We wish to thank G. E. Brown and A. Sjölander for helpful discussions. This work was supported by an Australian Research Commission Grant.

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Ripplon-Limited Mobility of a Two-Dimensional Crystal of Electrons: Experiment

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(Received 24 August 1981)

A measurement of the mobility of a two-dimensional Wigner lattice supported by a liquid-helium surface is reported, and the results are compared with theory. The data exhibit a minimum in the mobility as a function of temperature. A narrow excess scattering peak located at or near the melting temperature is reminiscent of excess scattering associated with the dissociation of vortex-antivortex pairs in two-dimensional helium films and superconductors. Quantum corrections to the melting curve are observed.

PACS numbers: 73.20.-r, 63.20.-e, 64.70.Dv, 68.10.-m

A two-dimensional (2D) electron lattice supported by a liquid-helium surface forms an interesting and novel system. The simplicity of the interparticle interaction, the freedom from impurities, and a substrate which does not impose a regular potential on the lattice make this system an ideal prototype for the study of 2D lattices.

The scattering of phonons of the 2D electron lattice by liquid-helium ripplons can be investigated by measuring the mobility of an electron lattice supported by a liquid-helium substrate. Such data complement the information on the phonon-ripplon coupling provided by the experiment of Grimes and Adams[1] and the theory of Fisher, Halperin, and Platzman.[2] This measurement further probes the mean square electron displacement, $\langle x^2 \rangle$, which affects the coherence between ripplon crests and the electronic coordinates. Finally, an abrupt change in mobility upon crystallization allows for a precise measurement of the melting curve.

A cross section of our experimental cell, consisting of a plane parallel capacitor with a plate separation of $s = 2$ mm, is shown in Fig. 1. Each plate has dimensions 18×25.4 mm^2 and is divided into three sections of equal area. A dc voltage is applied to the bottom plate to hold electrons against the helium surface, and the top plate is surrounded by guard electrodes used to shape the electron density profile. Electrons are deposited onto the surface from a glow discharge. All electrodes except B_1 and B_3 are at ac ground. An ac voltage of angular frequency ω applied to electrode B_1 causes a redistribution of the electrons, and a change in the density above electrode B_3 occurs after a time delay determined by the

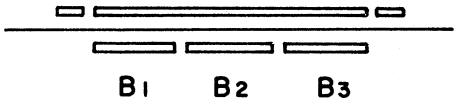

FIG. 1. A cross section of the experimental cell. The line between the plates represents the liquid level.

amount of scattering and the effective mass of the electrons. The mobility is deduced by comparing the phase of the signal induced on electrode B_3 with the phase of the driving signal. This technique was first used by Sommer and Tanner.[3]

The relative phase is measured with a lock-in detector and a feedback loop which employs a voltage-controlled phase shifter to maintain the signal and reference 90° apart. To determine the correct zero phase, a metallic plate located below the bottom electrodes is floated electrically to simulate a zero-resistance electron sheet. Prior to loading of the surface with electrons, this primary reference is used to calibrate a secondary reference circuit which is used to detect phase shifts in the electronics during an experiment. We operated with a driving voltage of 1.25 mV rms and frequencies of 0.75 and 1.0 MHz. Our experiment is capable of measuring time delays between the signal and driving voltage with a precision of 100 psec, and with an accuracy of $\simeq 1$ nsec.

An electrical analog of the sample is a transmission line with per unit length values of resistance $R = (ne\mu w)^{-1}$, inductance $L = m^*/ne^2w$, and capacitance, in the limit $\omega \to 0$, $C = w(\epsilon + 1)/2\pi s$. Here n is the electron density, μ is the dc mobility, w is the width of the lattice, m^* is the effective mass of the electrons, and ϵ is the dielectric constant of the liquid. Note that the dispersion relation follows from the relation $\omega = k/(LC)^{1/2}$. The value of L follows by equating $\frac{1}{2}LI^2 = \frac{1}{2}m^*v^2nw$ and writing the current as $I = nevw$. The value of m^* is enhanced over the electronic mass since a small dimple forms on the helium surface under the electrons.[2] The phase φ of the signal is a function of ωRC and ω^2LC. The values of R and L, and therefore μ and m^*, can be determined by measuring the phase shift at more than one frequency.

