Lett. 31A, 424 (1970).

 $^{15}$ K. van Hulst, C. J. M. Aarts, A. R. de Vroomen, and P. Wyder, J. Magn. Magn. Mater. 11, 317 (1979).  $^{16}$ Jos A. A. Perenboom, Physica (Utrecht) 107B, 589 (1981).

 $^{17}$ G. Barnea, J. Phys. F 7, 315 (1977).  $^{18}$ A. J. Arko and J. E. Schirber, J. Phys. (Paris).

Colloq.  $40, C4-9 (1979)$ .

 $^{19}$ P. Hertel, J. Appel, and D. Fay, Phys. Rev. B 22, 534 (1980).

## Evidence for Interaction Effects in the Low-Temperature Resistance Rise in Ultrathin Metallic Wires

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New measurements are reported of the low-temperature resistance rise in ultrathin wires of Cu, Ni, and AuPd, which confirm the proportionality to  $T^{-1/2}$  predicted by the interaction model. Moreover, these results and those in the literature show an absolute magnitude consistent within a factor of  $\sim 2$  with the predictions of this model, using independently determined parameters of similar accuracy. It is inferred that interaction effects are at least as important as localization effects in these systems.

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Recently there has been much theoretical<sup>1-5</sup> and experimental $^{6-9}$  work concerning the resistance rise at low temperatures observed in metallic samples of reduced dimensionality, attributed to either "localization" or "interaction" effects. In two-dimensional (2D) samples, the relative importance of the two mechanisms can be sorted out by application of a magnetic field. In the one-dimensional (1D) case of interest here, magnetic effects are smaller and less helpful; thus, one must put greater weight on the absolute magnitude of the effect and its dependence on material parameters and on temperature and sample size. On the other hand, the 1D regime has the advantage that the effect scales linearly with the characteristic length rather than only logarithmically as in the 2D case.

In this Letter, we report new experimental results on ultrathin wires of copper, nickel, and AuPd alloy, and also the results of a careful reanalysis of the data in the published literature. In all cases, the quantitative prediction of the interaction model of Altshuler  ${et\,al}_\bullet, ^4$  using independently determined parameter values, consistently accounts for much of the observed resistance rise, and in the case of Cu (our most reliable results), it accounts for essentially all of it. Accordingly, we infer that interaction effects are at least of comparable importance with, and may dominate over, localization effects in the metallic samples reported to date.

In either theory, so long as the resistance increase is small, it can be written as

$$
\delta R/R \approx \Lambda/L_q. \tag{1}
$$

The length  $\Lambda$  has a different meaning in the two models, while the length  $L_q = (A/\rho)(4\hbar/e^2)$  is the length of conductor having the characteristic quantum resistance  $4\hbar/e^2 \approx 16400 \Omega$ . In the freeelectron model,  $L<sub>g</sub>$  can be expressed in more microscopic terms as  $L_q \approx k_F^2 A l$ , so that it would be of the order of the mean free path  $l$  in the case of a "wire" made up of a single chain of metal atoms, but it is proportionally larger for wires of realistic cross sections.

In the localization model,<sup>1-3</sup> the length  $\Lambda$  is essentially the inelastic diffusion length<sup>10</sup>  $\Lambda_{R}$  $=(D\tau_E)^{1/2}$ , where D is the electronic diffusion coefficient and  $\tau_E$  is the inelastic scattering time.  ${\tau}_E$  is usually determined by the electron-phono coupling strength, which can vary markedly between metals, and  $\tau_E$  is normally expected to vary as  $T^{-p}$ , where  $p \approx 3$ . Alternatively, Abrahams  $et al.^5$  have recently argued that if electronelectron effects dominate the inelastic relaxation time,  $p$  should be  $\frac{1}{2}$  in 1D wires.

Although not derived in this way, the result of the interaction model of Altshuler  $et al.^4$  is equiva lent to taking  $\Lambda \approx \xi_N \approx (D\hbar/kT)^{1/2}$ , where  $\xi_N$  is the "normal-metal coherence length," a measure of the smallest wave packet that can be constructed from electron waves having energies within  $\nu kT$ of the Fermi surface. This model involves no poorly known, mechanism-dependent, or highly variable parameters, such as  $\tau_{\rm g}$ .

