately conserves $\vec{L}_x$, $\vec{L}_y$, and $\vec{L}_\lambda$ [see (2)]. We assume that hyperfine mixing will not badly violate these conservation laws in the ground-state wave functions by taking them to have the form

$$\psi_\alpha(x,y,\lambda) = \sum_{j=1}^{j_{\max}} \prod_{i=1}^{3} \sum_{k=1}^{k_{\max}} c_{\alpha ijk} \exp(-\tfrac{1}{2}\beta_{\alpha ijk}^{\;2}\xi_i^{\,2}), \tag{6}$$

where $(\xi_1, \xi_2, \xi_3) \equiv (\vec{x}, \vec{y}, \vec{\lambda})$.[7] Associated with the states $|\psi_\pm\rangle$ are flavor states $|\underline{3}^*{}_{12}\underline{3}_{34}\rangle$ and $|\underline{6}_{12}\underline{6}^*{}_{34}\rangle$ (in a notation analogous to our color notation) so that the $|\psi_+\rangle$ state will be the ground state of 0^{++} in what has been dubbed the cryptoexotic[3] sector (since $\underline{3}^* \times \underline{3} = 1 \oplus 8$ is not exotic); conversely, since $\underline{6} \times \underline{6}^*$ contains exotic flavor representations, $|\psi_-\rangle$ is the ground state of 0^{++} in the exotic sector.

We have looked for minima of (1) with (3) in a 62-parameter variational wave function, explored in a sequential fashion. The studies were all done with use of Hamiltonians which had their parameters fixed from fits to ground-state mesons; this was done both by taking the limit $x \to \infty$ with the states (3) and by using (1) in the $q\bar{q}$ sector.[8]

As stated earlier, with $\alpha_s = 0$ we never find binding. In the case $\alpha_s \neq 0$ required to fit observed meson masses, we never find binding in the exotic state $|\psi_-\rangle$, nor do we find binding in the cryptoexotic state $|\psi_+\rangle$ if the common quark mass m is greater than about 600 MeV (the hyperfine interaction, which is proportional to m^{-2}, has become too weak). However, for m in the range of the masses of the SU(3) constituent quarks (300 MeV $\leq m \leq$ 600 MeV) we do normally find a single weakly bound state. Typical results are shown in Table I. All of our meson fits required $m_d \sim 330$ MeV and $m_s \sim 550$ MeV, so we adopted these values since they also give good fits to baryon spectroscopy, magnetic moments, etc.[1] Given values for ω_0 and σ then determine α_s and e_0, and we have checked that our results are not qualitatively changed by reasonable variations in these parameters.[2]

As a model for nuclear physics, we believe that these results are interesting in their own right: The bound states we find look much like the mesonic analogs of nuclei with weak binding and (nearly) color-singlet clusters separated by several cluster radii. It is our view, however,

TABLE I. The cryptoexotic $qq\bar{q}\bar{q}$ system.[a] μ_P and μ_V are pseudoscalar and vector masses, respectively; r_s is the meson-meson separation and r_m the meson radius.

m (MeV)	μ_P (MeV)	μ_V (MeV)	$E-2\mu_P$ (MeV)	r_s/r_m	θ_+ (deg)
$330 \equiv m_d$	141	771	−52	3.2	1.9
440	508	874	− 7	4.7	0.4
$550 \equiv m_s$	···	1019	unbound	∞	0.0

[a] $\omega_0 \equiv (k/m_d)^{1/2} = 250$ MeV (so that $\omega_{\text{meson}} = 408$ MeV), $\alpha_s = 2.4$, $e_0 = -445$ MeV, $\sigma = 2\ \text{fm}^{-1}$.

that these states are also interesting because they are themselves actually found in nature. To help substantiate this opinion, we must examine what we believe to be the most severe flaw of our model.

