Supersymmetry, Cosmology, and New Physics at Teraelectronvolt Energies

Heinz Pagels
The Rockefeller University, New York, New York 10021

and

Joel R. Primack
Physics Department, University of California, Santa Cruz, California 95064
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If one assumes a spontaneously broken local supersymmetry, big-bang cosmology implies that the universe is filled with a gravitino ($g_{3/2}$) gas—possibly its dominant constituent. From the observational bound on the cosmological mass density it follows that $m_{g_{3/2}} \lesssim 1$ keV. Correspondingly, the supersymmetry breaking parameter F satisfies $\sqrt{F} \lesssim 2\times 10^3$ TeV, requiring new supersymmetric physics in the teraelectronvolt energy region. An exact sum rule is derived and used to estimate the threshold and cross section for the production of the new states.

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Several promising features of supersymmetry,[1] in addition to its intrinsic elegance, have recently attracted increased attention. Supergravity[2] possibly leads to a renormalizable—indeed, finite—quantum field theory of gravity, and may also explain the absence of a vacuum energy density (cosmological term).[3,4] While extended ($N \geq 2$) supergravity theories may unify gravitation and grand unified theories (GUT) in the vicinity of the Planck mass, $M_{\rm Pl} = G^{-1/2} = 1.2\times 10^{19}$ GeV, only simple ($N=1$) supersymmetry or supergravity is apparently relevant at ordinary energies.[5] Fairly realistic $N=1$ models have been constructed incorporating electroweak, GUT, or hypercolor phenomenology in which spontaneous supersymmetry breaking is assumed to occur in the teraelectronvolt energy range or below.[5-7] Of particular interest is the possibility that supersymmetry may protect the electroweak Higgs boson from acquiring a GUT-scale vacuum expectation value, thereby preserving a gauge hierarchy put in by hand. There is even the more attractive possibility that supersymmetry breaking may actually solve the notorious hierarchy problem.

Here we shall be concerned with $N=1$ exact supersymmetry[5] for which the fundamental anticommutator is (in $+---$ metric)

$$\{Q_\alpha, \overline{Q}_\beta\} = 2\gamma_{\alpha\beta}{}^\mu P_\mu . \tag{1}$$

Correspondingly,

$$\{Q_\alpha, \bar{J}_\beta{}^\nu\} = 2\gamma_{\alpha\beta}{}^\mu \theta_\mu{}^\nu , \tag{2}$$

where $\theta_{\mu\nu}$ is the energy-momentum tensor ($P_\mu = \int \theta_{\mu 0}\, d^3x$) and $J_\alpha{}^\mu$ is the supercurrent ($Q_\alpha = \int J_\alpha{}^0\, d^3x$). Since the Wigner-Weyl realization of supersymmetry is phenomenologically unacceptable we will assume that the symmetry is spontaneously broken, $Q_\alpha|0\rangle \neq 0$. Then Goldstone's theorem requires the existence of a massless spin-$\frac{1}{2}$ Majorana particle, the Goldstone spinor. The vacuum expectation value of $\theta_{\mu\nu}$,

$$\langle \theta_{\mu\nu}\rangle_0 = \tfrac{1}{2}F^2 g_{\mu\nu} , \tag{3}$$

defines the order parameter, F (analogous to f_π in chiral-symmetry breaking), which sets the scale of the spontaneously broken supersymmetry. Indeed, (2) and (3) imply for the current–Goldstone-spinor coupling

$$\langle 0|J_\alpha{}^\mu|\psi_\beta\rangle = F\gamma_{\alpha\beta}{}^\mu , \tag{4}$$

where $|\psi_\beta\rangle$ is the Goldstone spinor state.