The ultrathin wires used in this experiment were fabricated by a step-edge shadowing tech-<br>nique.<sup>11</sup> In this technique, the thickness of a w nique. In this technique, the thickness of a wire is equal to the reactive ion etching (RIE) depth and its width is determined by the depth of the evaporated film and the shadowing angle, all readily controlled parameters. In addition, wires with different cross sectional areas can be made in the same evaporation by varying the RIE depth on a single wafer. For this work, we made wires of copper, nickel, and Aupd alloy with crosssectional areas as small as  $220 \times 200$   $\AA^2$  and length  $\sim$  520  $\mu$ m.

A11 of our wires were measured with a fourterminal ac  $(17 \text{ Hz})$  arrangement. A broad thin film evaporated at the same time as the wire array was also measured and served as a reference. In both cases, a small ac current  $(\leq 10^{-6} \text{ A})$  was applied to the sample and the ac voltage was measured using a lock-in amplifier. At these low currents, the resistance was found to be independent of the current to within the experimental er-



FIG. 1. Reconstruction of measured resistance variation in ultrathin Cu wires as sum of a rise  $(\sim T^3)$  characteristic of bulk material and a rise  $({\sim}T^{-1/2}A^{-1})$  characteristic of the one-dimensional interaction effect. The wide error bars at the higher temperatures reflect data taken under transient conditions during cooldown. The data points at low temperatures, taken under steady-state conditions, are much more accurate.

ror.

To facilitate a quantitative comparison with experimental data, we have manipulated the result of Altshuler  $et al.^4$  to the following form

$$
\frac{\delta R}{R} = \frac{2 - F}{\sqrt{6}} \left(\frac{\rho}{\hbar/e^2}\right)^{1/2} \left(\frac{k_{\rm B}}{\gamma_V * T}\right)^{1/2} \frac{1}{A} \tag{2}
$$

Here  $\gamma_V * T$  is the electronic specific heat per unit volume at temperature  $T$  after removal of the enhancement factor  $(1+\lambda)$  due to electron-phonon interaction (which is not effective in the transport coefficients<sup>12</sup>).  $\rho$  is the resistivity of the material in the wire,  $A$  is its cross-sectional area, and  $F(0 \leq F \leq 1)$  is a screening parameter expected  $t^2$  ( $0 \le t \le 1$ ) is a screening<br>to be  $\sim \frac{1}{2}$  for these metals

In testing the fit of (2) to experimental data, care is required to obtain quantitative results, for reasons made clear from Fig. 1. This shows a fit to the raw data on three sets of Cu wires as a sum of a constant residual resistance, plus a term  $\sim T^3$  to represent the ordinary resistance rise at higher temperatures in a bulk sample,



FIG. 2. (a) Data of Fig. 1, plotted vs  $1/\sqrt{T}$  to isolate the interaction effect. Only the low-temperature points, where the  $T^3$  term is negligible, are fitted. (b) Plot of slopes found in (a) vs  $1/A$  to separate the one-dimensional effect from any bulk effect also giving a resistance rise at low temperatures. The point at  $1/A = 0$  is an *upper bound* set by the experimental accuracy.



FIG. 3. Values of  $2-F$  required to fit resistance-rise data on ultrafine wires of various metals and resistivities to the interaction model, Eq.  $(2)$ . (*F* is expected to lie between <sup>0</sup> and 1.) Error bars are somewhat subjective, especially for materials for which directly measured values of  $\gamma$  are not known to us.

plus a term of the form (2), which is size dependent. From this reconstruction, it is clear that the minimum-resistance temperature and the magnitude of the resistance rise at low temperatures from that minimum depend strongly on the relative magnitudes of the two temperature-dependent terms. Moreover, the magnitude of the actual rise from the minimum is only a fraction of the total rise given by (2). To isolate the desired term, in Fig. 2 we plot the data vs  $1/\sqrt{T}$ , and take the slope in the low-temperature region

where the  $T^3$  term is negligible. Next, the slopes determined in this way are plotted against  $1/A$  to isolate the true 1D effect from any background effects, yielding an experimental determination of the coefficient of  $T^{-1/2}A^{-1}$  in (2). The agreement with the theoretically expected value was so impressive that we were emboldened to make similar analyses of our less complete data on Ni and AuPd wires, and to reexamine the data in the literature from the same perspective.