Our calculated values of the effective length of our cell and edge corrections for C, L, and R gave an exact fit of the fluid-phase mobility at an areal density of $5\times 10^7/\text{cm}^2$ to the theory of Monarkha.[4] Our data are reduced by drawing a smooth curve through a plot of the phase shift versus temperature and using data from this curve along with a theoretical expression for the effective mass to deduce the mobility. The theoretical effective mass is used instead of measurements at two frequencies because of uncertainties in the absolute phase shift. These uncertainties result in a ± 10% error in the values of the inverse mobility.

A plot of inverse mobility versus temperature at an areal density of 5.5×10^8 cm^{-2} is presented by the heavy solid curve in Fig. 2. The light lines represent theoretical curves for two sets of the undetermined parameters, ν and η, of the weak-coupling theory.[5] We experienced an unexplained loss of electrons at low temperatures (for $n = 5\times 10^8$ cm^{-2}, below $\simeq 200$ mK). The data were taken chronologically with increasing temperature, and the density at the lowest temperatures may have been (5–8)% larger. Since μ^{-1} is approximately proportional to $n\varphi$, a correction for an enhanced density would increase μ^{-1} at the lowest temperatures by $\leq 8\%$.

In Fig. 3 we show the experimentally measured phase shift as a function of temperature in the region of the melting transition for $n = 4.2\times 10^8$ cm^{-2}. In order to elucidate the nature of the excess scattering, we subtract from the data a background which is an extrapolation of the low-temperature data in the crystal phase, a constant value above the transition, and a linear interpolation between these values in the transition region. The location and width of the transition was chosen to yield a symmetric peak. The theoretical phase shift for the gaseous state,[4] shown by the solid curve at the lower right, assumes that m^* is equal to the electronic mass.

The data presented in Figs. 2 and 3 exhibit the following features: (a) an initial rapid rise in the scattering rate with increasing T, (b) a maximum in the inverse mobility at approximately one-

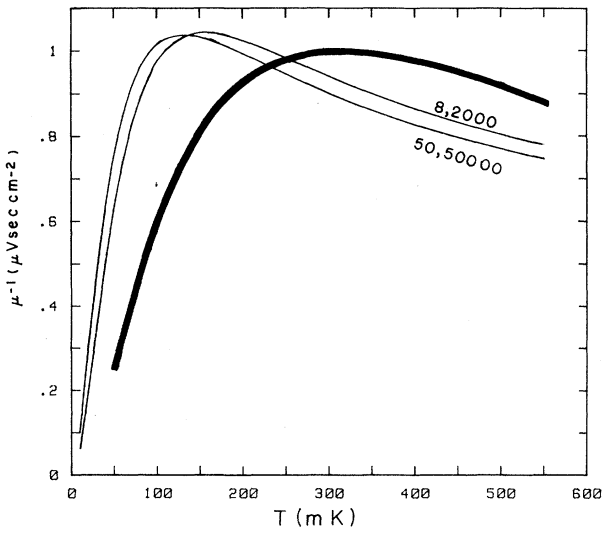

FIG. 2. Inverse mobility vs temperature represented by the thick curve for $n = 5.5\times 10^8$ cm^{-2} and $E_\perp = 540$ V/cm. The fine curves are theoretical curves.

