it is unlikely that the effect of this small population could be observed.

From our data we have shown how $P_{f}(\bar{L})$ varies with $L$ for ${ }^{200} \mathrm{~Pb}$ and how the ( $\mathrm{HI}, x n$ ) cross sections are limited by fission. For ${ }^{200} \mathrm{~Pb}$ the maximum angular momentum for which $\gamma$-ray studies can be performed is $\sim 35 \hbar$. This fission limit rises rapidly for lighter compound systems for which other limits, due, e.g., to incomplete fusion, ${ }^{12}$ may be relevant. Statistical-code calculations have indicated that $E_{f}(L) \approx 0.8 E_{f}{ }^{\mathrm{RLD}}$ in agreement with measurements for lighter systems.
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# Third Discontinuity in the Yrast Levels of ${ }^{158} \mathrm{Er}$ 

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#### Abstract

Discrete yrast transitions from states with spins up to $I=38$ have been observed in ${ }^{158} \mathrm{Er}$ via the reaction ${ }^{122} \mathrm{Sn}\left({ }^{40} \mathrm{Ar}, 4 n \gamma\right)$. In addition to the pronounced backbend at $I=14$ and the upbend at $I=28$ previously known, the start of a third yrast discontinuity at $I=38$ is indicated. Candidates for the $I=40$ state are suggested and possible causes of the yrast discontinuity are discussed.


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In rare-earth nuclei states with angular momenta up to $I \simeq 70$ can survive particle evaporation following heavy-ion fusion reactions. ${ }^{1}$ However, the nuclear structure of states of very high angular momentum can be studied only through unresolved deexcitation $\gamma$ rays. ${ }^{2}$ Resolved $\gamma$ lines from collective states have usually been observed only for $I<30$, although for noncollective states spins as high as $I=38$ have been identified. ${ }^{3}$ Many attempts have been made to extend these angular momentum limits because discrete $\gamma$ rays can give more detailed nuclear structure information. The discovery ${ }^{4}$ of "backbending," where the regular frequency increase with spin is temporarily reversed, produced many investigations of collective states by (HI, $x n$ ) reactions and by Coulomb excitation. The explanation usually given for this discontinuity in the yrast levels is
the alignment of the angular momentum of two high-j nucleons with the rotational angular momentum. ${ }^{5}$ In ${ }^{158} \mathrm{Er}$ the backbend at $I=14$ has been attributed to the alignment of a pair of $i_{13 / 2}$ neutrons, ${ }^{6,7}$ and the upbend at $I=28$ has been considered as due most probably to alignment of a pair of $h_{11 / 2}$ protons. ${ }^{8}$ The aim of the present investigation was to study still higher members of the yrast sequence.

The high-spin yrast levels in ${ }^{158} \mathrm{Er}$ have been produced in the reaction ${ }^{122} \operatorname{Sn}\left({ }^{40} \mathrm{Ar}, 4 n\right)$ with use of $170-\mathrm{MeV}{ }^{40} \mathrm{Ar}$ projectiles from the $88-\mathrm{in}$. cyclotron of the Lawrence Berkeley Laboratory. The lifetimes of the states with spins $\geqslant 20 \hbar$ observed in the present experiment are expected to be comparable to the stopping time of the excited recoiling nuclei in the target. Hence, with a thick target, $1-\mathrm{MeV} \gamma$ lines will have a width
corresponding to the full Doppler shift (2\%) of 20 keV . The broadening can be minimized by using a sufficiently thin target and placing the detector in a direction parallel to the beam, where the effect due to the finite acceptance angle is the smallest. To insure narrow lines without unduly reducing the yield, the target consisted of four self-supporting ${ }^{122} \mathrm{Sn}$ foils, $0.5 \mathrm{mg} / \mathrm{cm}^{2}$ thick, separated from each other by 0.5 mm (ample for most of the states to decay in flight).

