Institute on Phase Transitions, edited by M. Levy *et al.* (Plenum, New York, 1981), and to be published.
[6]D. S. Gaunt and M. F. Sykes, J. Phys. A 12, L25 (1979).
[7]Uncertainties are in units of the last decimal place.
[8]See, e.g., W. J. Camp *et al.*, Phys. Rev. B 14, 3990 (1976).
[9]J. C. Le Guillou and J. Zinn-Justin, Phys. Rev. B 21, 3976 (1980).
[10]G. A. Baker, Jr., *et al.*, Phys. Rev. Lett. 36, 1351 (1976).
[11]B. G. Nickel, Physica (Utrecht) 106A, 48 (1981).
[12]See J. J. Rehr *et al.*, J. Phys. A 13, 1587 (1980).
[13]R. Z. Roskies, Phys. Rev. B 24, 5305 (1981).
[14]Also J. J. Rehr and B. G. Nickel, to be published.
[15]J. Zinn-Justin, J. Phys. (Paris) 42, 783 (1981).
[16]J. R. Klauder, Ann. Phys. (N.Y.) 117, 19 (1979).
[17]J.-H. Chen and M. E. Fisher, to be published.
[18]M. E. Fisher and D. F. Styer, to be published. The main requirements are met here by J, L, M, and N arrays flush to the right and bottom with $j_{\max}=l_{\max}=n_{\max}=m_{\max}-2$ while K is flush to left and top.

Dynamic Form Factor, Polarizability, and Intrinsic Conductivity of a Two-Dimensional Dense Electron Gas at Zero Temperature

M. Howard Lee and J. Hong
Department of Physics, University of Georgia, Athens, Georgia 30602
(Received 9 September 1981)

The relaxation function for a two-dimensional dense electron gas is obtained by solving the generalized Langevin equation due to Mori. The dynamic form factor, dynamic polarizability, and conductivity are calculated with use of linear response theory. The conductivity arises from density fluctuations existing at finite wave vectors. The possibility of observing such an effect is considered, especially in the recent work of Allen, Tsui, and DeRossa.

PACS numbers: 72.10.Bg, 72.20.-i, 73.40.Qv

Recent experimental studies of inversion and accumulation layers of the metal-oxide-semiconductor system have stimulated considerable interest in two-dimensional dense-electron-gas models.[1] We report here our solution for the zero-temperature relaxation function for a two-dimensional dense electron gas obtained *exactly* from the generalized Langevin equation (GLE) due to Mori.[2] Our solution is valid if wave vectors k for density fluctuations ρ_k are small compared with the Fermi wave vector k_F. From this knowledge of the relaxation function, we use linear response theory[3] to obtain the dynamic form factor, dynamic polarizability, and conductivity.

Our system is represented by the two-dimensional Sawada Hamiltonian H_0,[4] imposed under an external perturbing potential H_{ext} defined by

$$H_{ext}=\sum_k \rho_k(t)v_k e^{i\omega t}, \qquad (1)$$

where v_k is the Fourier component of the external electric field such as to permit the use of linear response theory, $\rho_k(t)=\exp(iH_0t)\rho_k\exp(-iH_0t)$, $\rho_k=\sum_p c_{p+k}{}^\dagger c_p$, with $c_p{}^\dagger$ and c_p the fermion creation and annihilation operators, respectively. Mori[2] has given a formal solution for the relaxation function $\Xi_k(t)=(\rho_k(t),\rho_k)/(\rho_k,\rho_k)$ in a continued-fraction representation, viz.,

$$\Xi_k(z)=\frac{1}{z+\varphi_k(z)}=\cfrac{1}{z+\cfrac{\Delta_1}{z+\cfrac{\Delta_2}{z+\dots}}}, \qquad (2)$$