The results for our ultrathin wires of Cu, Ni, and AuPd are shown in Fig. 3, together with the results of our reanalysis of the data of Giordano and co-workers<sup>6,7</sup> on Pt and AuPd, and of the data of Chaudhari and co-workers<sup>3,9</sup> on WRe. To compare with theory, we have calculated the value of  $2-F$  required in (2) to reproduce the observed increase in resistance, using values of  $\gamma_{V}$ <sup>\*</sup> from the literature, insofar as they are known (see Table I), and values of  $\rho$  determined from the measured resistance and the known geometry. Because of the remark of Giordano' that the observed effect appeared to scale as  $\rho$  instead of the theoretically expected  $\rho^{1/2}$ , we have presented the data vs  $\rho$  in a logarithmic plot. In such a plot. the Giordano conjecture corresponds to a line with slope of  $\frac{1}{2}$ . Although such a slope fits the data well at the high values of  $\rho$  where Giordano's results lie, it is not consistent over the broader

TABLE I. Electronic specific-heat parameters. The parameters  $\gamma_{V}^{*}$  were obtained from experimental values of  $\gamma$  for the elements quoted by Kittel [C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1971), 4th ed., p. 254] by converting to unit volume  $(\gamma_v)$  and then dividing out the phonon enhancement factor  $(1 + \lambda)$ , as tabulated here. Values quoted in parentheses are estimated from values for other metals in the absence of knowledge of directly determined values.

Metal		$\gamma$ (mJ/mol K <sup>2</sup> ) $\gamma_V$ (ergs/cm <sup>3</sup> K <sup>2</sup> )		$\gamma_{\gamma}$ * (ergs/cm <sup>3</sup> K <sup>2</sup> )
Cu	0.695	980	$0.15 \pm 0.02^{\text{a}}$	852
Ni	7.02	7701	$(0.7 \pm 0.01)^{b}$	4530
P <sub>t</sub>	6.8	7462	$0.6 \pm 0.1^{\circ}$	4664
AuPd	$(0.73)^{d}$	714	$(0.16)$ <sup>e</sup>	616
WRe	$(1.9)$ <sup>f</sup>	2100	$(0.9 \pm 0.2)^8$	1106

<sup>a</sup>A. G. M. Jansen, A. P. van Gelder, and P. Wyder, J. Phys. C 13, 6073 (1980).

 $b$ Value from (c) for Pd, which is just below Ni in periodic table.

 ${}^{c}G.$  S. Knapp and R. W. Jones, Phys. Rev. B  $6.$  1761 (1972).

Value for Au, by analogy with the work on AgPd alloys by H. Montgomery,

G. P. Pells, and E. M. Wray, Proc. Boy. Soc. (London), Ser. <sup>A</sup> 301, 261 (1967). <sup>e</sup>Value for Au from (a); see (d).

 $f$  Weighted average of values for W and Re, which are adjacent in periodic table.

 $v^g$ Value from (c) for Ta, which is adjacent to W in periodic table.

range of  $\rho$  which our new results make available in Fig. 3. It may be significant that the apparent rise at high  $\rho$  values occurs where  $k_F l \rightarrow 1$  and the model of Altshuler  $et al.$  is expected to break down. We caution that in basing our analysis on (2), not only have we had to use estimated parameter values in some cases, but also (2) itself is based on a simplified spherical-Fermi-surface approximation. Still, we emphasize that the  $2-F$ value found for Cu (for which  $\delta R/R$  and  $\gamma_{\nu}$ <sup>\*</sup> have relatively small uncertainties) falls essentially where it would if the interaction effect were the only one present.

