Dominance of Monopole and Quadrupole Pairs in the Nilsson Model

Takaharu Otsuka
Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-11, Japan

and

Akito Arima and Naotaka Yoshinaga
Department of Physics, University of Tokyo, Hongo, Tokyo 113, Japan
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The intrinsic state of the Nilsson model is analyzed in terms of nucleon pairs coupled to spins 0^+, 2^+, 4^+, 6^+, ... It is shown that 0^+ and 2^+ pairs dominate the Cooper pair which constitutes the intrinsic state with large quadrupole deformation.

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The rotational band can be described in terms of its intrinsic state. In the description of Bohr and Mottelson,[1] the structure of the band is determined by the deformation of the nuclear surface. The intrinsic state is constructed microscopically from nucleons moving in the deformed single-particle field, which is produced by the deformation of the nucleus. A prescription to calculate the deformed field is presented in the Nilsson model.[2]

We analyze in this Letter the intrinsic state in terms of nucleon pairs, restricting ourselves to the ground-state rotational band. The intrinsic state is the n-nucleon ground state in the deformed single-particle field, which is expressed in terms of energies of deformed single-particle orbits (the Nilsson orbits). Since the pairing correlation is important,[3] the BCS calculation is often performed, and the intrinsic state is obtained as the condensate state of N Cooper pairs ($N=n/2$). The Cooper pair is written in general as a linear combination of nucleon pairs. Since the deformed field is not a scalar, the Cooper pair contains pairs of various spins. We rewrite it as a linear combination of pairs of spin J, and calculate the amplitudes of these spin-J pairs. It has been widely believed that pairs of various spins are needed to describe the intrinsic state.[4,5] This Letter will, however, demonstrate that the Cooper pair in the Nilsson orbits turns out to be comprised primarily of the 0^+ and 2^+ pairs in regions of large quadrupole deformation. This remarkable feature has not been reported previously.

Recently the interacting boson model (IBM) has been proposed and used to describe quadrupole collective states including rotational ones.[6,7] This model describes the collective states having 0^+ (s) and 2^+ (d) bosons. Phenomenological studies on rotational nuclei yielded results which fitted well to experimental data,[8,9] suggesting that the application of the IBM to rotational nuclei seems to be successful.

A major assumption of the IBM from the microscopic point of view is that the 0^+ (S) and 2^+ (D) collective nucleon pairs play dominant roles in the quadrupole collective states.[7,10-12] Quite recently, it was suggested in Refs. 4 and 5 that this assumption does not hold in strongly deformed nuclei. The consequence of this Letter as described above, however, seems to support the assumption.

The intrinsic state is the lowest eigenstate of the intrinsic Hamiltonian, h. In the Nilsson model, this Hamiltonian consists of the deformed single-particle field, h_D, and the monopole pairing interaction, h_P; $h=h_D+h_P$. The pairing interaction is defined as $h_P=-GS_+S_-$, where G is a constant, and

$$S_+=\sum_j (2j+1)^{1/2}\tfrac{1}{2}[a_j{}^\dagger a_j{}^\dagger]^{(0)}, \qquad S_-=(S_+)^\dagger. \tag{1}$$

The single-particle field h_D consists of scalar (ξ_0) and quadrupole (ξ_2) terms; $h_D=\xi_0+\xi_2$. The ξ_0 term stands for the spherical potential.[1,2] The ξ_2 term, being a quadrupole operator, originates in the quadrupole interaction. Its strength depends on the deformation parameter,[1,2] δ;

$$\xi_2=-\tfrac{2}{3}\,\delta M\omega^2 r^2 C_0{}^{(2)}(\theta), \tag{2}$$

where M and ω denote the nucleon mass and the oscillator frequency for the spherical potential ($\hbar\omega=41A^{-1/3}$ MeV), and r and θ are coordinates in the intrinsic frame.

We shall consider two examples, one in which the single-nucleon levels (of $\delta=0$) are degenerate and one in which they are not. In the first case the formulas are easy to understand. However, the example with nondegenerate single-nucleon levels is more realistic. We begin by considering the degenerate-level case in which the nucleons are filling one single j orbit. The ξ_0 term is then omitted, since it is equivalent to a constant. Utilizing the formula $\langle r^2\rangle=(N+\frac{3}{2})\hbar/M\omega$, and evaluating the harmonic-oscillator quanta N by $N+2\sim(\frac{3}{2}A)^{1/3}$ (Ref. 13), we rewrite Eq. (2) as $\xi_2\simeq-31\delta C_0^{(2)}(\theta)$. Since, for large j, $\langle jm|C_0^{(2)}\times|jm\rangle\simeq-\frac{1}{4}[3(m/j)^2-1]$, the energy shift caused by ξ_2 is calculated approximately as $\langle jm|\xi_2|jm\rangle\simeq8\delta[3(m/j)^2-1]$. For $\delta<0$, orbits with $m=\pm j$ are lowest, and the single-particle energy becomes higher as $|m|$ decreases. The level ordering is reversed for $\delta>0$. The total splitting caused by this ξ_2 is ~24δ MeV, which becomes 7.2 MeV for $|\delta|=0.3$.

