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Determination of the Fine-Structure Constant Using GaAs-Al_xGa_{1-x}As Heterostructures

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The fine-structure constant α has been determined from precision measurements of quantized Hall resistances R_H of three different GaAs-Al_xGa_{1-x}As heterostructures. The result, $\alpha^{-1}=137.035\,968(23)$ (0.17 ppm), is in excellent agreement with the 0.11-ppm value obtained from the gyromagnetic ratio of the proton, γ_p', and $2e/h$ via the Josephson effect. Our R_H value can be combined with γ_p' and $2e/h$ to yield a more accurate value of α^{-1} independent of the ohm: $\alpha^{-1}=137.035\,965(12)$ (0.089 ppm).

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The work of von Klitzing, Dorda, and Pepper[1] first demonstrated the possibility that measurements of quantized Hall resistances R_H of the two-dimensional (2D) electron gas formed in certain solid-state devices could be used to obtain a precise value for the fine-structure constant α. This possibility was subsequently verified to an accuracy of 1.3 ppm by Braun, Staben, and von Klitzing,[2] and to 0.88 ppm by Yamanouchi *et al.*[3] All these experiments used silicon metal-oxide-semiconductor field-effect transistors (MOSFETs), which require a magnetic field $B\sim 13$ T to reach the high-field quantization regime. Since the effective mass of the electrons in the 2D gas in GaAs formed at the interface of GaAs-Al_xGa_{1-x}As heterojunctions is three times less than in Si, this regime can be reached with $B<10$ T. In this Letter we report the first precision measurements of R_H in this 2D electron system. Our result gives the most accurate determination of α from any quantized Hall resistance experiment to date.

In GaAs-Al_xGa_{1-x}As heterojunctions, the quantized Hall effect is observed as a series of flat steps in plots of R_H as a function of B.[4] At a step, the Fermi energy is between two Landau levels, with all the conducting states filled in the lower level and empty in the upper level. The quantized Hall resistance (the ratio between the Hall voltage across the sample and the current I) is given by

$$R_H=\frac{V_H}{I}=\frac{h}{e^2 i}=\frac{\mu_0 c}{2\alpha i}\approx\frac{25\,813\ \Omega}{i}, \tag{1}$$

where $\mu_0=4\pi\times10^{-7}$ H/m is the permeability of vacuum; $c=(299\,792\,458\pm1.2)$ m/s (0.004 ppm) is the speed of light in vacuum[5]; h is Planck's constant, e is the electron charge, and the quantum number i is the number of completely filled Landau levels.

The GaAs-Al_xGa_{1-x}As ($x=0.29$) heterostructure

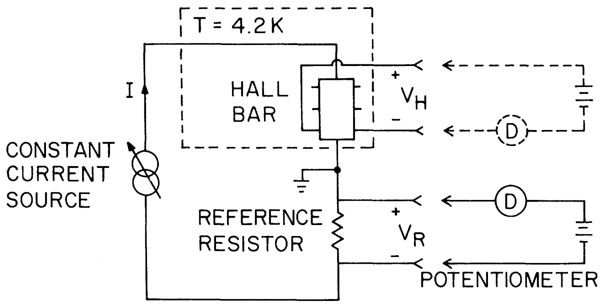

FIG. 1. A simplified schematic of the measurement circuit.

samples were prepared by molecular-beam epitaxy.[4,6] They are 4.6 mm long and 0.38 mm wide, and have three sets of potential probes with two sets symmetrically displaced ± 1.0 mm along the channel from the center set. Mobilities for the three samples are 23.5, 11.5, and 7.9 m^2/V s, respectively, at zero magnetic field and 4.2 K. Our measuring circuit is indicated in Fig. 1. Room-temperature reference resistors R_R were constructed with values nominally equal to that of the Hall resistance, that is, 12 906.40 Ω for $i=2$, 6453.20 Ω for $i=4$. Therefore, $R_H=(V_H/V_R)R_R$, with $V_H/V_R\approx 1$. The potentiometer voltage is made almost equal to the voltage drop across V_H or V_R, and an electronic detector, D, with an input current less than 10^{-15} A, amplifies the difference-voltage signal. The current source, potentiometer, and electronic detector are all battery operated.

The potentiometer does not require calibration in this arrangement. Thermally induced voltages and linear drifts in the current source and the potentiometer are canceled by reversing the current through the sample and the reference resistor. A series of reversals in the order +--+ and +--+ is made for each of two measurements of V_H which bracket in time one measurement of V_R in order to obtain a single datum point. Since V_H was measured with the center potential-probe set, no geometrical corrections[7] were required.

