in this case a larger degree of alignment towards H_{ext} can be achieved by the interaction of (larger) Fe cluster moments and H_{ext}.

Our results are in qualitative agreement with a recent theory[13] which predicts the coexistence of spin-glass and ferromagnetic ordering. This "mixed phase" is characterized by the coexistence of a spontaneous (longitudinal) magnetization (ferromagnetic order) and a spin-glass ordering of the transverse components of the spins. Our observations in an applied field suggest this behavior, i.e., ferromagnetic spin correlation for $T > T_f$, and preferentially ferromagnetic spin correlation combined with the development of a transverse spin component due to long-range antiferromagnetic interaction for $T < T_f$.

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[1] B. R. Coles, B. V. B. Sarkissian, and R. H. Taylor, Philos. Mag. B37, 489 (1978).

[2] B. H. Verbeek and J. A. Mydosh, J. Phys. F 8, L109 (1978).

[3] G. J. Nieuwenhuys, B. H. Verbeek, and J. A. Mydosh, J. Appl. Phys. 50, 1685 (1979).

[4] S. Crane and H. Claus, Phys. Rev. Lett. 46, 1693 (1981).

[5] P. A. Beck, Solid State Commun. 34, 581 (1980), and Phys. Rev. B 23, 2290 (1981).

[6] A. P. Murani, Solid State Commun. 34, 705 (1980).

[7] U. Gonser, R. W. Grant, C. J. Meechan, A. H. Muir, Jr., and H. Wiedersich, J. Appl. Phys. 36, 2124 (1965).

[8] B. Window, Phys. Rev. B 6, 2013 (1972).

[9] B. Window, J. Phys. E 4, 401 (1971).

[10] J. Lauer, W. Marschmann, and W. Keune, unpublished. An intensity increase of the "shoulder" in the left Mössbauer line with increasing Fe concentration can also be observed in the Mössbauer spectra at 4.2 K of Ref. 8. This shoulder is connected to the high-field shoulder in $P(H)$.

[11] See, e.g., G. K. Wertheim, *Mössbauer Effect: Principles and Applications* (Academic, New York, 1964).

[12] R. J. Borg, Phys. Rev. B 1, 349 (1970).

[13] M. Gabay and G. Toulouse, Phys. Rev. Lett. 47, 201 (1981).

Pinning of a Vortex Line to a Small Defect in Superconductors

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It is shown that quasiparticle scattering by a defect in a superconductor leads to large elementary pinning energies of flux lines. In the case of a small radius (R) point defect the new mechanism outweighs the traditional volume effect by the factor ξ_0/R.

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It is a long-standing problem in the context of critical current densities of type-II superconductors that the measured volume pinning forces turn out larger than predicted by theory.[1] Type-II superconductors are resistanceless only as long as the magnetic-flux vortex lattice which penetrates them remains stationary. Current tries to push the vortex lattice into motion and the critical current is determined by the pinning force which holds the lattice back. This force is a volume effect arising from a large number of elementary contributions from individual pinning centers. The vortex lattice deforms elastically to accommodate itself to the presence of pinning centers. The problem of determining the energy barriers between different equilibrium vortex configurations for a random array of pinning centers is called the statistical summation problem. This statistical theory needs the elementary pinning potentials and the elastic properties of vortices as input. A great deal of work on the statistical summation problem[1] has not been able to resolve the dilemma of the pinning forces appearing too large in experiments. It seems that the root of the trouble is in the elementary pinning force to which we address ourselves in this Letter.

Traditionally, it is assumed that voids or similar defects pin because they prohibit superconducting condensation in their locality. Hence a void attracts normal regions such as a vortex core in order to avoid the loss of condensation energy.[2-4] Elementary pinning energies from this source are on the order of the condensation energy density $\mu H_c^2/2$ times the volume of the pinning center $4\pi R^3/3$ and a more sophisticated calculation can only supply a multiplicative factor not much different from unity. The quasiparticle scattering effect considered in this Letter pins more strongly by one or two orders of magnitude. The physical basis of the new mechanism is a nonlocal effect that a scattering center has on Cooper pairs in its immediate environment. A scattering center helps a superconductor to sustain deformations of the order parameter up to distances on the order of the zero-temperature coherence length ξ_0. Hence, it is energetically advantageous for a region where the order parameter varies strongly, e.g., a vortex core, to coincide with a scattering center. In the case of small impurities (of size R^3), in particular, the new mechanism leads to elementary pinning energies proportional to $\mu H_c^2\xi_0 R^2$, larger by the factor ξ_0/R than the $\mu H_c^2 R^3$ from the volume effect.

