Discrepancy in the Heat Capacity of Liquid ^{3}He

Pat R. Roach and Mark W. Meisel[a]
Solid State Science Division, Argonne National Laboratory, Argonne, Illinois 60439

and

Yakov Eckstein
Solid State Science Division, Argonne National Laboratory, Argonne, Illinois 60439, and Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

(Received 23 March 1981)

High-precision $P_v(T)$ measurements have been made on liquid ^{3}He at low temperatures; the results yield values for the derivative of the heat capacity, $\partial C_v/\partial v$, which can be compared with those from actual heat-capacity measurements. The comparison shows agreement between the present data and $\partial C_v/\partial v$ from heat capacities of Wheatley but disagreement with $\partial C_v/\partial v$ from recent heat capacities of Haavasoja.

PACS numbers: 67.50.Dg, 65.20.+w, 65.70.+y

According to Fermi-liquid theory,[1] the heat capacity of liquid ^{3}He at low temperatures should be just directly proportional to the temperature

$$G_v/R = \gamma(v)T, \tag{1}$$

where G_v is the molar heat capacity, R is the gas constant, and γ is the (volume-dependent) proportionality constant. This heat capacity has been measured a number of times[2-6] and its low-temperature behavior is found to follow Eq. (1) quite well. From the value of γ obtained from these measurements comes one of the most important parameters in Fermi-liquid theory, the effective mass, m^*, of the quasiparticles; most of the effect of the strong interaction between ^{3}He atoms has been absorbed into this term.

It was very disturbing, therefore, when recent very-low-temperature measurements of G_v by Haavasoja[7] and by Zeise *et al.*[8] yielded values for γ that are 30%–40% lower than those obtained in the only other very-low-temperature measurements.[4-6] This discrepancy is shown in Fig. 1 where the curve labeled Wheatley shows smoothed values of $\gamma(v)$ as derived[9] from the earlier measurements[4,5] (represented by the points). The discrepancy between the two sets of data is much larger than their combined uncertainties. If the recent data[7,8] were correct then the previously accepted value of m^* would have to be lowered 30%–40%.

Also significant in the present context is the fact that the derivatives of the two curves in Fig. 1 are substantially different; in order to resolve the discrepancy between the two heat-capacity measurements, we have developed an experiment that determines the volume derivative of the heat capacity. Although it cannot determine C_v itself, the technique has the advantage of being nonthermal in nature. This means that it does not suffer from the drawbacks of a standard heat-capacity measurement such as uncertain calorimeter background heat capacity and it is not nearly so sensitive to the effects of heat leaks and long equilibrium times. Comparison of our measurements of $\partial C_v/\partial_v$ with the volume derivatives of the data of Wheatley[9] and of Haavasoja[7] should shed light on the source of the discrepancy between the two experiments.

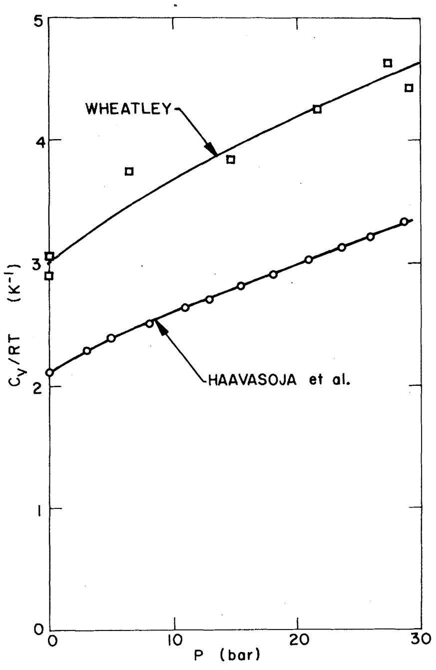

FIG. 1. Low-temperature heat-capacity coefficient of liquid ^{3}He showing disagreement between data of Wheatley (Ref. 9) and Haavasoja (Ref. 7).