Figure 2 shows the Hall steps for the three samples for $I=+10$ μA. Two of the steps are for $R_H=12\,906.40$ Ω ($i=2$) and one is for $R_H=6453.20$ Ω ($i=4$). The $i=4$ step occurred at such a low B that the sample was cooled to 1.4 K to ensure that the step was flat. Note that the structure at the edges of the Hall step is dramatically different in each sample. Although inhomogeneities[8] and thermoelectric effects[9] have been suggested as possible causes, there is no quantitative explanation of this structure at present. Since the step width decreased measurably for $|I|\geq 15$ μA, precision data were taken with $I=\pm 10$ μA. The magnet was operated in the persistent-current mode at the magnetic field values indicated by the arrows in Fig. 2.

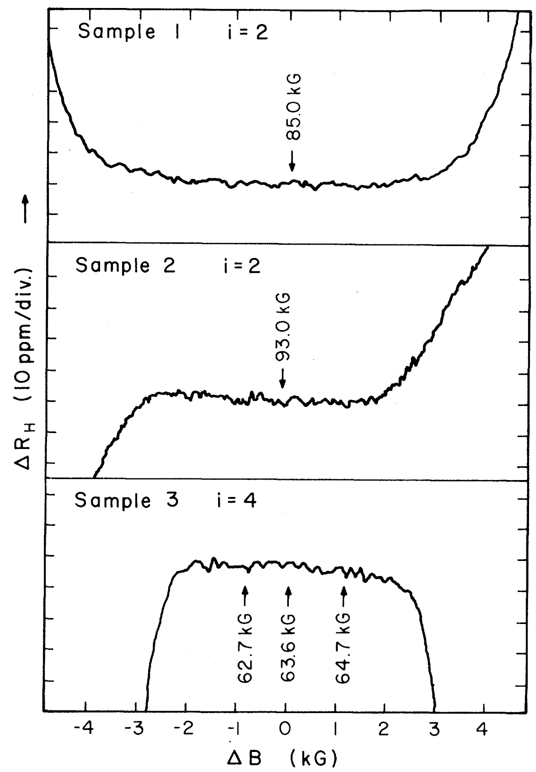

FIG. 2. High-sensitivity traces of the Hall steps for the three samples.

The quantities of Eq. (1) are in absolute (SI) units, but R_H is measured via R_R in terms of the National Bureau of Standards (NBS) as-maintained unit of resistance, Ω_{NBS}. Therefore, R_H and α^{-1} must be determined from $(R_H)_{NBS}$ by use of the equation

$$\alpha^{-1}=\frac{2iR_H}{\mu_0 c}=\frac{2i}{\mu_0 c}(R_H)_{NBS}\left(\frac{\Omega_{NBS}}{\Omega}\right)$$

$$=\frac{2i}{\mu_0 c}\left(\frac{V_H}{V_R}\right)(R_R)_{NBS}\left(\frac{\Omega_{NBS}}{\Omega}\right), \qquad (2)$$

where $(\Omega_{NBS}/\Omega)=1-(0.819\pm 0.075)\times 10^{-6}$ is the ratio of the NBS as-maintained ohm to the SI ohm,[10] and includes a 0.07-ppm uncertainty (one standard deviation estimate) due to a possible drift of the NBS ohm since its last absolute realization via the NBS calculable cross capacitor in 1974.

TABLE I. Measured values of $(R_{\rm H})_{\rm NBS}$.

Sample	i	$(R_{\rm H})_{\rm NBS}$ ($\Omega_{\rm NBS}$)	Uncertainty[a] (ppm)
1	2	12 906.4112(21)	0.16
2	2	12 906.4128(21)	0.16
3	4	6 453.2050(31)	0.20

[a]One standard deviation (68% confidence level) estimates based upon the root sum square of the uncertainties given in Table II.

The results for $(R_{\rm H})_{\rm NBS}$ are given in Table I, with the uncertainties listed in Table II. Expressing the simple average of the three results of Table I in terms of an $i=4$ step yields

$$(R_{\rm H})_{\rm NBS}=6453.2057(10)\ \Omega_{\rm NBS}\ \ (0.16\ \text{ppm}), \tag{3}$$

where the assigned uncertainty is that of one of the two 12.9-kΩ samples because the uncertainties for all three samples are highly correlated.

Using Eqs. (2) and (3) and the value given above for $\Omega_{\rm NBS}/\Omega$, we obtain

$$(R_{\rm H})_{i=4}=6453.2004(11)\ \Omega\ \ (0.17\ \text{ppm}) \tag{4}$$

and

$$\alpha^{-1}=137.035\,968(23)\ \ (0.17\ \text{ppm}). \tag{5}$$

This α^{-1} value is in good agreement with the 137.035 840(180) (1.3 ppm) result[2] and the 137.035 890(120) (0.88 ppm) result[3] obtained from previous quantized Hall resistance measurements on Si MOSFETs.

