Does Gravitation Resolve the Ambiguity among Supersymmetric Vacua?

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Globally supersymmetric theories often have several degenerate supersymmetric vacua. Gravitation splits this degeneracy in such a way that at most one of these vacuum solutions has energy density and cosmological constant equal to zero, while all the rest have negative energy density. Nevertheless, the vacuum with vanishing energy density is stable against decay into the others.

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It is common in supersymmetric theories to find several degenerate vacuum states in which supersymmetry is unbroken and gauge or global symmetries are broken in different ways. For instance, in a supersymmetric SU(N) gauge theory with a single left chiral superfield in the adjoint representation, there are various degenerate vacua in which supersymmetry is unbroken and SU(N) is broken down to SU(M)$\otimes$SU($N-M$)$\otimes$U(1) or to SU($N-1$)$\otimes$U(1) or not at all. Of course, in the real world supersymmetry *is* broken, but the vacuum ambiguity is nevertheless important for superunified theories in which the scale $\sqrt{F}$ of supersymmetry breaking is much less than the scale M at which the grand gauge group is broken. These ambiguities are not removed by higher-order corrections to the vacuum energy.[1]

One may hope that these ambiguities would be resolved when globally supersymmetric theories are coupled to gravitation. (For theories with scalar field expectation values of order $M \approx 10^{15}$–10^{17} GeV, the gravitational terms in the vacuum energy will be of order GM^6, which is much greater than the energy $F^2/2$ associated with supersymmetry breaking if $\sqrt{F} \ll 10^{13}$ GeV.) It will be shown here that this hope is only partly fulfilled—the different supersymmetric vacua are no longer degenerate, and only one is likely to have vanishing cosmological constant, but most or all of them are stable.

Before we consider the effects of gravitation, it will be useful to recall the reason why there tend to be several degenerate vacua in globally supersymmetric theories. The potential in such theories has the general form[2]

$$V = J_{ab}^{-1}(z,z^*)F_a(z)F_b(z)^* + \tfrac{1}{2}\textstyle\sum_A |D_A(z,z^*)|^2, \tag{1}$$

where

$$J_{ab}(z,z^*) = \partial^2 d(z,z^*)/\partial z^a\, \partial z^{b*}, \tag{2}$$

$$F_a(z) = \partial f(z)/\partial z^a, \tag{3}$$

$$D_A(z,z^*) = [\partial\, d(z,z^*)/\partial z^a](t_A)_{ab} z^b. \tag{4}$$

Here z^a are the complex scalar components of left chiral superfields S^a; t_A is the representation of the Ath gauge generator on these scalars, including a coupling-constant factor; and $f(S)$ and $d(S,S^*)$ are the arbitrary functions whose $\theta_L\theta_L$ and $\theta_L\theta_L\theta_R\theta_R$ terms (F and D terms) appear in the Lagrangian. For renormalizable theories the superpotential $f(z)$ is a cubic polynomial and $d(z,z^*) = |z|^2$; a general d function is included here because renormalizability will not be maintained when we include the effects of gravitation. (With such a general d function, the kinetic term for the scalars is $-J_{ab}\partial_\mu z^a \partial^\mu z^{b*}$.) The gauge group is assumed here to be semisimple, so no Fayet-Iliopoulos terms[3] appear in D_A. A supersymmetric solution is one for which all F_a and all D_A vanish.

Any gauge group G will generally have several "big" subgroups H_n, with the property that if G is spontaneously broken to H_n, then all D_A vanish solely as a consequence of the remaining symmetries in H_n. Inspection of Eq. (4) shows that all D_A vanish if there are no broken generators of G that are neutral under H_n. For example, this is the case if G is SU(N) and H_n is either SU(N) itself, or SU($N-1$)$\otimes$U(1), or SU(M)$\otimes$SU($N-M$)$\otimes$U(1), or SU(L)$\otimes$SU(M)$\otimes$SU($N-M-L$)$\otimes$U(1)$\otimes$U(1), etc., irrespective of the representations of SU(N) provided by the chiral scalar superfields of the theory. Suppose we constrain the scalar field expectation values to be invariant under any one of such big subgroups H_n. The conditions for a vacuum with unbroken supersymmetry are then just $F_a = 0$. Now, the constraint

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of invariance under H_n reduces the number of free complex parameters z^a available to satisfy these conditions, but since $f(z)$ is invariant under $G \supset H_n$, this constraint reduces the number of independent F_a that need to be made to vanish, and by precisely the same amount. With equal numbers of equations and complex variables, we expect at least one supersymmetric solution for each big subgroup H_n. This could only be avoided if $f(z)$ is subject to special constraints, which make the F_a dependent on only a limited set of combinations of the z^a, as in the O'Raifeartaigh model.[4] In the absence of such constraints, there will be at least one supersymmetric vacuum solution for each big subgroup H_n. It can happen that the solution which is invariant under a given one of the big subgroups is also invariant under a larger subgroup, and that only a few of the largest subgroups are realized as distinct vacuum symmetries. Even so, some ambiguity will arise if there are any symmetry-breaking solutions at all.