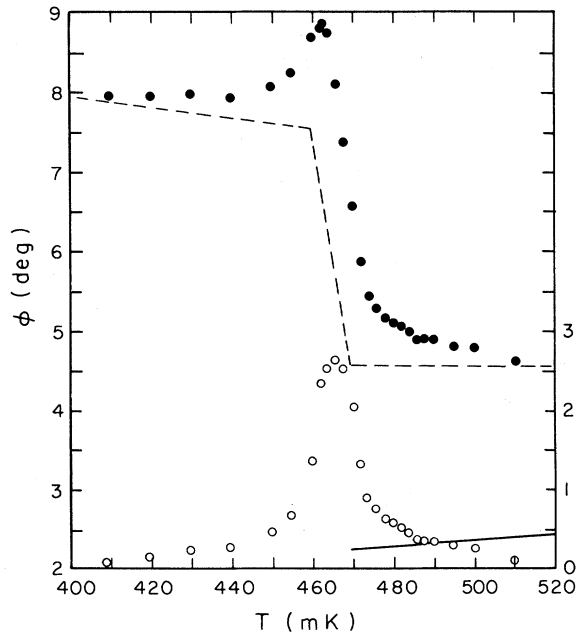

FIG. 3. Phase shift vs temperature in the transition region for $n = 4.2 \times 10^8$ cm^{-2} and $E_\perp = 430$ V/cm. Solid circles, raw data; open circles, excess scattering. The dashed curve represents the background, and the solid curve is the calculated phase for ripplon scattering from an electron gas. The scale on the right refers to the open symbols.

half of the melting temperature, T_m, (c) the melting transition region, defined by the linear portion of the sharp rise in φ, (d) an excess scattering peak approximately 12 mK in width at T_m, and (e) a phase shift above the scattering peak which is about twice the expected shift for electrons in the gaseous state. The scattering peak sets in at $n \simeq 10^8$ cm^{-2}, and its amplitude increases with increasing n to 5×10^8 cm^{-2}. At large n, the transition broadens and obscures these effects to some extent. We believe that this broadening may be artificial because of a change in the surface profile under the electronic pressure.

The width of the transition is critically dependent on the level of the capacitor plates with respect to the liquid-helium surface, and a width as narrow as 9 mK at $T_m = 500$ mK has been observed. It is not clear whether the minimum width observed is inherent or caused by a density variation across the cell.

The dominant features of the inverse mobility below the excess scattering peak are explained qualitatively by the weak-coupling theory.[5] The rapid rise at low temperatures results from the increase in the occupation factors for lattice phonons, and the maximum in the scattering rate is caused by an increase in $\langle x^2 \rangle$ which destroys the coherence between the ripplon and electronic coordinates. Mathematically, the Debye-Waller factor cuts off the larger ripplon wave vectors from participating in the scattering.

In the weak-coupling theory an approximation is used in relating the unperturbed phonon coordinates to the phonon-ripplon coupled-mode coordinates which is valid when the phonon modes are only slightly perturbed by ripplons ($n \lesssim 5 \times 10^6$ cm^{-2}). Another approximation used in the theory is that the inverse of the lattice-momentum relaxation time $\tau_{dc}^{-1} = e/m\mu$ is much less than the ripplon frequency at the smallest reciprocal lattice vector. We believe that the qualitative discrepancies between the data and the theoretical curves result from these approximations which are not valid at $n = 5.5 \times 10^8$ cm^{-2}. One observation may support this suggestion. We observed the phase at constant temperature as a function of the frequency of an rf voltage applied to electrode $T2$. The inhomogeneous rf field was large enough to drive the electrons out of the dimples in a portion of the lattice at the frequencies of the coupled-mode resonances, thus decreasing the coupling to the ripplons. The phase increased at the resonant frequencies even though the effective mass decreased, indicating an increase in scattering.