The here suggested binding energy via quasiparticle scattering is an effect on the scale ξ_0 not contained in any conventional Ginzburg-Landau approach or other schemes based on gradient expansions. Its quantitative evaluation has become possible with the refined techniques of a recent new formulation of the BCS-Gorkov theory of superconductivity,[5] the quasiclassical method.[6,7] The quasiclassical method is equivalent to the WKB method of quantum mechanics or the ray-tracing approximation of optics. It exploits the short wavelength ($k_F^{-1} \simeq 1$ Å) of electrons at the Fermi surface compared with the characteristic length in a superconductor ($\xi_0 \simeq 10^2$–10^3 Å). The quasiclassical method is exact to leading order in $1/\xi_0 k_F$. Scattering from walls, impurities, etc., appears as boundary conditions [into which we count the source term in our Eq. (4)] in this theory. In this paper we use Eilenberger's formulation of the quasiclassical method in terms of ξ-integrated Green's functions, which formulation appears to be the most comprehensive and most efficient in practical calculations.

We apply the quasiclassical theory of a small defect (impurity, void, dislocation ring, etc.) in a superconductor.[8] For a small defect we only need the leading terms in the quantity σ/ξ_0^2, the cross section of the impurity divided by the square of the coherence length. There are no temperature restrictions on the validity of the theory. The numerical work presented in this Letter for illustration purposes is limited to defects whose scattering of electrons is primarily of s-wave character. More quantitative computations and evaluation of the full pinning potential as a function of the distance between the vortex and the defect are deferred to a later publication. A preliminary comparison of our qualitative data with experiments[9] suggests that the agreement of theory and the measured pinning force improves considerably. Even the new pinning force, however, does not meet the threshold criterion, which has been criticized recently anyway.[10,11]

It turns out that a major part of the problem can be lifted out of the literature. Pesch and Kramer[12] have worked out the self-consistent solution of the Eilenberger equation for the vortex without the defect:

$$[\{i\epsilon_n + v_F e\hat{k}\cdot\vec{A}(\vec{R})\}\hat{\tau}_3 - \hat{\Delta}(R), \hat{g}_{imt}(\hat{k},\vec{R};\epsilon_n)] + i\hbar v_F\hat{k}\cdot\nabla_R\hat{g}_{imt}(\hat{k},\vec{R};\epsilon_n) = 0, \tag{1a}$$

$$\hat{g}_{imt}(\hat{k},\vec{R};\epsilon_n)^2 = -(\pi\hbar)^2. \tag{1b}$$

The propagator $\hat{g}_{imt}(\hat{k},\vec{R};\epsilon_n)$ and the order parameter $\hat{\Delta}(\vec{R})$ are standard 2×2 Nambu matrices. The order parameter is here a function of the position $\vec{R}$; $\hat{g}_{imt}$ depends on the Matsubara frequencies $\epsilon_n = (2n+1)\pi kT$ and on a momentum direction unit vector $\hat{k}$ as well as on $\vec{R}$. We then proceed in three steps. First $\hat{g}_{imt}$ is used to calculate the t matrix of quasiparticle scattering off the defect. The next step is solving the Eilenberger equation now with a source term (inhomogeneity) representing the scattering via a t matrix. This delivers the Green's function $\hat{g}$ in the presence of the pinning defect. Finally the difference in energy with and without the defect can be calculated given $\hat{g}$ and $\hat{g}_{imt}$.

The quasiclassical t-matrix equation[8]

$$\hat{t}(\hat{k},\hat{k}';\epsilon_n)=\hat{v}(\hat{k},\hat{k}')+\frac{N(0)}{\hbar}\int\frac{d\Omega_k''}{4\pi}\hat{v}(\hat{k},\hat{k}'')\hat{g}_{\rm imt}(\hat{k}'',\vec{R}=0;\epsilon_n)\hat{t}(\hat{k}'',\hat{k}';\epsilon_n) \tag{2}$$