where $\Xi_k(z)$ is the Laplace transform of $\Xi_k(t)$, φ is the memory function or the kernel of the Langevin equation, $\Delta_1,\Delta_2,\dots$ are static correlation functions[2] related to moments, not depending on H_{ext}, and finally (A,B) denotes the Kubo scalar product of operators A and B.[3,5] For small k we find that $\Delta_1=(\omega_p{}^{cl})^2+2k^2\epsilon_F{}^2$, $\Delta_2=k^2\epsilon_F{}^2+O(k^4)$, $\Delta_3=k^2\epsilon_F{}^2+O(k^4)$, ..., $\Delta_n=k^2\epsilon_F{}^2+O(k^4)$, ..., where k is expressed in units of k_F, $\hbar=1$, ϵ_F is the $2d$ Fermi energy, and for the classical plasma frequency[6] we use $\omega_p{}^{cl}=(2\pi\rho e^2k/m)^{1/2}$, with ρ and m the electron number density and mass, respectively. We immediately note that to lowest order in k, $\Delta_n=\Delta$ for $n=2,3,\dots$, where we define $\Delta=k^2\epsilon_F{}^2$. Recently Lee and Dekeyser[7] have shown that solutions for the relaxation function can be given for certain forms of $\{\Delta_n\}$, one of which is being realized here in this two-dimensional dense-electron-gas model. Following this work, we obtain the desired solution for the relaxation function,

$$\Xi_k(t) = A_s \sum_{n=0}^{\infty} (-\alpha)^n (\partial/\partial \mu t)^{2n} J_1(\mu t)/\mu t + A_p \cos\omega_p t\,, \tag{3}$$

where $\mu = 2\Delta^{1/2}$, $A_s = 1-(1-\alpha)^{1/2}$, $A_p = [(1-\alpha)^{1/2} - (1-\alpha)]/(\frac{1}{2}\alpha)$, $\alpha = 4\Delta(1-\Delta/\Delta_1)/\Delta_1$, $\omega_p = \alpha^{-1/2}\mu$, and J_1 is the Bessel function of order 1.[8] The parameter α turns out to be a natural quantity having the following limits: $\alpha \to 0$ for $k \to 0$ and $\alpha = 1$ if the electrons are noninteracting.[9] Thus for the ideal degenerate electron gas at zero temperature the relaxation function reduces to

$$\Xi_k^{\text{ideal}}(t) = J_0(\mu t), \tag{4}$$

where J_0 is the Bessel function of order zero. For an interacting electron gas, one may do perturbation calculations with α as a small number. However, the Fourier transform of (3) gives a closed-form solution so that we are able to apply linear response theory exactly. Our solution (3) satisfies the moment sum rules to all orders.[3]

The density-density response function or the frequency-dependent polarizability $\chi_k(\omega)$ may be obtained from (3) via a linear response relation $\tilde{\chi}_k(t) = -(\partial/\partial t)\Xi_k(t)$ for $t>0$, where $\tilde{\chi}_k(t) = \chi_k(t)/\chi_k$ and $\chi_k = (\rho_k, \rho_k)$ the static response function. We find that

$$\mathrm{Re}\tilde{\chi}_k(\omega) = \begin{cases} 1 + A_s\omega^2(1-\alpha)^{1/2}/(\mu^2 - \alpha\omega^2), & 0<\omega<\mu, \\ 1 + A_s\omega^2[(1-\alpha)^{1/2} + (1-\mu^2/\omega^2)^{1/2}]/(\mu^2-\alpha\omega^2), & \mu<\omega<\infty, \end{cases} \tag{5a}$$

$$-\mathrm{Im}\tilde{\chi}_k(\omega) = \begin{cases} A_s\omega(\mu^2-\omega^2)^{1/2}/(\mu^2-\alpha\omega^2), & 0<\omega<\mu, \\ \frac{1}{2}\pi A_p\omega[\delta(\omega-\omega_p) + \delta(\omega+\omega_p)], & \mu<\omega<\infty. \end{cases} \tag{5b}$$

We note that $\mathrm{Re}\chi$ is symmetric in ω whereas $\mathrm{Im}\chi$ is antisymmetric. Also, for $\omega \to 0$, $\mathrm{Re}\tilde{\chi} = 1 + O(\omega^2)$,[3] and for $\omega \to \infty$, $\mathrm{Re}\tilde{\chi} = O(\omega^{-2})$.[10] In the high-frequency regime, (5a) agrees with the *high-frequency* result obtained by Rajagopal.[11] For $\alpha = 1$, the above polarizability agrees exactly with the celebrated result of Stern.[12] The dynamic form factor $S_k(\omega)$, which may be obtained from $\mathrm{Im}\chi$ by the fluctuation-dissipation theorem, is illustrated in Fig. 1. We use $k = 0.2$ in units of k_F and $r_s = 0.5$ for which $\omega_p = 0.64$ in units of ϵ_F.[6] As $\alpha \to 1$, the peak due to the collective mode moves rapidly towards the broad spectrum of single-particle scattering. At $\alpha = 1$, they combine to produce the dynamic form factor for the ideal degenerate electron gas shown in the backdrop. To our knowledge there are no measurements made for the dynamic form factors, with which our result may be compared.[13]