The starting point for our measurement comes from the determination of the pressure of the liquid at constant volume as a function of temperature, $P_v(T)$. By a standard Maxwell relation one has

$$(\partial P/\partial T)_v = (\partial s/\partial v)_T, \qquad (2)$$

where s is the molar entropy. By taking another temperature derivative of this equation and interchanging the order of the derivatives one gets

$$\left(\frac{\partial^2 P}{\partial T^2}\right)_v = \left[\frac{\partial}{\partial v}\left(\frac{\partial s}{\partial T}\right)_v\right]_T = \left[\frac{\partial(C_v/T)}{\partial v}\right]_T. \qquad (3)$$

Thus, the volume derivative of C_v (but not C_v itself) can be determined by a measurement of $P_v(T)$. The quantity $(\partial^2 P/\partial T^2)_v$ is readily obtained experimentally from the asymptotic low-temperature behavior of $P_v(T)$. In the region where the heat capacity is described by Eq. (1) [which is equivalent to $s/R = \gamma T$], one has, using Eq. (2)

$$(\partial P/\partial T)_v = (\partial \gamma/\partial v)_T T \qquad (4)$$

which goes to zero at $T=0$. Therefore, in the expansion of $P(T)$ in powers of T about $T=0$,

$$P(T) = P_0 + \left(\frac{\partial P}{\partial T}\right)_v^{T=0} T + \frac{1}{2}\left(\frac{\partial^2 P}{\partial T^2}\right)_v^{T=0} T^2 + \dots, \qquad (5)$$

the coefficient of the linear term is zero and in the low-temperature limit, with use of Eqs. (1) and (3),

$$P(T) = P_0 + \frac{R}{2}\left(\frac{\partial \gamma}{\partial v}\right)_T T^2. \qquad (6)$$

The slope of $P(T)$ versus T^2 is just the quantity we want.

The apparatus for this experiment is shown in Fig. 2. It uses a very sensitive capacitive pressure gauge[10] which can resolve at least 10^{-5}-bar pressure changes over the entire 30-bar range of the experiment. Since the flexible BeCu diaphragm of the pressure gauge exhibits a slight hysteresis for large pressure changes, at each new pressure the gauge is calibrated against a room-temperature pressure gauge.[11] The other critical part of the experiment is the low-temperature valve.[12] Because it is mounted close to the cell, it isolates the sample ^{3}He from external pressure fluctuations. In addition, virtually all of the ^{3}He contained in the constant-volume sample is at the same temperature and any correction for part of the sample at a different temperature is negligible. Both the valve and the pressure gauge are thermally anchored to the mixing chamber of our dilution refrigerator. The lines connecting them to the sample cell are much longer and thinner than shown schematically in the figure and thermally isolate the sample cell. The amount of ^{3}He contained in the valve and gauge and their connecting lines is a very small fraction of the total ^{3}He and requires no correction to the data.

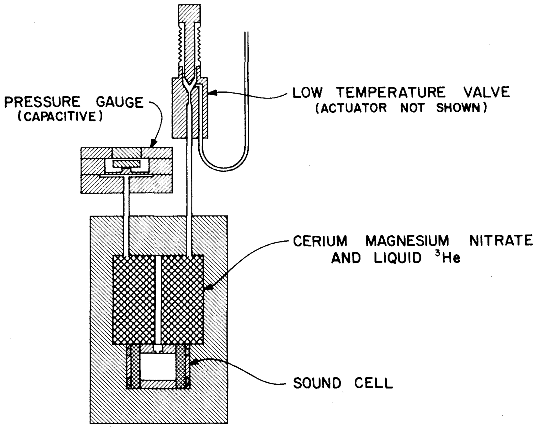

FIG. 2. Experimental apparatus for measuring $P_v(T)$ in liquid ^{3}He at low temperatures.

The cerium magnesium nitrate (CMN) contained in the main cell provides both refrigeration and thermometry. To reach the lowest temperatures (~2 mK) the cell is precooled to about 12 mK by our dilution refrigerator via a tin heat switch connected to copper wires imbedded in the inner wall of the sample cell. The heat switch is then opened and the CMN is demagnetized from 1 kG in a period of ~3 h. Once the magnetic field is zero, a set of coils on the outside of the cell measures the susceptibility of the CMN to determine the temperature. The constants C and M_0 in the equation $T = C/(M - M_0) + \Delta$ (where M is the reading of the susceptibility bridge) are determined by calibration against a germanium thermometer from 0.3 to 1.0 K. The constant Δ is determined by calibration against the superfluid transition temperature, T_c,[7] at high pressures; T_c is identified by observation of sound attenuation in the low-frequency acoustic cavity attached to the bottom of the cell. This sound cell was not otherwise used in the present experiment but was part of a related experiment to determine very accurate values of the thermal expansion of ^{3}He from 10 to 1100 mK and from 0 to 30 bars.[13]