involves $\hat{g}_{\rm imt}$ only at the center of the defect, $\vec{R}=0$. $\hat{v}(\hat{k},\hat{k}')$ is the defect scattering potential conveniently expressed in terms of its phase shifts δ_l which are input parameters of the theory. The partial-wave analysis is in fact the most efficient method of coping with Eq. (2). For instance, including only s-wave scattering,

$$\hat{v}(\hat{k},\hat{k}')=-[\pi N(0)]^{-1}\tan\delta_0, \tag{3}$$

makes Eq. (2) trivial [$N(0)$ is the quasiparticle density of states]. The inhomogeneous Eilenberger equation reads[8]

$$[\{i\epsilon_n+v_{\rm F}e\hat{k}\cdot\hat{A}(\vec{R})\}\hat{\tau}_3-\hat{\Delta}(\vec{R}),\ \hat{g}(\hat{k},\vec{R};\epsilon_n)]+i\hbar v_{\rm F}\hat{k}\cdot\nabla_R\hat{g}(\hat{k},\vec{R};\epsilon_n)$$
$$=[\hat{t}(\hat{k},\hat{k};\epsilon_n),\ \hat{g}_{\rm imt}(\hat{k},\vec{R}=0;\epsilon_n)]\,\delta(\vec{R}). \tag{4}$$

Far from the defect, the full propagator $\hat{g}$ approaches $\hat{g}_{\rm imt}$. The difference in free energy with and without the defect is given by[6]

$$\delta\Omega(\vec{r})=\int_0^1 d\lambda\,\frac{kT}{\hbar}\sum_n N(0)\int\frac{d\Omega_k}{4\pi}\int d^3R\,{\rm Tr}_2[\delta\hat{g}(\hat{k},\vec{R};\epsilon_n;\lambda)\hat{\Delta}_b(\vec{R})], \tag{5}$$

where $\delta\hat{g}(\hat{k},\vec{R},\epsilon_n;\lambda)$ is the difference $\hat{g}-\hat{g}_{\rm imt}$ calculated for the order parameter $\hat{\Delta}(\vec{R})=\lambda\hat{\Delta}_b(\vec{R})$, $\hat{\Delta}_b(\vec{R})$ being the actual order parameter of the superconductor without the defect. The result of the calculation, $\delta\Omega(\vec{r})$, is the elementary pinning potential of the vortex line in the vicinity of the defect. $\delta\Omega(\vec{r})$ is a function of the distance $\vec{r}$ of the defect from the center of the vortex line [the right-hand side of Eq. (5) depends on $\vec{r}$ through $\delta\hat{g}$]. At large $\vec{r}$ the t matrix commutes with $\hat{g}_{\rm imt}$. The inhomogeneity in Eq. (4) then disappears and the pinning potential vanishes at large $\vec{r}$ in agreement with Anderson's theorem[13] according to which a nonmagnetic defect in a homogeneous superconductor does not influence the condensation energy. Anderson's theorem, however, does not apply where the defect lies in a region of appreciable bending of the order parameter. Then the t matrix brings about substantial changes in $\hat{g}$ up to distances on the order of ξ_0 from the defect. The scattering in fact helps the order parameter adjust itself to the rapid changes required by the presence of the vortex. This leads to an increase in $|\Delta|$ as sketched in Fig. 1. The resulting gain in condensation energy is the source of the pinning mechanism discussed here.

Equations (1), (2), (4), and (5) can be solved on a computer with a modest effort. The constitutive equations of the semiclassical theory, here Eqs. (2) and (4), are ordinary linear differential equations along trajectories in the directions $\hat{k}$. With $\hat{\Delta}(\vec{R})$ obtainable from the literature[10] and a simple δ-function inhomogeneity in Eq. (1), we only have to find new solutions along trajectories that run through the defect. This is easily done with standard numerical routines.

With the impurity right at the core of the vortex, $\vec{r}=0$, the equations are particularly simple as the cylindrical symmetry limits the different trajectories to just those with different polar angles with respect to the vortex direction. In Fig. 2 numerical $\delta\Omega(\vec{r}=0)$ are displayed for the vortex structures of Ref. 12 in the weak scattering limit ($\delta_0\ll 1$) and in the unitary limit ($\delta_0=\pi/2$).

