states as well. In addition, it is expected that although the results presented are from model calculations, they are qualitatively correct. These extra zeros, which are absent in ground-state photoionization cross sections, depend primarily upon the phase shifts (quantum defects) of the various states. Thus, since our zero-energy phase shifts and discrete-state quantum defects substantially agree with experimental values,[11,12] it is believed that the zeros are at least semiquantitatively correct. Certainly more accurate calculations would alter their positions somewhat, but it is highly unlikely that the overall systematics would be substantially altered.

Finally, we note that the existence of these minima has not yet been tested experimentally. They could be looked for, not only in the total subshell photoionization cross section, but also in the photoelectron angular distribution and spin polarization, both of which will show rapid variations, as a function of $h\nu$, in the vicinity of these zeros. We urge laboratory exploration of this phenomenon.

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Generation of Squeezed Coherent States via a Free-Electron Laser

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Spontaneous radiation from a magnetic wiggler or undulator as used in a free-electron laser is shown to be in a squeezed state for a low-density electron beam. It is pointed out that the problem is formally analogous to radiation from a Josephson junction.

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Recent studies on the detection of gravitational radiation have pointed out that the detection of such weak forces demands taking care of quantum fluctuations in the measurement process.[1] One technique which might be indispensable for this purpose[2] is the use of the so-called squeezed states. In a squeezed state[3-7] the fluctuation of one variable is reduced below its symmetrical quantum limit at the expense of the conjugate one so that the uncertainty relation is not violated. Squeezed states of the radiation field could be used for improving the sensitivity of an interferometer at a given attainable cw laser power with applications to the detection of gravitational radiation in mind.[2] Squeezing, like antibunching, is a genuinely quantum-mechanical feature of the radiation field, in the sense that its occurrence requires a state having a Glauber representation with a nonpositive weight function. Unlike antibunching, squeezing has not been observed experimentally as yet, but has been predicted to occur in a degenerate parametric oscillator,[5,8] in degenerate four-wave mixing,[9] and in resonance fluorescence.[10] In this note, we point out that spontaneous radiation at low electron density from a wiggler (or an undulator in accelerator terminology), as it is used in a free-electron laser,[11] can be produced in a squeezed state.

Let A and $A^\dagger$ be the photon creation and annihilation operators, so that $[A, A^\dagger]=1$, and A_1 and A_2 the corresponding Hermitian dimensionless amplitudes, $A=A_1+iA_2$. We then have $[A_1, A_2]=i/2$ and the corresponding uncertainty relation is

$$\Delta A_1 \Delta A_2 \geq \tfrac{1}{4}. \quad (1)$$

We call a state squeezed if the uncertainty of either of the amplitudes A_i ($i=1$ or 2) satisfies

$$(\Delta A_i)^2 < \tfrac{1}{4}. \quad (2)$$

A squeezed state may or may not be a minimum-uncertainty state in the sense that in Eq. (1) the equality sign holds. In addition, a squeezed state may or may not be antibunched, the criterion for the latter property being ($n=A^\dagger A$)

$$\Delta n^2 - \langle n \rangle < 0. \quad (3)$$

We will now sketch the proof that Eqs. (2) and (3) can be fulfilled in the case of spontaneous emission from a free-electron amplifier. We shall employ methods developed earlier[12] in investigating the photon statistics of this device. We start with the one-electron nonrelativistic Bambini-Renieri Hamiltonian[13] which refers to a moving frame, where the laser and the wiggler frequency coincide with $\omega = ck/2$:

$$H = H_0 + H_I,$$
$$H_0 = p^2/2m + \hbar\omega A^\dagger A, \quad H_I = i\hbar g(A - A^\dagger), \quad (4)$$

where[14]

$$A = a e^{ikz}. \quad (5)$$

Here a is the annihilation operator of the laser field, p and z are the electron's momentum and coordinate with $[z, p] = i\hbar$, and $[A, A^\dagger]=1$, $[p, A] = \hbar k A$. m is the effective mass of the electron and

$$g = (e^2 B/mk)(2/V\epsilon_0 \hbar\omega)^{1/2}, \quad (6)$$

with V the quantization volume and B the magnetic field strength of the wiggler field in the moving frame. In Eq. (4) we have already taken the classical limit of the wiggler field. By transforming to the interaction picture we obtain

$$H_I(t) = ig\hbar\{\exp[-it(\hbar k^2 + 2kp)/2m]A^\dagger - \text{H.c.}\}. \quad (7)$$

Hence, the time-evolution operator for the electron-photon state is

$$S(T) = \mathcal{T}\exp[-(i/\hbar)\int_{-T/2}^{T/2} dt\, H_I(t)]. \quad (8)$$

$S(T)$ relates initial electron-photon states prior to the interaction mediated by the wiggler during the interaction time T, to final states after passage through the wiggler: $|\text{out}\rangle = S|\text{in}\rangle$. The symmetric integration limits in Eq. (8) have been chosen for convenience.