The results of the measurements at six different pressures are shown in Fig. 3. The data are

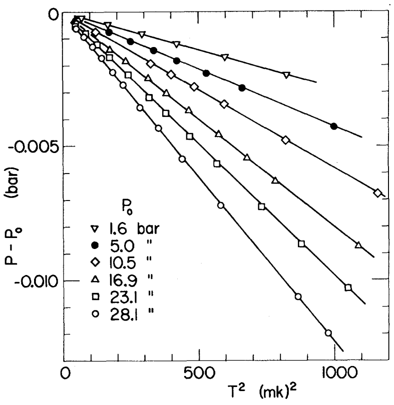

FIG. 3. Results of $P_v(T)$ for liquid ^{3}He at six different pressures plotted vs T^2.

plotted on a T^2 temperature scale to display the expected behavior according to Eq. (6). The data have been normalized to show only the changes in pressure from the value at $T = 0$. Clearly, the expected T^2 behavior is closely followed up to 30–35 mK, thus confirming the validity of Eq. (1). The slopes of these curves are, according to Eq. (6), the quantities to be compared to the volume derivatives of the experimental heat capacities. This comparison is made in Fig. 4. The volume derivatives of the C_v/RT data of Wheatley[9] and of Haavasoja[7] were obtained by first accurately fitting their data with smooth curves and then differentiating the equations describing the curves. In order to make the comparison with the present experiment, these derivatives are converted to units of μbar (mK)$^{-2}$. Error bars of ±10% are included with the derivatives of the data of Wheatley to reflect our estimate of the uncertainty of these numbers based on the uncertainties in his smooth values of C_v/RT. The data of Haavasoja are much more precise and we have not included the corresponding error bars for his data. Possible systematic errors, such as differing temperature scales between the two experiments, are not reflected in the error bars. Such effects would show up as an overall discrepancy between the two sets of data. The points shown in the figure are the results of the present experiment: The open circles are from the data shown in Fig. 3 and the closed circles are from various other runs on the same apparatus. Clearly, our points agree much better with the derivative of the heat-capacity data of Wheatley than with that of Haavasoja.

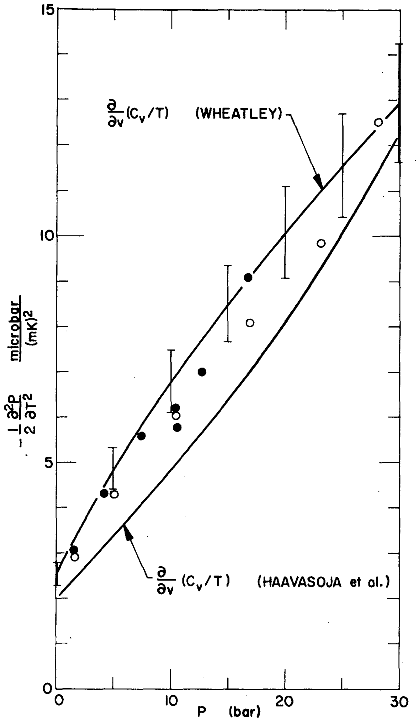

FIG. 4. Comparison of values of $-(\frac{1}{2})(\partial^2 P/\partial T^2)$ from the present experiment (open and closed circles) with values of $\partial(C_v/T)/\partial v$ from heat capacities of Wheatley (Ref. 9) and Haavasoja (Ref. 7).