To calculate the frequency-dependent conductivity $\sigma_k(\omega)$ we use the Kubo formula[14]

$$\sigma_k(\omega) = e^2 \int_0^\infty dt\, e^{-i\omega t} (j_k(t), j_k), \tag{6}$$

where j_k is the random current. The total current is connected to the density fluctuations by the continuity equation. The random current is the part of the total current which remains orthogonal to the density fluctuations $(\rho_k, j_k(t)) = 0$; that is, j_k is proportional to the random force defined by the Langevin equation.[2,14] From our knowledge of the relaxation function (3), it is possible to obtain an exact expression for the

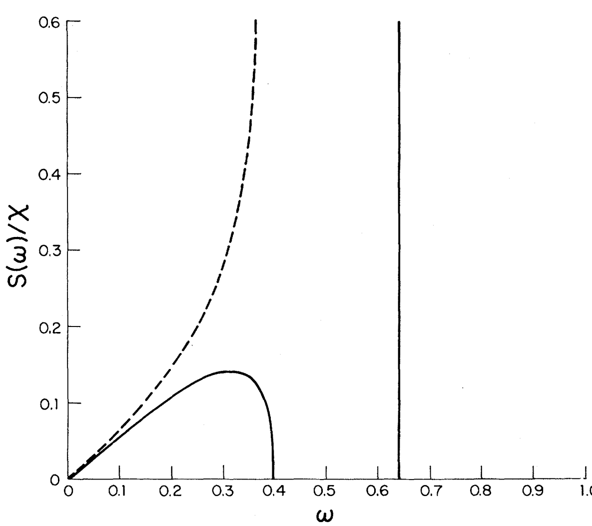

FIG. 1. Dynamic form factor vs frequency obtained from Eq. (5b) with $r_s = 0.5$, $k = 0.2$, and $\omega_p = 0.64$. The dynamic form factors for interacting and noninteracting electron gases are shown by solid and dashed lines, repsectively. To show both in one figure, the vertical scale used for the noninteracting electron gas is reduced by a factor of $\frac{1}{4}$. Observe that for the ideal gas $S(\omega) \to \infty$ as $\omega \to \mu = 0.4$.

time-dependent random current. In addition we note that because of the above-mentioned relationship between the random current and the random force, the current correlation function in (6) and the memory function φ are the same up to a multiplicative constant. The memory function also has the same continued-fraction expression [see Eq. (2)] but with $\Delta_2, \Delta_3, \ldots$. Thus, we obtain

$$(j_k(t), j_k)/(j_k, j_k) = 2J_1(\mu t)/\mu t. \tag{7}$$

The conductivity now follows from (6) and (7):

$$\mathrm{Re}\tilde{\sigma}_k(\omega) = \begin{cases} (\mu^2 - \omega^2)^{1/2}/\mu, & 0 < \omega < \mu, \\ 0, & \mu < \omega < \infty, \end{cases} \tag{8a}$$

$$-\mathrm{Im}\tilde{\sigma}_k(\omega) = \begin{cases} \omega/\mu, & 0 < \omega < \mu, \\ \mu[\omega + (\omega^2 - \mu^2)^{1/2}]^{-1}, & \mu < \omega < \infty, \end{cases} \tag{8b}$$

where $\tilde{\sigma}_k(\omega) = \sigma_k(\omega)/\sigma_k$ and $\sigma_k = 2e^2\rho/m\mu$.

The real part of the conductivity satisfies the conductivity sum rule[13,14]

$$\int_0^\infty d\omega\, \mathrm{Re}\sigma_k(\omega) = \pi e^2\rho/2m. \tag{9}$$

The above results (8a) and (8b) are illustrated in Fig. 2. We observe that $\mathrm{Re}\sigma_k(\omega)$ vanishes for $\omega > \mu$. This is the high-frequency regime where, ordinarily, impurities (absent in our system) are primarily responsible for the observed conductivity. $\mathrm{Re}\sigma_k(\omega)$ is finite for $0 < \omega < \mu$, arising purely from density fluctuations at small but nonzero wave vectors k. Following Kubo[14] we shall refer to it as the *intrinsic* conductivity. In the long-wavelength limit ($k \to 0$), $\mu \to 0$ so that there is no conductivity just as there is no current.

