Current Maintenance in Tokamaks by Use of Synchrotron Radiation

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A tokamak at fusion temperatures generates large amounts of synchrotron radiation. With proper configuration of the walls, this radiation can sustain the current. This is accomplished by a fish-scale wall that preferentially reflects radiation propagating in one direction while absorbing that going oppositely. The wall transfers momentum to the electrons through radiation pressure. A rough theoretical treatment shows that at high temperatures (30–50 keV) sufficient current for a steady-state tokamak may be driven.

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Tokamak operation in a steady state would undoubtedly improve its reactor prospects. Much theoretical effort and experimental work is being devoted to the possibility of driving the dc in a tokamak by using particle beams[1] and/or rf and microwave sources.[2] The conventional approach to tokamak current drive is an "active" one, viz., one injects *externally* generated rf and/or particle beams into the tokamak to introduce a relative streaming between electrons and ions. The same results may be obtained in a more convenient manner by using "passive" schemes. Here, one manipulates the natural loss processes from the plasma in such a way as to generate net plasma currents. Operationally, a passive scheme is more attractive because (a) it eliminates the inefficiencies associated with the external cycle for generating rf/particle beams, etc., and (b) it reduces the complexity of the current drive system.

One passive scheme that we investigate in some detail in this Letter is the one using synchrotron radiation. Through the mechanism of synchrotron radiation,[3] the plasma is generating large quantities of radiation in the 0.1- to 10^{-2}-cm wavelength range. This form of radiation becomes particularly important at high electron temperatures (30 to 50 keV) where it can be an important form of energy loss from the plasma. For such temperatures, a few percent of the fusion power can be lost as synchrotron radiation giving synchrotron powers in the tens of megawatts range for reactors of 10^9 W or more. These powers are comparable to those being considered for current drive. Since this radiation is not only strongly emitted by the plasma but is also strongly absorbed and because the walls of the tokamak can be made highly reflecting to it, this radiation forms a medium through which the plasma can interact with the walls of the reaction chamber. Because the walls and the plasma are not in thermal equilibrium, it is possible to use this interaction to drive a current in the plasma.

One method for driving current in the tokamak is to use an anisotropic wall like that shown in Fig. 1 to cause the synchrotron radiation to rotate preferentially around the device in one direction. The simplest form consists of a series of fins perpendicular to the wall which are reflecting on one side and absorbing on the other;

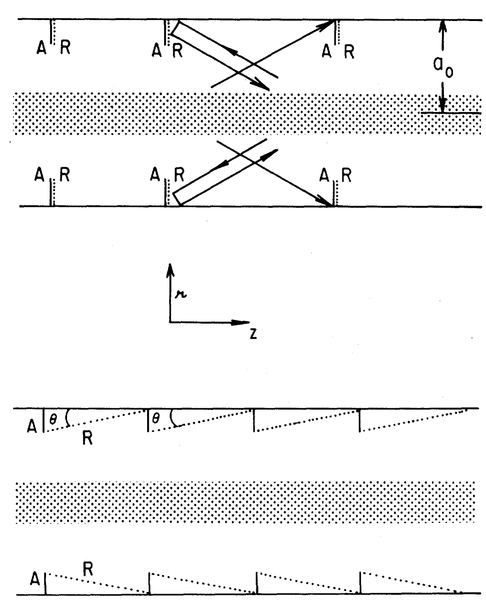

FIG. 1. (a) A simple configuration for achieving current drive by synchrotron radiation. (b) A more energy-efficient wall configuration.

this form is shown at the top of Fig. 1. A more efficient form is shown at the bottom; the wall has a sawtoothed or fish-scale structure with reflecting surfaces R and absorbing surfaces A. In the figure, radiation striking the reflecting surfaces gets deflected to the right while radiation that would reflect to the left off of the ends of the sawteeth is absorbed. There is a net radiation pressure pushing on the walls to the left and an equal and opposite push on the electrons to the right. As radiation repeatedly reflects from the walls, its direction of propagation rotates until finally it is propagating in a direction such that it strikes the absorbing ends of the sawteeth. As it does this, it repeatedly passes through the plasma and a fraction is absorbed. To optimize the momentum transferred to the electrons we choose the angle θ of the sawteeth so that on the average the radiation is reabsorbed by the plasma before being absorbed by the black edge of the sawteeth; i.e., we choose θ so that the average optical depth through the plasma of radiation at the frequency of the peak intensity of the synchrotron radiation is unity. The peak intensity of the synchrotron radiation depends on the reflectivity of the walls and occurs at approximately that frequency where the optical depth through the plasma is 1 for the number of reflections giving wall absorption. Since we want to minimize the plasma energy loss while maximizing the momentum transfer to the plasma, we should make these two lengths roughly equal or the angle θ should be approximately given by

$$\theta = \tfrac{1}{2}(1-R), \tag{1}$$

where R is the reflectivity of the reflecting surface.