In order to construct the intrinsic state, the BCS calculation is performed for the intrinsic Hamiltonian h. The wave function describing the intrinsic state is then $|\text{BCS-Nil}\rangle=\prod_{m>0}(u_m+v_m\times a_m^\dagger a_{\tilde m}^\dagger)|0\rangle$, where $a_{\tilde m}^\dagger$ creates the time-reversal state of a state m, and u_m and v_m denote the u and v factors in the BCS theory. This wave function does not conserve the number of nucleons. Its projection on a fixed nucleon number $2N$ is given by

$$|2N\rangle=\mathfrak{N}^{-1}[\sum_{m>0}(v_m/u_m)a_m^\dagger a_{\tilde m}^\dagger]^N|0\rangle, \tag{3}$$

where $\mathfrak{N}$ is a normalization constant. This state can be rewritten as

$$|2N\rangle=(\mathfrak{N}')^{-1}(\Lambda^\dagger)^N|0\rangle, \tag{4}$$

with

$$\Lambda^\dagger=\sum_{m>0}c_m a_m^\dagger a_{\tilde m}^\dagger, \tag{5}$$

where $\mathfrak{N}'$ is a normalization constant, and amplitudes c_m are given by $c_m\propto v_m/u_m$ with normalization $\langle0|\Lambda\Lambda^\dagger|0\rangle=1$.

The $\Lambda^\dagger$ operator can be expressed in a linear combination of spin-J pairs as

$$\Lambda^\dagger=\textstyle\sum_J x_J\,\mathfrak{A}^{\dagger(J)}, \tag{6}$$

where x_J denote amplitudes, and $\mathfrak{A}^{\dagger(J)}$ is given in the single j orbit by $\mathfrak{A}^{\dagger(J)}=\frac{1}{2}\sqrt{2}[a_j^\dagger a_j^\dagger]^{(J)}$. The x_J amplitudes are calculated by $x_J=\langle0|\times\Lambda\mathfrak{A}^{\dagger(J)}|0\rangle$, which becomes, in the single j orbit, $x_J=\sum_{m>0}c_m\sqrt{2}(j,j,m,-m|J,0)$. Clebsch-Gordan coefficients in this equation are approximated for large j by $(-)^{j-m}[(2J+1)/(2j+1)]^{1/2}\times P_J(m/j)$ with P_J being the Legendre polynomial. Thus, for large j,

$$x_J\simeq\left(\frac{2(2J+1)}{2j+1}\right)^{1/2}\sum_{m>0}c_mP_J(m/j). \tag{7}$$

We look into the x_J amplitudes in an analytic way, by omitting the pairing interaction from the intrinsic Hamiltonian. This case corresponds to the normal phase, giving rise to $v_m=1$, $u_m=0$ for $m\in R$, and $v_m=0$, $u_m=1$ for $m\notin R$, with R being the set of occupied orbits. We then obtain

$$c_m=1/\sqrt{N}\text{ for }m\in R;\quad c_m=0\text{ for }m\notin R. \tag{8}$$

Although the signs of the v_m's and c_m's are in principle arbitrary in the normal-phase case, we take uniform signs, since the normal-phase case is introduced as a limiting situation of general cases with nonvanishing monopole pairing.

We first consider $\delta<0$, since lower orbits have larger quadrupole moments than for $\delta>0$. The occupied-orbit set R is given by $R=\{m|m=\pm j,\pm(j-1),\ldots,\pm(j-N+1)\}$. The x_0 and x_2 amplitudes in Eq. (6) are then calculated as

$$x_0=(N/\Omega)^{1/2}, \tag{9a}$$

$$x_2\simeq\left(\frac{5}{4}\right)^{1/2}\left(\frac{N}{\Omega}\right)^{1/2}\left(1-\frac{N}{\Omega}\right)\left(2-\frac{N}{\Omega}\right), \tag{9b}$$

with $\Omega=j+\frac{1}{2}$.