Whether the intrinsic conductivity is measurable raises an interesting possibility. Allen, Tsui, and DeRossa[15] measured the frequency-dependent conductivity in the silicon inversion layer at metallic densities ($r_s = 2$–4). Their measured conductivities, especially at frequencies 10–40 cm^{-1}, follow Drude behavior, which Tzoar, Platzman, and Simon[16] have attributed to impurity scattering. In the neighborhood of frequencies 5–10 cm^{-1} the measured conductivities seem to show a small downward deviation from the Drude form. This particular behavior may represent no more than a systematic scatter in the measurements, but it is nonetheless suggestive of a change in the conduction mechanism from low-frequency density fluctuations to high-frequency impurity scattering. More striking is the comparison between our $\mathrm{Im}\sigma$ [see Fig. 2(b)] and the "measured" $\mathrm{Im}\sigma$ obtained from the measured

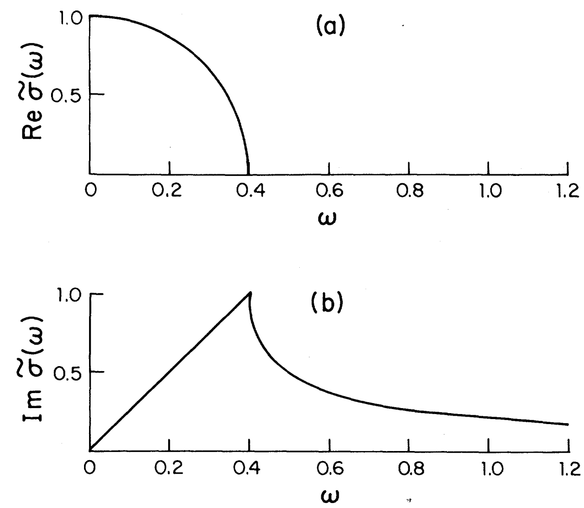

FIG. 2. Conductivity vs frequency obtained from Eqs. 8(a) and 8(b) at a fixed value of $k = 0.2$. The dc conductivity is given by $\mathrm{Re}\sigma_k(0)$. $\mathrm{Re}\tilde{\sigma}$ vanishes for $\omega > \mu = 0.4$. Observe that the maximum of $\mathrm{Im}\tilde{\sigma}$ occurs at $\omega = \mu$ and it decreases only gradually as ω increases.

$\mathrm{Re}\sigma$ by an application of the Kramers-Kronig relation.[17] The position of max $\mathrm{Im}\sigma$, which occurs at $\omega = \mu$ in our result, appears to be consistent with the location of changeover in the scattering mechanism indicated by $\mathrm{Re}\sigma$.[18]

Ron and Tzoar[19] have deduced a perturbative formula for calculating the conductivity due to impurity scattering. Their formula depends on the knowledge of the dielectric function $\epsilon_k(\omega)$ of a homogeneous system,[14]

$$\epsilon_k^{-1}(\omega) = 1 - \tilde{\chi}_k(\omega). \tag{10}$$

With our result for the polarizability [(5a) and (5b)], the conductivity for a slightly impure electron gas can be calculated.

We wish to thank Dr. D. C. Tsui, Professor N. Tzoar, and Professor J. E. Rives for very helpful discussions on conductivity. We are grateful to Professor A. K. Rajagopal and Professor K. S. Singwi for explaining their work to us. Our work was supported in part by the U. S. Department of Energy under Contract No. DE-AS09-77ER01023.

[1] See *Electronic Properties of 2D Systems*, edited by G. Dorda and P. J. Stiles (North-Holland, Amsterdam, 1978).

[2] H. Mori, Prog. Theor. Phys. 33, 423 (1965), and 34, 399 (1965).

[3] W. Marshall and R. D. Lowde, Rep. Prog. Phys. 31,

705 (1968).

[4]K. Sawada, Phys. Rev. 106, 372 (1957).

[5]G. Mukhopadyay and A. Sjölander [Phys. Rev. B 17, 3589 (1978)] and H. De Raedt and B. De Raedt [Phys. Rev. B 18, 2039 (1978)] applied the Mori formalism to three-dimensional (3D) electron systems approximately.