Chaudhari and co-workers' reached the opposite conclusion, namely that localization was the dominant mechanism in their samples. This inference was based on measurements of the differential resistance of phase-slip centers below the superconducting  $T_c$  of their WRe samples, which provide an estimate of the charge-imbalance relaxation length<sup>10</sup>  $\Lambda_{\Omega}$ . From this Chaudhari et al. inferred a value for the inelastic diffusion length  $\Lambda_{\kappa}$ , which was consistent with the value required to fit the resistance-increase data. Two remarks can be made: (1) Chaudhari  $et al.$  comment that their data in fact "agree quantitatively with the Coulomb interaction theory, " and so we agree on that point. (2) The agreement between the phaseslip data and the resistance-rise data noted by Chaudhari et al. could be explained equally as well by assuming that both are determined by a length related to  $\xi_N$  as by a length related to  $\Lambda_E$ . In fact, their published phase-slip data were taken at rather high voltages  $(0.3-2 \text{ mV})$ , where nonlinear effects of heating and charge imbalance could distort the data and also introduce a charge relaxation mechanism unrelated to the inelastic relaxation time  $\tau_E$  which enters in  $\Lambda_E$ . In any case, caution is in order in interpreting these observations.

In conclusion, the Coulomb interaction model is found to give an excellent fit to our best data (on Cu), and an acceptable fit (although typically too small by a factor  $\sim$  2) to the data obtained by three groups on the resistance rise in 1D samples of five metals with resistivities varying over more than two orders of magnitude and widely varying electronic densities of states. This suggests that the interaction model (or another model with a similar prediction) can account for much of the observed effect in these samples, while the localization model does not provide as natural an localization model does not provide as natura<br>explanation for the magnitude and  $T^{-1/2}$  depen dence of the data using current models' of inelastic scattering processes in 1D samples.

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 ${}^{1}D. J.$  Thouless, Phys. Rev. Lett. 39, 1167 (1977), and Solid State Commun. 34, 683 (1980).

 $E^2$ E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).

 ${}^{3}P.$  W. Anderson, D. J. Thouless, E. Abrahams, and D. S. Fisher, Phys. Rev. B 22, 3519 (1980).

<sup>4</sup>B.L. Altshuler, D. Khmel'nitzkii, A.I. Larkin, and P. A. Lee, Phys. Rev. B 22, 5142 (1980).

 ${}^5E$ . Abrahams, P. W. Anderson, P. A. Lee, and T. V. Ramakrishnan, Phys. Rev. B 24, 6783 (1981).

 ${}^6N$ . Giordano, W. Gilson, and D. E. Prober, Phys. Rev. Lett. 43, 725 (1979); N. Giordano, Phys. Bev. B

 $\frac{22}{7}$ , 5635 (1980).<br><sup>7</sup>N. Giordano, in *Physics in One Dimension*, edited by <sup>'</sup>N. Giordano, in *Physics in One Dimension*, edited by J. Bernasconi and T. Schneider (Springer-Verlag, New York, 1981), p. 310; J. T. Masden and N. Giordano, York, 1981), p. 310; J. T. Masden and Giordano, Physica (Utrecht) 107B, 3 (1981).

 ${}^{8}P$ . Chaudhari and H.-U. Habermeier, Solid State Commun. 34, 687 (1980), and Phys. Rev. Lett. 44, 40 (1980).

<sup>9</sup>P. Chaudhari, A. N. Broers, C. C. Chi, R. Laibowitz, E. Spiller, and J. Viggiano, Phys. Rev. Lett. 45, 930 (1980).

 $10$ W. J. Skocpol, M. R. Beasley, and M. Tinkham, J. Low Temp. Phys. 16, <sup>145</sup> (1974); W. J. Skocpol, A. M. Kadin, and M. Tinkham, J. Phys. Colloq. 39, C6-1421 (1978).

D. C. Flanders and. Alice E. White, J. Vac. Sci. Technol. 19, 892 (1981).

 ${}^{12}R$ . E. Prange and L. P. Kadanoff, Phys. Rev.  $134$ , A566 (1964).