The quantity x_J^2, which is denoted hereafter by $\Pi(J)$, means the probability that the spin-J pair is found in the Cooper pair. In the present case, $\Pi(0)$ is equal to N/Ω, giving rise to 50% at the middle of the shell. The probability $\Pi(2)$ is also proportional to N/Ω in the lowest order. The probability $\Pi(0)+\Pi(2)$ is denoted by $\Pi(0\text{-}2)$. From Eqs. (9a) and (9b), one clearly sees that $\Pi(0\text{-}2)$ increases very rapidly as N/Ω, and becomes about 86% at $N/\Omega\sim0.4$. We emphasize that this behavior is independent of the value of j, in spite of the fact that the number of possible pairs is given by Ω $(=j+\frac{1}{2})$.

The pairing interaction h_P is now turned on. Since the pairing interaction pushes down the 0^+ pair, $\Pi(0\text{-}2)$ naturally increases as the strength G. In Fig. 1, the probabilities $\Pi(J)$ $(J=0,2,4,6)$ and $\Pi(0\text{-}2)$ are shown for $j=\frac{41}{2}$ with $G=0.2$ MeV and $\delta=-0.3$. The value of G is evaluated from the formula $G\simeq30/A$ MeV in Refs. 1 and 14, taking $A\simeq150$. Again, the rapid increase of $\Pi(0\text{-}2)$ with N/Ω is clearly seen. Furthermore, the probabilities $\Pi(4)$ and $\Pi(6)$ decrease as N/Ω, and become quite small when $N/\Omega>\frac{1}{4}$. The intrinsic quadrupole moment $Q_0=2r^2C_0^{(2)}(\theta)$ is also shown as a function of N in Fig. 1, where

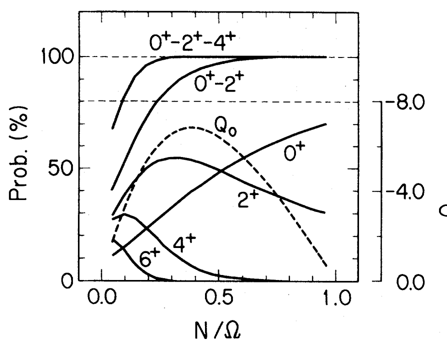

FIG. 1. Probabilities to find 0^+, 2^+, 4^+, and 6^+ pairs in the Cooper pair obtained in the Nilsson model. The single $j=\frac{41}{2}$ orbit is taken with the deformed single-particle field of $\delta=-0.30$ and the pairing force of $G=0.20$ MeV. The number of nucleon pairs (N) is varied from the beginning of the shell to the end, while its fraction of the total degeneracy Ω is shown. The sum of the probabilities of the 0^+ and 2^+ pairs (0^+-2^+) and the sum of the probabilities of the 0^+, 2^+, and 4^+ pairs (0^+-2^+-4^+) are also shown. The intrinsic quadrupole moment (Q_0) is indicated by the broken line. The moment has no units since r^2 in Q_0 is replaced by unity.

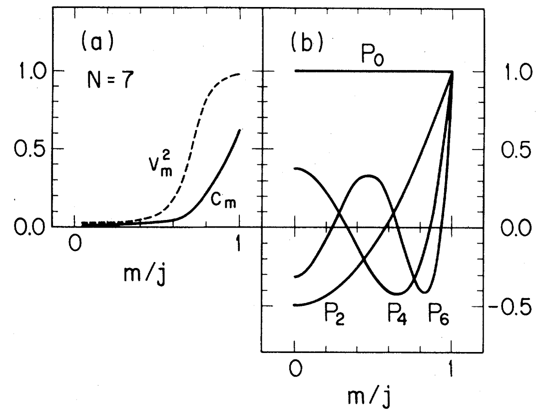

FIG. 2. (a) Amplitudes c_m in Eq. (5) and probabilities v_m^2 for $N=7$ in Fig. 1. (b) Legendre polynomials $P_J(m/j)$ in Eq. (7).

r^2 in Q_0 is replaced by unity because of the $j=\frac{41}{2}$ single j orbit. As is well known, the Nilsson model is applicable in regions of large quadrupole moments. Figure 1 shows that the 0^+ and 2^+ pairs are indeed dominant in such strongly deformed regions.[15] The probability $\Pi(0\text{-}2\text{-}4)=\Pi(0)+\Pi(2)+\Pi(4)$ is also shown in Fig. 1. This probability is already 98% around $N/\Omega=\frac{1}{4}$ where strong deformation starts. Thus, one can describe the Cooper pair almost perfectly by including only the 4^+ pair in addition to the 0^+ and 2^+ pairs. We mention that the trend seen in Fig. 1 is quite insensitive to the ratio G/δ. Although the description in terms of holes is more suitable for $N/\Omega \gg \frac{1}{2}$, this trend remains in the hole system.