[6]A. K. Rajagopal and J. C. Kimball, Phys. Rev. B 15, 2819 (1977); M. Jonson, J. Phys. C 9, 3055 (1976).

[7]M. H. Lee and R. Dekeyser, Physica (Utrecht) 86–88B+C, 1273 (1977). M. H. Lee, Lett. Appl. Eng. Sci. 4, 472 (1976).

[8]M. H. Lee (to be published) deduced three recurrence relations for GLE. If they are regular or well behaved, analytic solutions are possible. This particular solution depends on the density of states and the dimensionality among others.

[9]α as a function of Δ_1 has two solutions. For $\Delta_1 > 2\Delta$, $0<\alpha<1$, and for $\Delta_1 < 2\Delta$, $-\infty<\alpha<1$. We consider here only the first solution. The second solution gives a mathematical mechanism for the disappearance of the plasma oscillation.

[10]M. H. Lee, Phys. Rev. B 8, 3290 (1973).

[11]A. K. Rajagopal [Phys. Rev. B 15, 4264 (1977)] calculated the polarizability in powers of k/ω up to second order which thus represents a *high-frequency* polarizability. Rajagopal's result (excluding the exchange contribution) may be compared with the high-frequency limit of our *exact* result (5a), i.e., $\mathrm{Re}\tilde{\chi}(\omega\to\infty) = (\omega_p{}^{\mathrm{cl}}/\omega)^2\{1+2(k/\omega_p{}^{\mathrm{cl}})^2 + [(\omega_p{}^{\mathrm{cl}})^2+3k^2]/\omega^2 + O(\omega^{-4})\}$. Rajagopal agrees with the above excepting the second term $2k^2/\omega^2$, which arises from single-particle scattering, absent in Rajagopal's perturbative treatment. To the same order in ω, Rajagopal's dispersion relation, however, agrees identically with our exact *plasmon* dispersion relation $\omega_p = \alpha^{-1/2}\mu$.

[12]F. Stern, Phys. Rev. Lett. 18, 546 (1967). We must compare the two results in the regimes where both are valid. In the regimes of $\omega>\mu$ and $\omega<\mu$ but excluding $\omega=\mu$, we find that our $\chi_k(\omega)$ and Stern's are term-by-term identical. Also see P. F. Maldague, Surf. Sci. 73, 296 (1978). As $\omega\to\mu$ from below, $-\mathrm{Im}\tilde{\chi}(\omega;\alpha=1)\to\infty$ (a special 2D feature). Thus, one may regard $\omega=\mu$ as a quasi plasma frequency since $\omega_p=\mu$ for $\alpha=1$.

[13]For 3D electron gases, see D. Pines and P. Nozieres, *Theory of Quantum Liquids* (Benjamin, New York, 1966), p. 110.

[14]R. Kubo, Rep. Prog. Phys. 29, 255 (1966).

[15]S. J. Allen, D. C. Tsui, and F. DeRossa, Phys. Rev. Lett. 35, 1359 (1975).

[16]N. Tzoar, P. M. Platzman, and A. Simon, Phys. Rev. Lett. 36, 1200 (1976).

[17]S. J. Allen *et al.*, to be published.

[18]If we expand our $\mathrm{Re}\sigma_k(\omega)$ for small ω, the first term gives the Drude form. Thus it is possible to carry out a very crude comparison study by expressing the Drude parameters m^* and τ in terms of our quantities, i.e., $m^*=xm$ and $\tau^2=\frac{1}{2}\mu^{-2}$. Using the data of Allen, Tsui, and DeRossa [Ref. 15, Figs. 2(a) and 2(b)], we get $x\simeq 0.18$, which is to be compared with the experimental value of $x\simeq 0.2$. We also obtain $k=0.031$ and $\mu=7.8$ cm^{-1} for $r_s=2.07$ [Fig. 2(a)] and $k=0.062$ and $\mu=7.3$ cm^{-1} for $r_s=2.81$ [Fig. 2(b)]. In this work, the wave vectors were not reported. Presumably they were very small, if not zero. The frequency at which the intrinsic conductivity vanishes (i.e., $\omega=\mu$) comes out to be within our previous estimate 5–10 cm^{-1}. Because of the crudeness of our comparison these numbers of course cannot be taken very seriously, but they do indicate a certain amount of consistency.

[19]A. Ron and N. Tzoar, Phys. Rev. 131, 12 (1963).