It is gradually becoming understood how many relativistic effects are subsumed into the parameters of the nonrelativistic quark model[9]: The constituent quark mass m is, for example, undoubtedly a repository of such effects. However, we cannot expect to be able to hide relativistic effects in the same m for both $q\bar{q}$ and $qq\bar{q}\bar{q}$ and, in particular, we must consider the effects of the fact that our Hamiltonian is treating the weakly bound meson systems as though the mesons had mass $\mu = 2m$ [see the coefficient of p_x^2 in (2)]. We have studied this problem[2] by using our bound-state wave functions to define a meson-meson effective potential $V_{\text{eff}}(x)$. We then use V_{eff} in a meson-meson Schrödinger equation to study the sensitivity of our conclusions to μ. We find that the apparently relatively strongly bound ("$\pi\pi$") system observed for $m = 330$ MeV corresponding to $\mu = 660$ MeV becomes unbound long before μ is reduced (by almost a factor of 5) to the neighborhood of μ_π, but that the ("$K\bar{K}$") binding for $m = 440$ MeV persists nearly down to μ_K (a reduction by less than a factor of 2). We conclude that not only do very heavy quarks not bind (because their hyperfine interactions are too *weak*), but neither do very light quarks (since the corresponding pseudoscalar mesons are too light, i.e., because the intermeson hyperfine interactions are too *strong*). Thus if any such states exist at all, it seems likely that they would be weakly bound $K\bar{K}$ states.[10]

We can, incidentally, also use V_{eff} to conclude that, as would be expected, with such weakly bound ground states, no resonances of two ground-state mesons (i.e., states quasibound in an $l_x > 0$ centrifugal barrier) exist. On this basis we suspect (but have not yet proved) that apart from these possible $0^{++}K\bar{K}$ states, no trace of the original towers of T- and M-baryonia remains in any sector.[11]

It was precisely the two $K\bar{K}$-like states $S^*(980)$ ($IJ^{PC} = 00^{++}$) and $\delta(980)$ ($IJ^{PC} = 10^{++}$) that were the strongest phenomenological candidates for $qq\bar{q}\bar{q}$ cryptoexotics in Jaffe's original proposal.[3] From our present perspective which sees these states as having a quite different character from that originally imagined (e.g., in the bag model the system is not a weakly bound meson-meson system) we can adduce the following evidence (in addition to that already discussed by Jaffe[3] and others) in favor of this interpretation:

(1) The positions of the S^* and δ just below $K\bar{K}$ threshold is no longer an accident.[12]

(2) The absence of the other members of the cryptoexotic nonet is expected.

(3) The correct absolute widths of the S^* and δ emerge from this picture.[2] The decay of the S^*, for example, occurs in this view as a consequence of the inelastic collision $K\bar{K} \to \pi\pi$ of the two weakly bound kaons. The resulting narrow widths of the S^* and δ (we calculate $\Gamma_{S^*} \simeq 15$ MeV and $\Gamma_\delta \simeq 40$ MeV) are a consequence of the weak binding and may be contrasted both with the $q\bar{q}$ interpretation (where the S^* would have a width of the order of 1000 MeV) and with the bag-model interpretation (where the δ would have a very large width into $\eta\pi$ via a "fall-apart" mode).

Taken together, we believe the evidence in favor of the $qq\bar{q}\bar{q}$ interpretation of these two states is now quite strong.

This concludes our report on our principal findings in the simplest multiquark sector.[2] We believe that the picture of the $qq\bar{q}\bar{q}$ system outlined here in which the S^* and δ are the only $qq\bar{q}\bar{q}$ hadrons is appealing. It also seems reasonable to extrapolate the qualitative features of this system to multiquark systems in general. Not the least interesting feature of such an extrapolation lies in the encouragement that it provides to attempts to derive the properties of nuclei from quark potential models. It would certainly be satisfying to in this way bring the study of hadron physics full circle back to its origins.

[1]For a review, see, for example, Nathan Isgur, in *The New Aspects of Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1980), p. 107.

[2]Details are given in John Weinstein and Nathan Isgur, to be published.

[3]R. L. Jaffe, Phys. Rev. D 15, 267, 281 (1977).

[4]The confinement properties of (1) are discussed in Ref. 1. For further references, and especially for a discussion of possible problems with long-range van der Waals forces, see O. W. Greenberg and Harry J. Lipkin, to be published. We hope that by choosing Gaussian trial wave functions we have avoided these spurious long-range effects (see Ref. 2 for a discussion of this point).