To enhance the coincidence counting rate in order to see the weak high-spin transitions, five $\mathrm{Ge}(\mathrm{Li})$ coaxial detectors were used. Four of them (of $20 \%, 20 \%, 15 \%$, and $10 \%$ efficiency) were placed at the most backward angles available, $\pm\left(155^{\circ}\right.$ to $\left.160^{\circ}\right)$ with respect to the beam. The fifth detector ( $10 \%$ ) was placed alternatively at $0^{\circ}$ or $90^{\circ}$ for an angular distribution determination. All these detectors were about 10 cm from the target. A multiplicity filter consisting of five $7.6 \times 7.6 \mathrm{~cm}^{2} \mathrm{NaI}$ detectors surrounded the target, and the number (fold) of NaI detectors firing in coincidence with each event was recorded. Each event consisted of a $\gamma-\gamma$ coincidence between two of the five $\mathrm{Ge}(\mathrm{Li})$ detectors, gated by one or more detectors in the multiplicity filter. About $10^{8}$ events were recorded and subsequently ana-


FIG. 1. Coincident $\gamma$-ray spectra from the reaction ${ }^{122} \mathrm{Sn}\left({ }^{40} \mathrm{Ar}, 4 n\right){ }^{158} \mathrm{Er}$ obtained with indicated gates.
lyzed by placing gates on the $\mathrm{Ge}(\mathrm{Li})$ energy, the $\mathrm{Ge}(\mathrm{Li})-\mathrm{NaI}$ time-to-amplitude converter, and the fold spectra.
The upper part of Fig. 1 shows the sum of the $\geqslant 1$-fold $\gamma-\gamma$ coincidence spectra observed by the four backward $\mathrm{Ge}(\mathrm{Li})$ detectors gated by the sum of transitions that deexcite the $18^{+}$to $32^{+}$levels (after background subtraction). Below this are similar spectra gated by the three well-defined highest energy lines. It is clear that these three peaks, labeled $34^{+} \rightarrow 32^{+}, 36^{+} \rightarrow 34^{+}$, and $38^{+}$ $\rightarrow 36^{+}$, are in coincidence with the main sequence of the yrast band. The yields of the $\gamma$-ray transitions [ $\mathrm{Ge}(\mathrm{Li})$ singles] were measured at $0^{\circ}$ and $90^{\circ}$ for fold $\geqslant 3$, and the ratio of these is given in Table I. These anisotropies are consistent with stretched $E 2$ decays ( $\sim 1.4$ ). From the data in Table I the yrast level sequence can be established with reasonable certainty up to $I=38$.
In addition to the well-defined peaks, there are weaker lines from (1) other product nuclei, (2) sideband transitions in ${ }^{158} \mathrm{Er}$, and (3) higher members of the yrast band. In an attempt to find any higher members, fifteen weak lines in the region of 900 to 1300 keV were considered as

TABLE I. $\gamma$-ray energies, relative intensities, and anisotropies.