Now let us look at the effects of gravitation. The mysterious physics of the Planck scale induces small nonrenormalizable interactions in the effective Lagrangian that describes physics at lower energies. However, the above discussion shows that such terms cannot resolve the ambiguity between different supersymmetric vacua. For instance, there may be terms in $f(z)$ higher than cubic, and terms in $d(z,z^*)$ beyond $|z|^2$, all suppressed by factors of Newton's constant G; even so, the vacuum energy is still of the form (1), and we still expect solutions of the equations $F_a = D_A = 0$ invariant under each big subgroup.

But gravitation enters in the effective Lagrangian that describes physics at sub-Planck energies not only through the appearance of suppressed nonrenormalizable terms in f and d, but also through the appearance of light particles: the graviton and gravitino. Supersymmetry then requires that the graviton and gravitino terms in the Lagrangian be accompanied by other terms involving scalar fields. These change the form of the scalar potential to[5]

$$V = \exp(8\pi G d)[J_{ab}{}^{-1} F_a F_b{}^* - 24\pi G|f|^2] + \tfrac{1}{2}\textstyle\sum_A |D_A|^2 \qquad (5)$$

with J_{ab} and D_A the same as before, but now

$$F_a = \partial f/\partial z^a + 8\pi G f\, \partial d/\partial z^a. \qquad (6)$$

For each stationary point of the original potential (1), we expect (provided $G|z|^2 \ll 1$) a nearby stationary point of the corrected potential (5). Those stationary points that correspond to the supersymmetric vacuum solutions discussed earlier are quite easy to find. Again, constrain the scalar field values to be invariant under any one of the "big" subgroups H_n of the gauge group, so that all D_A vanish. In the near neighborhood of the solution of the equations $\partial f/\partial z^a = 0$, one expects to find a solution of the equations $F_a = 0$, with F_a now given by (6). It is easy to see that these solutions are stationary points of the corrected potential (5), the variation in $|f|^2$ being compensated by the variation in d. At any such stationary point $z^a = z_n{}^a$, the potential takes the value

$$V_n = -24\pi G|f(z_n)|^2 \exp[8\pi G d(z_n, z_n{}^*)]. \qquad (7)$$

To lowest order in $G|z_n|^2$, we can drop the exponential and any nonrenormalizable terms in $f(z)$, and take z_n as the solution of the original equations $\partial f(z)/\partial z^a = 0$ invariant under H_n, so that

$$V_n = -24\pi G|f(z_n)|_{G=0}{}^2 + O(G^2). \qquad (8)$$

The value of $f(z)$ is in general different for each supersymmetric vacuum solution z_n, so the degeneracy among the different solutions is split. For instance, for a renormalizable SU(N) gauge theory with a single chiral scalar superfield S in the adjoint representation,[6] the function $f(z)$ takes the form

$$[f(z)]_{G=0} = c + b\,\mathrm{Tr}z^2 + a\,\mathrm{Tr}z^3$$

with z now a traceless complex $N \times N$ matrix. A straightforward calculation shows that for SU(N) broken to the "big" subgroups SU(M)$\otimes$SU($N-M$)$\otimes$U(1) [including the cases $M=1$ and $M=0$, with $H_1 = \mathrm{SU}(N-1)\otimes\mathrm{U}(1)$ and $H_0 = \mathrm{SU}(N)$] there are supersymmetric vacuum solutions z_M with

$$[f(z_M)]_{G=0} = c + \frac{4b^3 MN(N-M)}{27a^2(N-2M)^2}.$$

We must take $b \neq 0$ [otherwise SU(N) could not be spontaneously broken] so these solutions all give different vacuum energies.

On observational grounds, the physical vacuum energy must be very close to zero. It is always possible to adjust an additive constant in $f(S)$ (such as c in the above example) to make $f(z)$ and the vacuum energy vanish for any *one* of the vacuum solutions. However, the vacuum energy will then be negative definite for all of the other solutions. We are left with the disturbing conclusion that in typical supersymmetric theories, our nearly flat and empty vacuum is not the state of lowest energy.

However, this does not mean that our vacuum is unstable. The difference in energy densities between the supersymmetric vacua are relatively small, of first order in G, which according to the work of Coleman and de Luccia[7] makes it possible that gravitation might stabilize the zero-energy vacuum.[8] In fact, this must be the case. General theorems[9] tell us that in supergravity, under the constraint of flat space and zero energy density at large distances, all states have positive energy. Thus although the energy would be lowered if the scalar field expectation value were *everywhere* at one of the zeros of $F_a(z)$ with $f(z)\neq 0$, there is no way to gain energy by making a transition to such a lower energy state through perturbations of finite size.