We approximate the rate of momentum transfer to the walls by employing the radiation pressure; for the purpose of estimation, we approximate it as an isotropic pressure. The thrust dT_z on an element of area of the wall dA (containing many sawteeth sections) for small θ is

$$dT_z = \tfrac{1}{2}P_r\,\theta\,dA. \tag{2}$$

$P_r\theta dA$ is the pressure that would be exerted on the absorbing areas and the factor of $\frac{1}{2}$ comes from the fact that the absorbing surfaces feel a pressure of $\frac{1}{2}$ the value felt by reflecting surfaces. Now the radiation pressure is given in terms of the radiation energy density $W_{\rm EM}$ by

$$P_r = \tfrac{1}{3}W_{\rm EM} \tag{3}$$

for isotropic radiation. To estimate the radiation energy density we equate the radiation energy power absorbed by the walls to the synchrotron emission power of the electrons:

$$W_{\rm EM}/\tau_{\rm EM} = n_e T_e/\tau_{\rm syn}. \tag{4}$$

Here $\tau_{\rm EM}$ is the wall absorption time for the synchrotron radiation in the chamber and $\tau_{\rm syn}$ is the synchrotron cooling time of the electrons (including effects of reflection from the walls). From Eq. (4) we find for $W_{\rm EM}$

$$W_{\rm EM} = n_e T_e \tau_{\rm EM}/\tau_{\rm syn}. \tag{5}$$

The radiation absorption time for the walls is given by

$$\tau_{\rm EM} \simeq (2a/c)1/(1-R), \tag{6}$$

where a is the radius (minor radius) of the tokamak.

The thrust equation per unit axial distance along the chamber can now be written as

$$dT_z/dz = \tfrac{1}{6}W_{\rm EM}\,\theta\, 2\pi a. \tag{7}$$

Making use of Eqs. (5) and (6) gives

$$\frac{dT_z}{dz} = \frac{2\pi}{3}\frac{a^2 n_e T_e}{\tau_{\rm syn}c}\frac{\theta}{(1-R)} \approx \frac{\pi}{3}\frac{a^2 n_e T_e}{c\tau_{\rm syn}}, \tag{8}$$

where we have used Eq. (1) for θ to obtain the last approximation. Equating dT_z/dz to the rate of transfer of momentum from the electrons to the ions (through collisions) per length dz gives

$$\frac{\pi a^2 n_e m_e V_D}{\tau_{ei}} = \frac{dT_z}{dz} = \frac{\pi a^2}{3}\frac{n_e T_e}{c\tau_{\rm syn}}. \tag{9}$$

From this we obtain the steady-state current

$$I = -\pi a^2 e n_e V_D = -e\frac{\pi a^2}{3}n_e\left\{\frac{T_e}{m_e c}\frac{\tau_{ei}}{\tau_{\rm syn}}\right\}. \tag{10}$$

The term in braces has a simple physical interpretation. The mass of the energy radiated in the synchrotron cooling time, $\tau_{\rm syn}$, is T_e/c^2. If all this energy could be channeled to go in one direction, the momentum it would carry would be T_e/c, and hence, the recoil velocity of the electron would be $T_e/m_e c$. However, because the electrons continually loses momentum to the ions they only gain a drift velocity given by the radiation momentum loss in one electron-ion collision time; i.e., $\tau_{ei}/\tau_{\rm syn}$ of this drift velocity.

To proceed we need $\tau_{\rm syn}$ and τ_{ei}. We obtain $\tau_{\rm syn}$ from the work of Trubnikov.[3] He gives the following approximate formula:

$$\tau_{\rm syn} = \left(\frac{2.6\times 10^8}{B^2}\right) \times \left\{ \frac{1}{60}\left(\frac{mc^2}{T_e}\right)^{3/2} \left(\frac{a\omega_p^2}{c\omega_c(1-R)}\right)^{1/2} \right\}. \qquad (11)$$

The first term is the radiation damping time for an isolated electron, the term in the braces gives the effect of reabsorption on increasing the cooling time.