|  |  |  |  |
| :--- | ---: | :---: | :---: |
| $I_{i} \rightarrow I_{f}$ | $E_{\gamma}(\mathrm{keV})$ | Relative <br> intensities | $I\left(0^{\circ}\right) / I\left(90^{\circ}\right)$ |
| $6^{+} \rightarrow 4^{+}$ | $443.4(3)$ | 100 | $1.13(6)$ |
| $8^{+} \rightarrow 6^{+}$ | $523.7(3)$ | $98(5)$ | $1.60(7)$ |
| $10^{+} \rightarrow 8^{+}$ | $579.8(3)$ | $64(3)$ | $1.46(7)$ |
| $12^{+} \rightarrow 10^{+}$ | $608.8(3)$ | $48(3)$ | $1.45(7)$ |
| $14^{+} \rightarrow 12^{+}$ | $510.4(6)$ | $47(3)^{\mathrm{b}}$ | b |
| $16^{+} \rightarrow 14^{+}$ | $473.3(3)$ | $54(3)$ | $1.75(8)$ |
| $18^{+} \rightarrow 16^{+}$ | $566.9(3)$ | $39(2)$ | $1.54(8)$ |
| $20^{+} \rightarrow 18^{+}$ | $659.4(4)$ | $23(1)$ | $1.29(8)$ |
| $22^{+} \rightarrow 20^{+}$ | $740.0(4)$ | $17(1)$ | $1.41(8)$ |
| $24^{+} \rightarrow 22^{+}$ | $806.2(4)$ | $11(1)$ | $1.39(8)$ |
| $26^{+} \rightarrow 24^{+}$ | $846.0(6)$ | $5.1(8)$ | $1.17(10)$ |
| $28^{+} \rightarrow 26^{+}$ | $859.2(6)$ | $4.1(8)$ | $1.51(15)$ |
| $30^{+} \rightarrow 28^{+}$ | $876.5(7)$ | $3.1(5)$ | $2.00(35)$ |
| $32^{+} \rightarrow 30^{+}$ | $906.7(7)$ | $1.9(5)$ | $1.26(35)$ |
| $34^{+} \rightarrow 32^{+}$ | $960.3(10)$ | $1.7(5)$ | $1.26(35)$ |
| $36^{+} \rightarrow 34^{+}$ | $1018.7(15)$ | $0.9(5)$ | $1.91(40)$ |
| $38^{+} \rightarrow 36^{+}$ | $1061.3(15)$ | $0.6(3)$ | $\cdots$ |
| Case $a^{\text {Case } b}$ | $1067.0(15)$ | $0.2(1)$ | $\cdots$ |

[^0]possible candidates. The data were resorted in coincidence with these fifteen lines. In two cases it appeared from the resulting spectra that the lines feed the whole sequence of the yrast band. To make this more quantitative, the intensity of each yrast line from these spectra (after background subtraction) was divided by the intensity of the same line obtained from the sum of gates on the transitions from the $34^{+}, 36^{+}$, and $38^{+}$ states. Examples of such ratios are given in Table II. The weighted averages of the transitions indicated in the third and fourth columns have been normalized to those from the $6^{+}$to $24^{+}$levels. If a line feeds the whole sequence from above the $38^{+}$state, the ratios should be constant (unity) up to the transition from the $32^{+}$state and exceed 1 for the three higher energy lines because of the suppression of their intensity in the summed spectrum used for the denominator. As can be seen, cases $a$ and $b$ comply with this requirement, whereas the remaining three, chosen as typical for all the other lines examined, do not. This result was found to be insensitive to reasonable changes in the background subtractions. The line at 1067 keV (case $a$ ) appears in Fig. 1 weakly, but as a distinct shoulder just above the $38^{+} \rightarrow 36^{+}$transition, and we tentatively consider it as the most likely $40^{+} \rightarrow 38^{+}$member of the yrast band. The other line at 1155 keV (case $b$ ) might be another $40^{+}$state that belongs to the previous yrast band. These $40^{+}$assignments, however, are quite tentative.

Figure 2 shows a plot of the moment of inertia versus the rotational frequency. The beginning of a discontinuity at the $38^{+}$state can be seen. This discontinuity would be continued by the tentative $40^{+}$member. From these data one can construct a plot of $I$ vs $\hbar \omega$ (center of Fig. 2) according to the prescription given by Bengtsson and Frauendorf. ${ }^{9}$ At the bottom of Fig. 2, the relative aligned angular momentum, $i$, is shown. For the first two discontinuities we find ( $\hbar \omega, \Delta i$ )

TABLE II. Evaluation of $40^{+}$candidates.