If in Eq. (7) the momentum operator p is replaced by a c number p_0, the formal solution (8) can be explicitly evaluated. This approximation provides for spread and spontaneous emission, and the resulting photon statistics is Poissonian for an initial field vacuum. It does not account, however, for gain. Hence, we will expand $S(T)$ to first order around the c-number value p_0, writing

$$S(T) = S_0(T) + S_1(T) + \cdots, \quad (9)$$

$$S_0(t_2, t_1) = \mathcal{T}\exp[-(i/\hbar)\int_{t_1}^{t_2} dt\, H_I(t)]|_{p=p_0}, \quad (10)$$

$$S_1(T) = \int_{-T/2}^{T/2} dt\, S_0(\tfrac{1}{2}T, t)[-(igk/m)t][(p-p_0)A^\dagger e^{-i\beta t} + A(p-p_0)e^{i\beta t}]S_0(t, -\tfrac{1}{2}T), \quad (11)$$

where $S_0(T) = S_0(\frac{1}{2}T, -\frac{1}{2}T)$ and $\beta = (\hbar k^2 + 2kp_0)/2m$ is the detuning parameter. This linear approximation covers the small-signal regime of the free-electron laser.

We now consider an initial state made up by an electron with momentum $\bar{p}$ and the field vacuum, $|\text{in}\rangle = |\bar{p}, 0\rangle$. Hence,

$$p|\bar{p}, 0\rangle = \bar{p}|\bar{p}, 0\rangle, \quad (12a)$$

$$A|\bar{p}, 0\rangle = 0, \quad (12b)$$

$$A^\dagger|\bar{p}, 0\rangle = |\bar{p} - \hbar k, 1\rangle. \quad (12c$$

$S(T)$ can be shown[12] to be independent of the expansion parameter p_0; we then fix it by $p_0 = p - \hbar k/2$, so that now $\beta = k\bar{p}/m$. The final-state expectation value of any operator $O(A, A^\dagger)$ is then

$$\langle \text{out} | O | \text{out} \rangle = \langle \bar{p}, 0 | S(T)^\dagger O S(T) | \bar{p}, 0 \rangle . \qquad (13)$$

The evaluation of Eq. (13) is straightforward along the lines given in Ref. 12, and we obtain

$$(\Delta A_1)^2 = \frac{1}{4} - \frac{\hbar k^2}{2m} j \frac{\partial j}{\partial \beta}, \qquad (14a)$$

$$(\Delta A_2)^2 = \frac{1}{4} + \frac{\hbar k^2}{2m} j \frac{\partial j}{\partial \beta}, \qquad (14b)$$

$$(\Delta A_1)(\Delta A_2) = \tfrac{1}{4}, \qquad (14c)$$

$$\Delta n^2 - \langle n \rangle = -\frac{2\hbar k^2}{m} j^3 \frac{\partial j}{\partial \beta}, \qquad (15)$$

where

$$j = (2g/\beta) \sin(\beta T/2) . \qquad (16)$$

In our notation the gain of the free-electron laser is proportional[12] to $-j\, \partial j/\partial \beta$. Hence, Eqs. (14a) and (14b) exhibit that, depending on the sign of the gain, either A_1 or A_2 is squeezed while, because of Eq. (14c), minimum uncertainty is maintained. This is in contrast to resonance fluorescence where the squeezed states are no longer minimum-uncertainty states.[10] Finally, Eq. (15) shows that we have antibunching for negative gain. Here we have defined squeezing with respect to the operator A defined in Eq. (5), instead of the annihilation operator a of the radiation field. This must be so because we employ electron-photon states and in view of the Hamiltonian (4), annihilation of a photon always comes up to increasing the momentum of the electron by $\hbar k$.

It is interesting to point out the complete analogy between our Hamiltonian (4) and the governing radiation from a Josephson junction,[15]

$$H = [(2e)^2/2C] N^2 + \hbar\omega A^\dagger A + i\hbar \hat{g}(A - A^\dagger), \qquad (17)$$

where now $A = a e^{i\varphi}$, N is the operator describing the difference in the number of Copper pairs on either side of the junction, φ is the operator of the relative phase, and C is the capacitance of the junction. In view of the commutation relation

$$[\varphi, N] = i, \qquad (18)$$

both Hamiltonians are equivalent. T would now be a time which is arbitrary but short in comparison with the Q/ν value of the cavity. Our first-order expansion of $S(T)$ now comes up to an expansion around a c-number phase, which is in the spirit of the usual treatment of Josephson radiation. Hence, for short times, when damping in the Josephson case can be neglected, we expect our present results for the free-electron amplifier to apply.

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