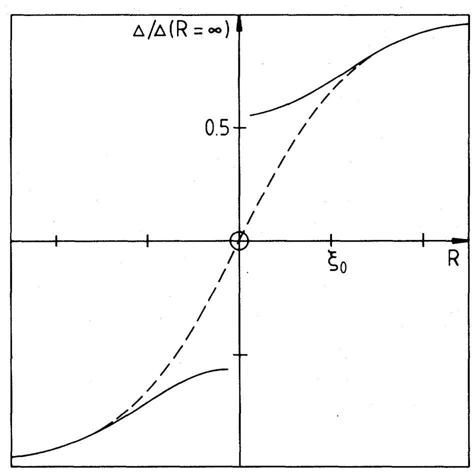

FIG. 1. Enhancement of the order parameter (grossly exaggerated) in the vicinity of a vortex with an impurity at its core. The dashed line represents the order parameter without the impurity.

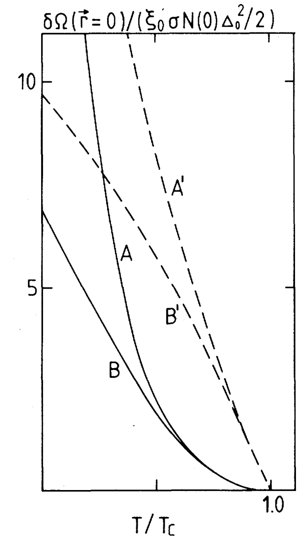

FIG. 2. Normalized binding energy to a small impurity (curve *A*) in the Born approximation and (curve *B*) in the unitary limit $\delta_0 = \pi/2$. The dashed lines A' and B' are the same in the zero-core-diameter model. The symbol σ denotes the scattering cross section of the impurity and ξ_0 is defined as $\hbar v_F/\pi\Delta_0$.

The results of a simple model calculation, a vortex with a vanishingly small core radius, are included. This model admits the analytic solution

$$\delta\Omega(\vec{r}=0) = -2kT \ln\left[\cosh\left(\frac{\Delta_b \sin\delta_0}{2kT}\right)\right]. \qquad (6)$$

One should notice that the Born approximation breaks down at low T and leads to an unphysical divergence of $\delta\Omega$ however weak the scattering.

In conclusion, we have shown that there is an elementary pinning mechanism of vortices to defects through quasiparticle scattering in addition to the classical volume effect. For defects small on the scale ξ_0, the new effect is stronger by the factor coherence length divided by the size of the defect as compared with the volume effect. In the case of a pinning center of 5 Å in a typical superconductor the factor can easily be as large as 100. The pinning potential can be calculated quantitatively with a reasonable numerical effort given the scattering data (phase shifts) of conduction electrons off the defect. For the interpretation of experimental results we need both the elementary pinning potential and the superposition theory of individual pinning forces into a bulk effect (summation theory). Both have been plagued by uncontrollable uncertainties. We cannot really trust the measured elementary pinning energies at present because of the uncertainties in the summation theory. It is fortunate that we have a perfect theory of superconductivity, the BCS-Gorkov theory, within which the elementary force can be calculated to lay a firm foundation on which statistical summation theories can be built, theories which will be applicable in other contexts as well such as pinning in ferromagnets, charge-density wave systems, etc. Indeed, the efficiency of the computer programs developed with the aim of evaluating the full pinning potential of a single vortex to an impurity holds the promise that the simulation of a vortex lattice with realistic pinning centers is within the range of modern computers.

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[1]R. Labusch, discussion contribution in E. J. Kramer, J. Nucl. Mater. 72, 5 (1978).

[2]A. M. Campbell and J. E. Evetts, Adv. Phys. 21, 199 (1972).

[3]H. Ullmaier, Springer Tracts Mod. Phys. 76, 1 (1975).

[4]For a more recent effort along these lines, see e.g., G. Zerweck, J. Low. Temp. Phys. 42, 1 (1981).

[5]G. Rickayzen, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), p. 51.

[6]G. Eilenberger, Z. Phys. 214, 195 (1968).

[7]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 55, 2262 (1968) [Sov. Phys. JETP 28, 1200 (1969)].

[8]E. V. Thuneberg, J. Kurkijärvi, and D. Rainer, J. Phys. C 14, 5615 (1981).

[9]E. J. Kramer, J. Appl. Phys. 49, 742 (1978).

[10]A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. 34, 409 (1979).

[11]H. R. Kerchner, Physica (Utrecht) 107B, 463 (1981).

[12]W. Pesch and L. Kramer, J. Low Temp. Phys. 15, 367 (1974).

[13]P. W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).