## Renormalization-Group Explanation of Nonuniversal Decay of Boundary-Spin Correlations

By allowing nearest-neighbor interactions which depend on the distance $z$ from the surface in a half-infinite triangular Ising model, Hilhorst and van Leeuwen[1] obtain nonuniversal decay of boundary-spin correlations. When deviations of the interactions from the critical bulk value decay as $A/z$ the boundary spin-spin correlation function $g(r)$ goes as $r^{-\eta}$ as $r \to \infty$ with an exponent $\eta$ which depends on $A$. In this Comment I show that this is an example of a very general type of behavior, expected on the basis of renormalization-group arguments introduced by Cordery and Griffin.[2]

Consider any lattice spin model with short-range interactions $\{J\}$. In the standard real-space renormalization-group scenario block spins are associated with $2^d$ site spins. The block-spin correlations are given by a set of renormalized short-range interactions:

$$\{J'\} \equiv R_2\{J\}. \qquad (1)$$

The renormalization-group transformation $R_2$ has a fixed point $\{J^*\}$. Linearizing around this fixed point there is, in ordinary critical phenomena, a single relevant even spin variable $\{j_1\}$ which for small values of $\lambda$ satisfies

$$R_2[\{J^*\}+\lambda\{j_1\}] \simeq \{J^*\} + 2^{1/\nu}\lambda\{j_1\}. \qquad (2)$$

Now consider a half-infinite version of this spin lattice model with an inhomogeneous Hamiltonian with interactions of the form

$$\{J^*\} + A\{z^{-y} j_1\}. \qquad (3)$$

The deviation of each local interaction from the bulk fixed point is proportional to $z^{-y}$, where $z$ is the average distance of spins in that term from the surface. Now apply a blocking $R_2$ to this system. For large enough $z$, using the fact that renormalized interactions only depend on nearby interactions, we obtain

$$R_2[\{J^*\}+A\{z^{-y}j_1\}] = \{J^*\} + 2^{1/\nu}A\{z^{-y}j_1\}. \qquad (4)$$

Lengths in the renormalized lattice are reduced by a factor of 2. Substituting $z' = z/2$ we see that the right-hand side of (4) has the same form as (3),

$$R_2[\{J^*\} + A\{z^{-y}j_1\}] = \{J^*\} + A'\{(z')^{-y}j_1\}, \qquad (5)$$

where $A' = 2^{1/\nu - y}A$. A Hamiltonian with interactions of the form (3) can be a fixed point only if $y = \nu^{-1}$. In that case we actually have a line of fixed points parametrized by $A$. In systems where there is a line of fixed points one expects critical exponents to be nonuniversal. In the special case of the two-dimensional Ising model this line of fixed points occurs for $y = 1$ exactly as found by Hilhorst and van Leeuwen.

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