[5]The literature is too vast to be listed here (see Ref. 2 for a more complete bibliography), but a sample of various approaches would include in addition to Ref. 3 the following: M. Gavela *et al.*, Phys. Lett. 79B, 459 (1978); H. M. Chan and H. Hogaasen, Nucl. Phys. B136, 401 (1978); I. M. Barbour and D. K. Ponting, Nucl. Phys. B149, 534 (1979); Archibald W. Hendry and Ian Hinchliffe, Phys. Rev. D 18, 3453 (1978); A. T. Aerts, P. J. Mulders, and J. J. de Swart, Phys. Rev. D 21, 1370 (1980).

[6]This problem has been addressed in the bag by R. L. Jaffe and F. E. Low, Phys. Rev. D 19, 2105 (1979), in terms of their P-matrix formalism. The resulting physical interpretation of $qq\overline{q}\overline{q}$ states in the bag is then more consonant with ours, but still very different from it.

[7]Hyperfine interactions cause a similar mixing in baryons where they create a small $[70,0^+]$ component in the nucleon [see Nathan Isgur, Gabriel Karl, and Roman Koniuk, Phys. Rev. Lett. 41, 1269 (1978), and 45, 1738(E) (1980)].

[8]To deal with, for example, the kaons, we used $m = \frac{1}{2}(m_d + m_s)$.

[9]H. J. Lipkin, in *Proceedings of the Fourth International Conference on Baryon Resonances, Toronto, 1980*, edited by Nathan Isgur (University of Toronto, Toronto, 1980), p. 461; Cameron Hayne and Nathan Isgur, Phys. Rev. D (to be published).

[10]We should remind the reader that our results are only applicable to $qq\overline{q}\overline{q}$ systems with four equal (or nearly equal) masses (see Ref. 8). Further work will be required to draw conclusions on, for example, the existence of stable charm-strange exotics [see, e.g., H. J. Lipkin, Phys. Lett. 70B, 113 (1977)].

[11]This suspicion is based on the presumption that sectors describing the possible states of excited mesons will once again display a weak effective potential. The only indication we know which runs counter to these conclusions is the observation by M. B. Gavela *et al.* in Ref. 5 that in the harmonic limit high-l_λ states would be stable. With the known anharmonicities and hyperfine interactions in effect, we discount this possibility.

[12]The importance of this observation for the possbile existence of stable charm-strange exotics (see Ref. 10) was discussed by Nathan Isgur and H. J. Lipkin, Phys. Lett. 99B, 151 (1981).

Short-Distance Behavior of Spontaneously Broken Quantum Chromodynamics

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Light, colored scalars are shown to preserve all presently observed short-distance properties of quantum chromodynamics even when belonging to large multiplets of $SU(3)_c$. However, light-scalar pair production by gluon fusion dominates hadronic high-p_T single-particle inclusive production cross sections at $\bar{p}p$ collider energies ($\sqrt{s} = 540$ GeV) and two-jet production even at $\sqrt{s} \approx 50$–60 GeV. Embedding in a unified SU(5) theory requires many scalars to be superheavy in order to preserve asymptotic freedom below the unification mass.

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There is by now growing evidence[1] for the existence of free fractionally charged particles. On the other hand there is growing theoretical evidence[2] for quark confinement in unbroken quantum chromodynamics (QCD). There are essentially two ways to reconcile these seemingly contradicting observations: Either QCD is spontaneously broken[3,4] or there exist in nature some unconventional heavy, fractionally charged, color-neutral hadrons and/or leptons. A heavy color-neutral hadron could result, for example, from the binding of an exotic heavy neutral color-triplet of quarks (or scalars) with the ordinary fractionally charged quarks. As to the first option, one conventionally introduces a scalar Higgs multiplet which, under certain circumstances, breaks the symmetry of the vacuum and causes some or all gluons to become massive. The gluon mass m_g is expected to be small compared to the mass scale Λ of QCD. Then the mass of colored hadrons, being[3] $O(\Lambda^2/m_g)$, is much higher than the mass of ordinary hadrons $O(\Lambda)$.