A value of α^{-1} with even smaller uncertainty can be obtained by combining Eq. (2) with Eq. (1) of Ref. 11 for the gyromagnetic ratio of the proton, $\gamma_p'(\text{low})_{\rm NBS}$:

$$\alpha^{-1}=\left[\frac{i}{2\mu_0 R_\infty}\,\frac{\mu_p'}{\mu_{\rm B}}\left(\frac{\Omega_{\rm NBS}{}^{1981}}{\Omega_{\rm NBS}{}^{1978}}\right)\frac{(2e/h)_{\rm NBS}(R_{\rm H})_{\rm NBS}}{\gamma_p'(\text{low})_{\rm NBS}}\right]^{1/3}, \tag{6}$$

where R_∞ and $\mu_p'/\mu_{\rm B}$ are, respectively, the Rydberg constant and the proton magnetic moment in units of the Bohr magneton. Using $(R_{\rm H})_{\rm NBS}$ given in Eq. (3), assuming that $(\Omega_{\rm NBS}{}^{1981}/\Omega_{\rm NBS}{}^{1978})=1\pm0.03\times10^{-6}$, and using values listed in Table I of Ref. 11 for the other constants, we obtain

$$\alpha^{-1}=137.035\,965(12)\ \ (0.089\ \text{ppm}). \tag{7}$$

This value should be more reliable than that given in Eq. (5) because $\Omega_{\rm NBS}{}^{1981}$ is closer in time to $\Omega_{\rm NBS}{}^{1978}$ than to $\Omega_{\rm NBS}{}^{1974}$.

It must be noted that we have not exhaustively addressed the question of the dependence of $R_{\rm H}$

TABLE II. Estimated one standard deviation (68% confidence level) uncertainties in $(R_{\rm H})_{\rm NBS}$.

Sources of uncertainty	Uncertainties (ppm) Samples 1 and 2	Sample 3
Random measurement uncertainty[a]	0.06	0.05
Circuit leakage currents[b]	0.02	0.02
Reference-resistor temperature correction	0.05	0.04
Transportation shift of reference resistor	0.09	0.16
Calibration of reference resistor[c]	0.10	0.10
Root-sum-square total (ppm)	0.16	0.20

[a]Standard deviation of the mean of 20 experimental data points.

[b]Upper limit based upon intercomparison of two 10-kΩ resistors in place of the sample and reference resistor.

[c]Uncertainty in stepup from $\Omega_{\rm NBS}$ (maintained via five 1 Ω resistors) to the reference resistor.

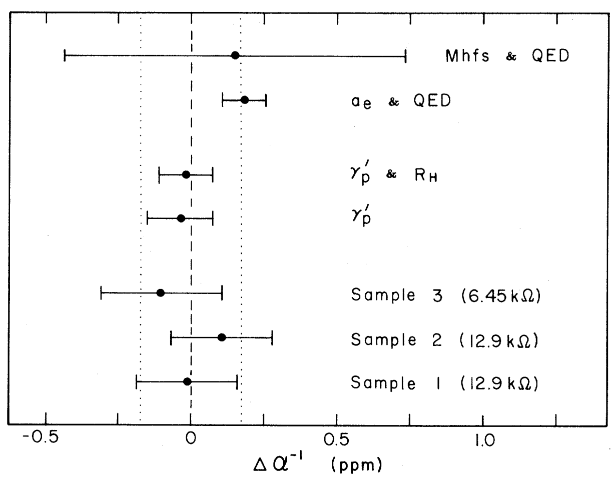

FIG. 3. A comparison of α^{-1} values relative to the average value for our three samples, $\alpha^{-1}=137.035\,968(23)$ (0.17 ppm). This value is indicated by the vertical dashed and dotted lines. The value marked γ_p' is the $\gamma_p'(\text{low})_{\rm NBS}$ result (Ref. 11); the value marked γ_p' & $R_{\rm H}$ is the $\gamma_p'(\text{low})_{\rm NBS}$ and $(R_{\rm H})_{\rm NBS}$ combined result, Eq. (7); the a_e & QED value is the QED theory-dependent result (Ref. 15), $\alpha^{-1}=137.035\,993(10)$ (0.073 ppm), from the anomalous magnetic moment of the electron (Ref. 16); and the Mhfs & QED value, $\alpha^{-1}=137.035\,988(80)$ (0.58 ppm), is the preliminary result from the most recent muonium hyperfine splitting measurements (Ref. 17) and includes the presently estimated 5-kHz uncertainty in the QED theory due to uncalculated terms (Ref. 18).

on sample parameters; we can only state that there is no detectable dependence for the three samples reported here. Nevertheless, these measurements, previous results,[1-3] and recent theoretical advances[12-14] support the validity of Eq. (1). Our α^{-1} results are compared in Fig. 3 with other determinations that have uncertainties smaller than 0.6 ppm. They are in good agreement.

The largest uncertainties in our experiment arise from transport of the reference resistors and their calibration with respect to the NBS ohm. Improved instrumentation now under construction should allow measurements of R_H to an accuracy of a few parts in 10^8.

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