Both the vacuum energy and the massless Goldstone spinor can be eliminated by gauging the supersymmetry and coupling the matter action to supergravity as discussed by Deser and Zumino.[3] By the super-Higgs mechanism,[8] the gravitino, the self-conjugate spin-$\frac{3}{2}$ partner of the graviton, absorbs the Goldstone spinor and becomes massive. The vacuum energy (3) associated with the matter action is exactly cancelled by the cosmological term incorporated into the gravitational action if the gravitino mass is given by

$$m_{g_{3/2}} = \kappa F/\sqrt{6} , \tag{5}$$

where $\kappa = (8\pi G)^{1/2} = 4.1\times 10^{-19}$ GeV^{-1}. It has been suggested that this cancellation of the effective cosmological term follows from a symmetry of the action (checked through terms of order κ),[4] and we assume its validity here.

In the context of the above assumptions we can now summarize our conclusions. (1) The major material constituent of the universe could be massive gravitinos. Gravitinos could also provide the dark matter required in galactic halos and small clusters of galaxies. (2) The gravitino mass is bounded from above by the bounds on the cosmological matter density in the standard big-bang cosmology. This implies from (5) an upper bound on the supersymmetry order parameter, $F \lesssim (10^3\ \mathrm{TeV})^2$, a remarkable connection between high-energy physics and cosmology. Hence if supersymmetry is at all relevant to high-energy physics it will be dynamically manifested at energies well below the Planck mass—the GUT "desert" must be populated. (3) To reinforce this conclusion we derive an exact sum rule using supersymmetry current algebra for the mass of any particle as an integral over the total gravitino-particle cross section. Analysis of this mass sum rule enables us to estimate the threshold and cross section for the new physics.

First we turn to cosmology. If a spontaneously broken local supersymmetry actually corresponds to physical reality, then the universe will be filled today with a gas of primordial gravitinos in addition to photons and neutrinos. According to standard hot big-bang cosmology,[9] weak-interaction cross sections were sufficiently large to keep left-handed neutrinos in thermal equilibrium with the primordial plasma until the temperature dropped to a few megaelectronvolts. The subsequent e^+e^- annihilation at $T \simeq m_e$ heated the remaining interacting particles but not the neutrinos, with the result that thereafter $T_\nu/T_\gamma = (\frac{4}{11})^{1/3}$. The number density of photons today is $n_{\gamma 0} = (400\ \mathrm{cm}^{-3})\,[T/(2.7\ \mathrm{K})]^3$; that of each species of neutrino plus antineutrino is $n_{\nu 0} = \frac{3}{4}(T_\nu/T_\gamma)^3 n_{\gamma 0} = \frac{3}{11} n_{\gamma 0}$. The corresponding expression for gravitinos is

$$n_{g_{3/2}0} = \frac{3}{4}\left(\frac{g}{2}\right)\frac{43}{11}\,\frac{1}{g_{g_{3/2}d}}\,n_{\gamma 0}. \qquad (6)$$

Here $g_{g_{3/2}d}$ is the effective number of degrees of freedom of thermally interacting relativistic particles [= (number of boson spin states) + $\frac{7}{8}$(number of fermion spin states)] at the temperature of gravitino ($g_{3/2}$) decoupling, and $g = 4$ is the number of spin states of the gravitino. As we discuss below, we expect $g_{g_{3/2}d}$ to be at least as large as, but not necessarily much larger than, the effective number of degrees of freedom g_{ew} just above the electroweak symmetry restoration temperature [$g_{\mathrm{ew}} = 106\frac{3}{4}$ including the SU(3) ⊗ SU(2) ⊗ U(1) gauge bosons, the Higgs boson, and three generations of quarks and leptons].

Using (6) and bounds on the average mass density we can obtain a bound on the gravitino mass. Measurements of the Hubble parameter and isotope and stellar age determinations imply that ρ_0, the average mass density today, is not more than $\rho_{\mathrm{max}} = 2\times 10^{-29}\ \mathrm{g\ cm^{-3}} = 11\ \mathrm{keV\ cm^{-3}}$ if there is no cosmological constant.[10] This implies an upper bound on the masses of primordial particles[11]

$$n_{g_{3/2}} m_{g_{3/2}} + n_\nu \sum m_\nu \leq \rho_{\mathrm{max}},$$

or

$$m_{g_{3/2}} \leq g_{g_{3/2}d}\,(5\ \mathrm{eV} - \tfrac{2}{43}\textstyle\sum m_\nu). \qquad (7)$$