This can be seen in detail by considering the formation of a thin-walled spherical bubble containing space with negative energy density $-\epsilon$ in a flat-space background with zero energy density. Reference 7 shows that the appearance of such a bubble is impossible if $\epsilon \leq \epsilon_c$, where ϵ_c is a critical energy density

$$\epsilon_c = 6\pi G S_1^2 \tag{9}$$

with S_1 the bubble surface tension. One way to understand this is by considering the total energy of the bubble and its gravitational field; a straightforward calculation gives[10]

$$E = -(4\pi/3)\epsilon R^3 + 4\pi R^2 S_1[1+8\pi\epsilon G R^2/3]^{1/2} - 8\pi^2 G S_1^2 R^3, \tag{10}$$

where R is the bubble radius, limited by the condition that

$$8\pi G R^2(\epsilon_c - \epsilon)/3 \leq 1. \tag{11}$$

(This condition is needed to avoid a coordinate singularity in the bubble wall, which would make the whole space two sheeted.) For $\epsilon \leq \epsilon_c$, E remains positive definite for all nonzero R satisfying (11), so formation of such a bubble is energetically impossible. On the other hand, for $\epsilon > \epsilon_c$ Eq. (11) is satisfied for all R, and for R sufficiently large E vanishes and then becomes negative; hence bubble formation is here energetically possible, and once formed, the bubble will release energy by continual expansion.

Now we must evaluate the critical energy (9) for supergravity. The general results of Ref. 7 give in our case

$$S_1 = \int ds \sum_a \left[\left|\frac{dz^a}{ds}\right|^2 + \left|\frac{\partial f(z)}{\partial z^a}\right|^2 \right], \tag{12}$$

the integral over proper radial distance s being taken from deep inside the bubble to far outside it, with $z^a(s)$ the function that minimizes (12) subject to the boundary conditions that $z^a(s)$ approaches the stationary points z_1^a and z_0^a of $f(z)$ deep inside and far outside, respectively, at which $f(z_0)=0$ and $f(z_1)\neq 0$. Equation (12) can be rewritten as

$$S_1 = \int ds \sum_a \left|\frac{dz^a}{ds} + \xi\left(\frac{\partial f(z)}{\partial z^a}\right)^*\right|^2 + 2|f(z_1)| \tag{13}$$

with ξ the phase factor $\xi \equiv f(z_1)/|f(z_1)|$. This yields a lower bound of $2|f(z_1)|$ for S_1, which together with Eq. (8) sets an upper bound on ϵ just equal to ϵ_c:

$$\epsilon = 24\pi G|f(z_1)|^2 \leq 6\pi G S_1^2 = \epsilon_c. \tag{14}$$

The inequality in (14) becomes an equality if and only if there exists a solution of the differential equation

$$dz^a/ds = -\xi[\partial f(z)/\partial z^a]^* \tag{15}$$

such that $z\to z_0$ outside the bubble and $z \to z_1$ inside it. If there is no such solution, then $\epsilon<\epsilon_c$, so bubbles of negative energy density are energetically forbidden! On the other hand, in many cases [e.g., if $f(z)$ is a cubic polynomial in a single complex scalar] there actually is a solution of Eq. (15) with appropriate boundary conditions. In such cases $\epsilon=\epsilon_c$ to order G, and it is necessary to carry these calculations to higher order to check the stability of flat space. The general theorems of Ref. 9 indicate that here also $\epsilon \leq \epsilon_c$, and bubble formation is energetically impossible.

We conclude that a given supergravity theory will generally have at most one flat-space vacuum solution, and that this state will be stable against decay into the negative-energy vacua. The negative-energy vacua are also stable.[11] The question of which vacuum state in a given model will actually be occupied may perhaps be answered by following the history of the universe at early times, when the various vacua have substantially different thermal energies.[12] However, this raises a problem. At very high temperature thermal effects favor the most symmetric phase, and it has been suggested that the transition to broken symmetry occurs through the growth of the grand unified coupling constant with decreasing temperature, which might alter the coefficient of T^4 in the free energy density.[12] But at temperatures below about $(GM^6)^{1/4}$ (10^{13} to 10^{15} GeV for $M=10^{15}$ to 10^{17} GeV) these thermal energies are negligible compared with the gravitational splitting, which as we have seen must make our present flat-space broken-symmetry phase the one of *highest* energy density. Thus if supersymmetry is really broken only at energies below 10^{13} GeV, then it is difficult to see how a universe that starts in a grand-unification-symmetric phase can ever get out of it.[13]

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