The electron-ion momentum-transfer time as obtained from Spitzer[4] is roughly given by

$$n_e \tau_{ei} = 6.6\times 10^4 T_e^{3/2}/\bar{Z}, \qquad (12)$$

where $\bar{Z}$ is the mean ionic charge on the plasma,

$$\bar{Z} = \sum_i M_i Z_i^2 / \sum_i M_i Z_i. \qquad (13)$$

Equation (12) is the value obtained from the conductivity of a Lorentz gas. Using Eqs. (11) and (12) for $\tau_{\rm syn}$ and τ_{ei} gives for the current

$$I = \alpha 9.4\times 10^{-7} a^{3/2} \left(\frac{T_e}{m_e c^2}\right)^{5/2} \times \frac{T_e^{3/2} B^{5/2} (1-R)^{1/2}}{\bar{Z} n_e^{1/2}}. \qquad (14)$$

The coefficient α comes about because the more energetic electrons radiate and absorb the synchrotron radiation most strongly and collide with the ions less frequently so that they carry more current. In other words, the particles carrying most of the current have an energy greater than T_e and so have a lower effective resistivity. Because the current is such a strong function of the energy, this is an important factor. We can roughly estimate the size of α by using the expression for the emissivity of a single electron (in vacuum) as a function of frequency, weigh it with the Boltzmann factor and compute the γ for those particles contributing most at the peak harmonic for synchrotron radiation. The emissivity is[5]

$$\epsilon(\omega) \propto \frac{1}{2}\left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma\omega/\omega_{ce}} \exp\left(\frac{-\gamma m_e c^2}{KT_e}\right). \qquad (15)$$

Computing the γ which maximizes this function for $\omega/\omega_{ce} \approx 15$ (roughly the harmonic of maximum intensity at 50 keV) gives an energy, E, of between 250 and 300 keV or 5–6 times the thermal energy. The collision rate (proportional to $E^{3/2}$) for such electrons is 10–15 times slower than for thermal electrons. However, these high-energy electrons also collide with thermal electrons which in turn transfer their momentum rapidly to the ions reducing the conductivity by about 2 so that our estimate of the value of α is between 5 and 10. A detailed and involved calculation is required to give a more accurate answer.

As an example, consider a plasma with the following conditions: $n_e = 10^{14}$ cm^{-3}, $T_e = 5\times 10^4$ eV, $B = 10^5$ G, $a = 150$ cm, $R = 0.9$. Equation (14) predicts a current of

$$I = 6.0\times 10^6 \alpha/\bar{Z}, \qquad (16)$$

which with an α of 5 and a $\bar{Z}$ of 2 gives 15×10^6 A.

We now present a brief discussion of the results. Since the current is a very strong function of T_e, T_e^4, synchrotron current drive works best at high temperatures. It also works best for high magnetic fields. These are the conditions that would be found in a deuterium or D-^{3}He burning tokamak and so this method of current drive should work best there.

The current as calculated is simply a momentum-balance calculation. If tokamaks generate the so-called bootstrap current,[6] then this current may be amplified and more margin would be provided. There is some controversy about whether or not the bootstrap current exists[7] and the issue seems to be unsettled at present. Of course, one should also take into account other neoclassical effects on the current[8]; magnetic particle trapping of particles with high perpendicular energies will reduce the magnitude of the effect. Let us estimate what we are paying for this current drive scheme. For sawtooth angle $\theta \simeq \frac{1}{2}(1-R)$, wall absorption of synchrotron radiation roughly doubles. Thus, conditions on reactor wall reflectivity will be more stringent than usual. Furthermore, defining $j = J/n_0 e v_{\rm th}$ and $p = P/n_0 T_e \nu_{ei,\,\rm th}$ and using (10), one finds approximately for efficiency of current drive

$$\frac{j}{p} \simeq \frac{v_{\rm th}}{c} \frac{\tau_{ei}(\text{energetic})}{\tau_{ei}(\text{th})} \approx \frac{c^2}{v_{\rm th}^2}. \qquad (17)$$

Thus, the ratio of current to power generated is comparable to that in lower hybrid and other rf schemes. One can clearly do better in the efficiency factor, if radiation absorbed through the sawtooth angle at the wall could be reemitted in the appropriate direction (to the right in Fig. 1) at a lower frequency and then sent back into the plasma. In this case, the wall acts as a Maxwell demon which soaks up momentum of electrons going one way via the intermediary of synchrotron radiation with hardly any consumption of energy.

The problem of controlling the current is one which will certainly be of importance. While we have not investigated this in detail, it appears that control could be achieved by ports in the wall that could be opened or closed to control the level of synchrotron radiation in the device.

Finally, we should like to point out that this form of current drive might be used with other devices such as stellarators. Here large toroidal currents are not required; in fact, one of the major arguments for stellarators is that they require no toroidal current and hence could be made steady state. However, here we see the possibility of the plasma maintaining a steady current and this would allow these devices to carry current. One possibility that this would provide is that of balancing the book forces through the use of a vertical B and the $\vec{j} \times \vec{B}$ force; this in turn could eliminate the secondary currents and minimize the Pfirsch-Schluter[8] diffusion. Such current could be used to optimize confinement or other aspects of the device as, for example, the engineering design. It gives one more parameter for the designer to work with. Such a current need not be large (say, a few hundred kiloamperes), which would be much easier to sustain.

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