Figure 2(a) shows c_m's and v_m^2's for $N=7$ as an example. Note that c_m is proportional to v_m/u_m. In Fig. 2(b), the Legendre polynomials P_0, P_2, P_4, and P_6 are shown as a function of m/j. The Legendre polynomial $P_J(x)$ has $J/2$ nodes between $x=0$ and $x=1$ for even J. Therefore, $P_J(x)$ changes its sign very frequently if J is large. The c_m amplitude, on the other hand, has the same sign (taken positive) because of the monopole pairing, and is a smooth function of m. Equation (7) therefore yields a small value of $|x_J|$ for large J because of cancellation among the $c_m P_J(m/j)$ terms. This cancellation arises at smaller J as N increases.

We have discussed so far cases of $\delta<0$. Probabilities $\Pi(0\text{-}2)$ for $\delta>0$ are remarkably larger than the corresponding ones for $\delta<0$ as a result of narrower energy spacings between lower orbits. This will be shown in a forthcoming paper.

Realistic cases with many nondegenerate single-particle orbits are considered next. The BCS calculation is performed for the Nilsson orbits where several j orbits are admixed as a result of the deformed field. The $\Lambda^\dagger$ operator in Eq. (5) is obtained from the u and v factors given by this BCS calculation. Extracting $\mathfrak{A}^{\dagger(J)}$ operators in Eq. (6) by the angular momentum projection of $\Lambda^\dagger$, we calculate x_J in Eq. (6).

An example of such a calculation is shown in Fig. 3. Single-particle orbits $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ are included. $\delta=0.25$, $G=0.20$ MeV, and $A\sim150$ are assumed. The spherical part of the single-particle field, which contains the spherical harmonic-oscillator potential, the (ls) and the (ll) terms, is the same as adopted in Refs. 1 and 16. In Fig. 3, probabilities $\Pi(J)$, $\Pi(0\text{-}2)$, and $\Pi(0\text{-}2\text{-}4)$ are shown as functions of N and N/Ω $[\Omega=\sum_j(j+\frac{1}{2})]$. The intrinsic quadrupole moment Q_0 is also plotted in the figure. Similarly to Fig. 1, the probabilities $\Pi(0)$, $\Pi(2)$, $\Pi(0\text{-}2)$, and $\Pi(0\text{-}2\text{-}4)$ increase with N, and become large in the region of large quadrupole moment. The probability $\Pi(0\text{-}2)$, for instance, is already 85% at $N=6$, i.e., $N/\Omega\simeq0.27$. Thus, the major trends observed in the single-j-orbit case are also seen in this realistic case. The present case is just an example, and the dominant roles of the 0^+ and 2^+ pairs are found in general.

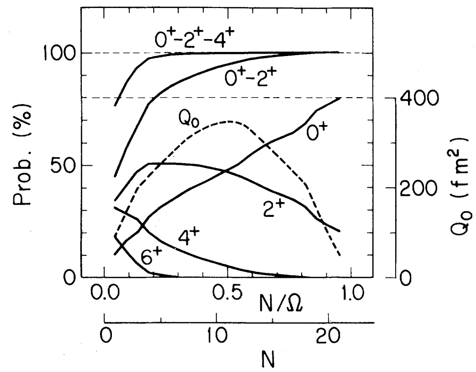

FIG. 3. Same as Fig. 1 except that a realistic system with single-particle orbits $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $i_{13/2}$ is considered and $\delta = 0.25$ is taken. The r^2 dependence of Q_0 is included with $A \simeq 150$.

We here introduce a 2^+ pair operator by the commutation relation[11] $\mathcal{F}^{\dagger(2)} \propto [\mathcal{A}^{\dagger(0)}, \xi_2]$, where ξ_2 is introduced in Eq. (2). The overlap between $\mathcal{A}^{\dagger(2)}$ and $\mathcal{F}^{\dagger(2)}$ is very large in general. For instance, in the present system, it is > 0.99 for $1 \lesssim N \lesssim 17$. This similarity between $\mathcal{A}^{\dagger(2)}$ and $\mathcal{F}^{\dagger(2)}$ implies that, in the Cooper pair, the 2^+ pair absorbs almost all quadrupole strength from the 0^+ pair. Thus, the 0^+ and 2^+ pairs are coupled strongly by the quadrupole field, dominating the Cooper pair.[12]

In summary, it has been shown that the Cooper pair in the Nilsson model consists mainly of the 0^+ and 2^+ pairs in regions of large quadrupole deformation. If the 4^+ pair is included, the Cooper pair is described almost perfectly in those regions.[17]

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Note added.—After completion of this work, we learned that a similar conclusion has been obtained in an investigation of Ref. 12 by R. A. Broglia and E. Maglione.[18]

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