If one assumes for definiteness that $g_{g_{3/2}d} \lesssim 200$, the bound becomes $m_{g_{3/2}} \leq 1$ keV. If $\rho_0 < \rho_{\mathrm{max}}$ or the neutrinos account for most of ρ_0, the bound is lowered.[12]

Although a neutrino-dominated universe is becoming increasingly attractive to cosmologists,[10,13] it is worth considering the possibility that much of the missing mass is gravitinos. Neutrinos cluster[13] on mass scales $\sim M_{\mathrm{Pl}}^3/m_\nu^2$, which for $m_\nu = 30$ eV is $\sim 10^{15} M_\odot$, i.e., the scale of large clusters of galaxies. Since the mass bound on gravitinos is higher, they could cluster on a smaller mass scale. If the mass of the gravitino is comparable to our bound, then gravitinos will play a role in the formation of galaxies and small groups of galaxies.

Most interesting from the viewpoint of high-energy physics is that the bound on the gravitino mass implies that the order parameter F of spontaneously broken supersymmetry is also bounded. Using (5) and (7) we obtain

$$F \leq (5\ \mathrm{eV})(\sqrt{6}/\kappa)\,g_{g_{3/2}d} = (1.7\times 10^2 g_{g_{3/2}d}^{1/2}\ \mathrm{TeV})^2. \qquad (8)$$

This implies that dynamical features associated with supersymmetry must appear in the teraelectronvolt energy range and possibly at much lower energies. The GUT desert must be populated.

In order to make this more precise we return to the supersymmetry algebra (1) to derive a current-algebra sum rule (the analogue of the Adler-Weissberger sum rule) for the mass of any particle. The amplitude

$$A_{\alpha\beta}^{\mu\nu}(p,q) = i\int d^4x\, e^{iq\cdot x}\langle p|\,T\{J_\alpha^{\ \mu}(x), \bar{J}_\beta^{\ \nu}(0)\}\,|p\rangle \qquad (9)$$

corresponds to the scattering of gravitinos of momentum q on a spin-averaged target of momentum p, $p^2=M^2$. Let $p\cdot q=M\omega$. By Eq. (2) and standard current-algebra manipulations we find, using $\langle p|\theta_{\mu\nu}|p\rangle=p_\mu p_\nu$, the Ward identity

$$q_\mu A_{\alpha\beta}{}^{\mu\nu}=-2(\not p)_{\alpha\beta}p^\nu \qquad (10)$$

from the equal-time anticommutator. The gravitino scattering amplitude (9) is related to the corresponding Goldstone spinor scattering amplitude $A_{\alpha\beta}$ by (4):

$$A^{\mu\nu}=(F\gamma^\mu)(i/\not q)A(i/\not q)(F\gamma^\nu)+\text{nonpole terms}. \qquad (11)$$

In the $\omega\to 0$ limit (10) and (11) give the low-energy theorem $A\to 2M\omega\not p/F^2$. Fayet[6,14] first pointed out that the couplings of a light helicity-$(\pm\frac{1}{2})$ gravitino are much larger than gravitational in strength, consistent with this theorem. The corresponding theorem for the forward Goldstone-spinor–particle scattering amplitude is

$$T(\omega)\underset{\omega\to 0}{=}M\omega^2/2\pi F^2+O(\omega^4). \qquad (12)$$

Using the optical theorem, $\operatorname{Im}T(\omega)=-\omega\sigma_T(\omega)/4\pi$ where $\sigma_T(\omega)$ is the total cross section for scattering helicity-$(\pm\frac{1}{2})$ gravitinos on the spin-averaged target, and an unsubtracted dispersion relation for $T(\omega)/\omega^2$, we deduce the supersymmetry mass sum rule

$$M/F^2=\pi^{-1}\int_{\omega_T}^{\infty}(d\omega/\omega^2)\sigma_T(\omega), \qquad (13)$$

where ω_T is the production threshold. The Froissart bound $\sigma_T(\omega)\leq c\ln^2\omega$ guarantees that (13) converges. This sum rule can be generalized to the nonforward direction and one obtains a rule for $\langle p'|\theta_{\mu\nu}|p\rangle$, $p'\neq p$.