The data shown in Fig. 3 are qualitatively identical to the temperature-dependent losses associated with the superfluid transition in thin helium films.[6] These data were interpreted as losses associated with the dissociation of vortex-antivortex pairs.[7] This same mechanism explains the anomalous rf absorption in thin superconducting films.[8] We conjecture that the excess scattering near T_m is caused by the dissociation of dislocation pairs associated with the Kosterlitz-Thouless melting mechanism.[9-11] The lack of an abrupt melting transition may result from the dynamic response of the system to a finite driving frequency.[6,7]

The large discrepancy in the phase between experiment and theory above the transition might result from scattering associated with free dislocations and disclinations in a hexatic liquid-crystal phase.[11]

We have also measured the melting curve. We define T_m as the temperature at which the phase in the fluid state intercepts the rapid rise in the transition region since we assume that this point corresponds to the measured density (the maxi-

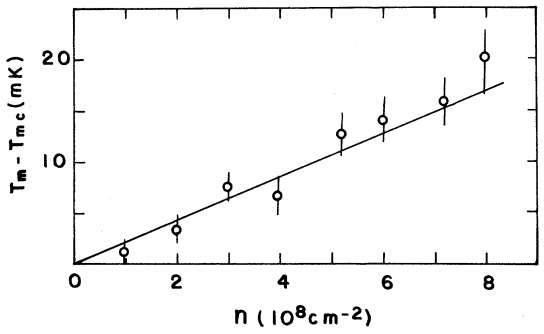

FIG. 4. Deviations from the classical melting curve plotted as $T_m - T_{mc}$ vs electron density.

mum density in the cell). The melting curve is characterized by a parameter Γ which is a measure of the ratio of the potential to kinetic energy of the electrons and is defined as

$$\Gamma = \pi^{1/2} e^2 n^{1/2} / \bar{\epsilon} k_B T_M,$$

where $\bar{\epsilon} = (\epsilon_v + \epsilon_l)/2$ is the average of the vapor and liquid dielectric constants ($\epsilon_v = 1$). We observe a deviation from a constant value of $\Gamma = \Gamma_0$ describing the classical melting curve.

We plot in Fig. 4 this deviation $T_m - T_{mc}$ vs n. Here $T_{mc} = \pi^{1/2} e^2 n^{1/2} / \bar{\epsilon} k_B \Gamma_0$. The error bars result from a 1% uncertainty in the relative measurement of n. The solid line represents the best fit to the single (Fermi-liquid) phase theory of Platzman and Fukuyama,[12] which can be written in the low-density limit as

$$\Gamma = \Gamma_0 (1 + 9 \times 10^{-7} n^{1/2})$$

with a value of $\Gamma_0 = 122.4$. The correction term $\propto n^{1/2}$ results from quantum corrections to the kinetic energy of the electrons. The quantum corrections calculated by Fukuyama[13] do not fit our data well. We also realize that this particular formula for $\Gamma(n)$ may not apply to a Kosterlitz-Thouless melting transition. We obtain a value for Γ_0 of 124 ± 4 which is more exact than, and within the range of error of, other experimental values.[1,14] This value may be compared to a value[15] of 128 based on the Kosterlitz-Thouless theory and a value[16] of 125 ± 15 from Monte Carlo calculations. The uncertainties result from an imprecise absolute measurement of the density which was obtained from the relation $E_\perp = 2\pi n e$, where $E_\perp$ is the holding field. The height of the liquid level, which enters the expression for $E_\perp$, was determined from capacitance measurements of the cell with the cell empty, partially filled, and completely filled with liquid.

We have presented mobility data for the 2D electron crystal which have the qualitative features predicted by the weak-coupling theory. Our data near the melting temperature may be explicable in terms of the dislocation-mediated melting model, and we are testing this model by measuring the excess scattering peak at various frequencies. We have also reported quantum corrections to the melting curve.

We wish to thank Y. Z. Ruan and C. J. Guo for technical assistance and wish to acknowledge fruitful discussions with T. Eck, C. C. Grimes, D. H. Keefe, D. R. Nelson, and J. Van Himbergen. This work was supported by the National Science Foundation under Grant No. DMR 79-20833.

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