This suggests that, of the possible reasons for the discrepancy in the heat capacities, gross error in Wheatley's heat-capacity technique is not one of the possibilities. The problem must lie either in a difference in the temperature scales between the two experiments or perhaps in a fundamental difference in the behavior of ^{3}He in the two experiments. The fact that our results confirm Wheatley's measurements does not necessarily reinforce the validity of his temperature scale. Our thermometry and that of Wheatley are almost identical. We use CMN calibrated from 0.3 to 1.0 K and he uses CMN calibrated from 1.1 to 2.2 K. This is contrasted with the Haavasoja experiment where La-diluted CMN was calibrated in the millikelvin region against the nuclear susceptibility of platinum. A discrepancy between the temperature scales could arise if the Curie constant of our CMN changed between high and low temperatures. It is unlikely that such a problem existed for the temperature scale used by Wheatley since that scale was compared against a noise thermometer to below 10 mK (Ref. 14) and no unusual behavior was found. The scale used by Haavasoja has been compared by Lhota *et al.*[15] with a scale developed by the National Bureau of Standards and agreement is found to within a few percent in the 16–23-mK region. Nevertheless, when Haavasoja directly compared[7] his temperature scale with a recent scale by Paulson *et al.*[16] he found $T_P^* = 0.900 T_H^* + 0.003$ mK where T_P^* and T_H^* are the magnetic temperatures of Paulson *et al.* and of Haavasoja, respectively. While the Paulson scale, which is based on La-diluted CMN calibrated between 0.3 and 1.1 K, is not necessarily similar to the Wheatley scale used in the heat-capacity experiments, the comparison does lend support to the idea that significant disagreement can arise between scales calibrated at high and low temperatures. A specific example of a magnetic thermometer with a different Curie constant at high and low temperatures has been reported by Zeise.[17]

Another possible reason for the heat-capacity discrepancy is that the heat capacity of ^{3}He is modified by the presence of finely powered CMN. An interaction between ^{3}He and CMN could affect the results of the present experiment but would not affect the results of Haavasoja[7] or of Zeise *et al*,[8] where the ^{3}He being measured is not in contact with CMN. It should not affect the results of Wheatley,[4] either, however, because his results were derived from the difference of measurements with different amounts of ^{3}He but a constant amount of CMN.

It seems that there is no completely plausible explanation for the heat-capacity discrepancy. It is now fairly clear, though, that the discrepancy cannot be ignored. It does not appear to be an artifact of one particular set of measurements but seems to arise from a more fundamental problem.

This work was supported by the U. S. Department of Energy.

[a] Present address: Department of Physics, Northwestern University, Evanston, Ill. 60201.

[1] See, for example, G. Baym and C. Pethick, in *The Physics of Liquid and Solid Helium, Part II,* edited by K. H. Bennemann and J. B. Ketterson, (Wiley, New York, 1978), Chap. 1.

[2] D. F. Brewer and J. R. G. Keyston, Nature (London) 191, 126 (1961).

[3] M. Strongin, G. D. Zimmerman, and H. A. Fairbank, Phys. Rev. 128, 1983 (1962).

[4] W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Phys. Rev. 147, 111 (1966).

[5] A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev. 130, 495 (1963).

[6] W. P. Halperin, F. B. Rasmussen, C. N. Archie, and R. C. Richardson, J. Low Temp. Phys. 31, 617 (1978).

[7] T. Haavasoja, Ph.D. thesis, Helsinki University of Technology (unpublished). See also T. A. Alvesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, Phys. Rev. Lett. 44, 1076 (1980).

[8] E. K. Zeise, J. Saunders, A. I. Ahonen, C. N. Archie, and R. C. Richardson, Physica (Utrecht) 108B, 1213 (1981).

[9] J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).

[10] G. C. Straty and E. D. Adams, Rev. Sci. Instrum. 40, 1393 (1969).

[11] Quartz bourdon gauge, product of Texas Instruments, Inc., Houston, Texas.

[12] P. R. Roach, J. B. Ketterson, and M. Kuchnir, Rev. Sci. Instrum. 43, 898 (1972).

[13] P. R. Roach, M. W. Meisel, and Y. Eckstein, to be published.

[14] R. A. Webb, R. P. Giffard, and J. C. Wheatley, J. Low Temp. Phys. 13, 383 (1973).

[15] E. Lhota, M. T. Manninen, J. P. Pekola, and A. T. Soinne, Phys. Rev. Lett. 47, 590 (1981).

[16] D. N. Paulson, M. Krusius, J. C. Wheatley, R. S. Safrata, M. Kolac, T. Tethal, K. Svec, and J. Matas, J. Low Temp. Phys. 34, 63 (1979), and 36, 721(E) (1979).

[17] E. K. Zeise, unpublished.