The sum rule (13) can be used to obtain both a rough upper bound for the threshold ω_T for the new physics and a rough bound on the cross section. For the first purpose we make the plausible assumption that σ_T is dominated by one or a few low partial waves for which unitarity implies $\sigma_T(\omega)\leq 4\pi/\omega^2$ up to a factor of order unity. The sum rule (13) and bound (8) then imply (for $M\sim\text{GeV}$)

$$\omega_T\leq(4F^2/3M)^{1/3}\leq g_{g_{3/2}d}{}^{2/3}(10^4\ \text{TeV}). \qquad (14)$$

In supersymmetry theory there is a reflection symmetry R under which ordinary particles are even and their symmetric partners are odd,[6] so that ω_T is the threshold for R-odd states. It is unlikely that we can experimentally reach these states in the near future if their threshold is near the bound (14); however, the threshold for R-odd states could easily be far below our bound. In either case it is clear the GUT desert must be populated with R-odd states. We emphasize that this conclusion is model independent.

If we presume that the threshold ω_T for R-odd states is at least of the order 100 GeV (as is plausible from the phenomenological success of the electroweak theory and will soon be checked by experiment) then the sum rule (13) allows us to estimate the cross section

$$\bar\sigma\simeq\frac{\pi M\omega_T}{F^2}=(10^{-37}\ \text{cm}^2)\left(\frac{\omega_T}{100\ \text{GeV}}\right)\left(\frac{1\ \text{TeV}^2}{F}\right)^2$$
$$\geq\frac{10^{-46}\ \text{cm}^2}{g_{g_{3/2}d}{}^2}\left(\frac{\omega_T}{100\ \text{GeV}}\right). \qquad (15)$$

For ω_T and F not too large the R-odd states might be experimentally accessible.[5-7]

Especially interesting is another conclusion that follows from (15): The cross section for helicity-$(\pm\frac{1}{2})$ gravitino scattering is large enough to keep these gravitinos in thermal equilibrium down to $T\simeq\omega_T$ or 100 GeV, whichever is higher. The relevant criterion[9] is $n\sigma vt>1$ where $n=\frac{1}{2}gn_\gamma$ is the density, $v\simeq c$, and t is the characteristic expansion time. This is why we expect $g_{g_{3/2}d}$ to be of the order of $g_{\text{ew}}\simeq 10^2$ as mentioned above. What about helicity-$(\pm\frac{3}{2})$ gravitinos? They interact only with gravitational strength and, like the primordial gravitons and (if they exist) right-handed neutrinos,[15] they presumably dropped out of equilibrium at much higher temperature. If the helicity-$(\pm\frac{3}{2})$ decoupling temperature $T_{3/2d}$ is greater than the GUT symmetry restoration temperature, the primordial helicity-$(\pm\frac{3}{2})$ gravitinos would be much less dense than the helicity-$(\pm\frac{1}{2})$ [in this case the factor $g=4$ in Eq. (6) should be replaced by 2, with small numerical changes in subsequent equations]. The fact that the helicity-$\frac{3}{2}$ and -$\frac{1}{2}$ components of the massive gravitino effectively couple differently to matter has been discussed in detail in Fayet's review.[6] If Guth's "inflationary universe" hypothesis[16] is right and $T_{3/2d}>T_{\text{GUT}}$ then the density of primordial helicity-$(\pm\frac{3}{2})$ gravitinos is infinitesimal.

In summary we find that if the idea of spontaneously broken local supersymmetry applies to the real world then (1) possibly the dominant component of the universe is a gas of helicity-$(\pm\frac{1}{2})$ gravitinos, and (2) cosmological bounds on the gravitino mass density imply that we must have R-odd states below about 10^5 TeV. An interesting question, unanswered here, is what dynamical mechanism sets the scale for F.

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