| $40^{+}$candidates <br> (keV) | $6^{+}-24^{+}$ | $26^{+}-32^{+}$ | $34^{+}-38^{+}$ |
| :--- | :--- | :--- | :--- |
| 1033 | $(1.00)$ | $0.60(11)$ | $0.76(18)$ |
| $1067(a)$ | $(1.00)$ | $0.96(10)$ | $1.17(15)$ |
| 1080 | $(1.00)$ | $0.95(11)$ | $0.70(16)$ |
| 1091 | $(1.00)$ | $1.06(17)$ | $0.51(28)$ |
| $1156(b)$ | $(1.00)$ | $1.09(14)$ | $1.37(21)$ |

to be $(0.28,10)$ and $(0.43,6)$, respectively. The first of these sets is in agreement with the previous determinations for ${ }^{158} \mathrm{Er}$ and the second is similar to that found for ${ }^{160} \mathrm{Yb}$ (Ref. 10). They support the previous suggestions that $i_{13 / 2}$ neutrons and $h_{11 / 2}$ protons, respectively, are the particles responsible for the effects. ${ }^{6,8}$

It is interesting to consider the origin of the third discontinuity indicated at $\hbar \omega \sim 0.53 \mathrm{MeV}$. It seems most likely to be the alignment with the rotation axis of an additional high- $j$ particle pair, thus continuing the trend for the nucleus to share its angular momentum between the collective rotation and the alignment of high- $j$ particles. At the first backbend two $i_{13 / 2}$ neutrons contribute $\sim 10 \hbar$ of aligned angular momentum to a total spin of $16 \%$. At the second discontinuity (an upbend), the pair of $h_{11 / 2}$ protons adds another $6 \hbar$ of aligned angular momentum out of a total spin of $28 \hbar$. Although the angular momentum alignment is not yet determined, it seems most likely that the third discontinuity is due to either $h_{9 / 2}$ neutrons or $h_{9 / 2}$ protons. Of these, the calculations ${ }^{11,12}$ for $h_{9 / 2}$ neutrons give closer agreement with the experimental $\hbar \omega$.
The present work indicates a third discontinuity in yrast levels at about spin $40 \hbar$ in ${ }^{158} \mathrm{Er}$.


FIG. 2. Plots of the moment of inertia (top), yrast spin (middle), and spin alignment (bottom) vs the rotational frequency.

This has been observed (1) by using ${ }^{40} \mathrm{Ar}$ projectiles to bring in high angular momenta, (2) by minimizing the Doppler broadening without compromising the coincidence counting rate by employing a target of four thin foils and using four $\mathrm{Ge}(\mathrm{Li})$ detectors at backward angles, and (3) by enhancement of a particular reaction channel with a $\gamma$-ray multiplicity filter. The discontinuity is probably due to alignment of a third pair of particles, most likely $h_{9 / 2}$ neutrons.

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# Double Escape of Two Electrons at Threshold: Dependence on $L, S$, and $\pi$ 

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#### Abstract

The Wannier theory for the double escape of two electrons from a positive ion is extended to general values of the total orbital and spin angular momentum and parity of the pair. Except for ${ }^{3} S^{e}$ and ${ }^{1} P^{e}$, all states are described by the same threshold law. Specific results on the correlations in energy, angle, and spin of the electrons are derived and compared with experimental data.


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One aspect of the general three-body problem that is of interest in many areas of physics is the threshold law for three-body processes. The problem in atomic physics when all three partners are charged has particular subtleties because of the feature that long-range forces are present. When two electrons escape from a positive ion with a small energy excess $E$ above the threshold for this process, one can expect strong correlations to develop between the escaping particles, given the competing long-range forces
and the large time of interaction over which correlations can develop. The Wannier theory ${ }^{1-3}$ for this threshold process provides, besides the threshold law (the exponent in the relation $\sigma \propto E^{m}$, where $\sigma$ is the cross section for this process), many details on these correlations, ${ }^{4}$ such as the angular distribution of the electrons, the way they partition the available energy between them, and the correlations between their spins. Increasingly, experimental tests of these results are becoming available. ${ }^{5-7}$


[^0]:    ${ }^{\text {a }}$ For transitions from states with $I \geqslant 26 \hbar$, summed coincidence spectra with gates on $18^{+}$through $24^{+}$were used.
    ${ }^{\mathrm{b}}$ May contain a contribution from annihilation radiation.

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