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Atomic Cascade of Muonic and Pionic Helium Atoms

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The cascade of muonic and pionic helium atoms in targets of arbitrary density is investigated. The calculation does not use any free parameters except for strong-interaction effects. All measured x-ray intensities are reproduced, in particular the K_β/K_α intensity ratios in pionic helium.

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Energies and intensities of x rays emitted in the cascade of muonic and pionic helium atoms were determined in several experiments[1-5] but as yet there appears to be no adequate quantitative understanding of the cascade process. In contrast to heavy exotic atoms the exotic helium ion $(\text{He-}M)^+$ has no electron cloud of its own and therefore experiences, in collisions with neighboring helium atoms, strong and fluctuating electric fields and electron densities inducing Stark and Auger transitions, respectively.[6] The situation is also different from that of the mesonic hydrogen problem,[7,8] where the exotic atom forms a neutral system. Earlier cascade calculations on exotic helium atoms[3,9] were based on a phenomenological picture using free parameters for Stark mixing and Auger rates, which were separately determined for each system and for different densities. The purpose of this Letter is to describe a calculation of the cascade of exotic helium atoms *without any free parameter* for these effects.

The atomic cascade in helium starts after the capture of the free negative particle M with mass m which proceeds generally via the Auger effect. Then the meson is bound in an atomic orbit around the alpha particle. It is assumed that it is captured into orbits with principal quantum numbers $n_c = (m_M/m_e)^{1/2}$; n_c is about 14 for muons and 16 for pions. The second electron is ejected during the first deexcitation step via the internal Auger effect unless Auger transitions are suppressed in almost circular atomic states. Recombination of the protonlike ion $(\text{He-}M)^+$ with electrons of colliding helium atoms is prohibited because of energy conservation.

Deexcitation of the $(\text{He-}M)^+$ ion can occur by radiation or by the external Auger effect. For high n and for experimentally accessible helium densities the deexcitation proceeds exclusively via the external Auger effect with a rate increasing linearly with density. For levels below $n = 4$–7, depending on the system and the density, the rate for radiative transitions becomes comparable to or larger than the external Auger rate. Another important process during the cascade is nuclear capture. This occurs predominantly from s and p states, and is important even for states of nominally large n and l because strong electric fields (10^8–10^9 V/cm) occurring during

collisions cause Stark mixing between states of different l.

To eliminate the need for free parameters the phenomenological picture of Auger process and the Stark mixing during collisions had to be replaced by quantitative calculations. The collision process was investigated under the assumption that the $(\mathrm{He}\text{-}M)^+$-He complex can be described by the known electronic wave functions of the H^+-He system.[10] The wave functions yield the potential energy, the electric field, and the occupation number for the electronic K orbital of the exotic ion[11] and thus the electron density experienced by the exotic ion in a distance R from a helium atom.

The dynamics of a collision was studied numerically by Monte Carlo–type simulations of the motion of the ion through the target. The classical trajectories of N helium atoms ($N=10-30$) interacting with each other and with the exotic ion by van der Waals forces inside a volume of constant matter density were calculated using the known He-He potential[12] and as approximation the He-H^+ potential[10] for the helium-ion interaction. Since the Born-Oppenheimer approximation was applicable, the static values for the electric field $E(r)$ and the electron density $\rho(r)$ could be used. 200 simulations were performed with different initial kinetic energies of the ion (0.1–10 eV), with various helium densities ($10^{-2}d_0$–d_0; d_0 is the density of liquid helium), and with random initial distributions of all particles. The maximum initial kinetic energy of 10 eV is due to the recoil after an external Auger effect: The electron moves rapidly away within a time $<10^{-17}$ sec and the exotic ion finds itself in the intense electric field of the residual He^+ ion in approximately 1 Å distance, beginning to recoil.

The program gave information about the relative distance of the ions to all other particles, their kinetic and potential energies, the time dependence of the electric field, and the electron density experienced by the ions and their integrals with respect to time. The accuracy of the solution was checked by observing that the sum of potential and kinetic energies of all particles changed less than 1% after 1000 iteration steps. The equations were solved for time intervals of 10^{-15} sec with the following results:

(1) Moderation of the ion from 10 eV below 1 eV takes place after 6 to 10 collisions.

(2) "Typical" collisions occur for relative energies below 1 eV and impact parameters below 1.5 Å. Then the ion penetrates into the electron cloud of the helium atoms, crosses the potential minimum (1.85 eV) at 0.75 Å, and is repulsed at 0.5 Å. The average value of the integrated electron density is only weakly dependent on the actual values of relative energy and impact parameter:

$$\rho\tau = \int_{\mathrm{collision}} \rho(t)dt = (4\times10^{-15}\ \mathrm{sec})\rho_0$$

[$\rho_0 = (\pi a_8^3)^{-1}$ is the electron density at the center of the hydrogen atom]. With increasing energy in the range between 1 and 10 eV the integrated electron density per collision decreases but the collision rate increases.

(3) The mean electron density defined as collision rate N multiplied with the integrated electron density per collision adopts the value

$$N_c(d)\rho\tau = (0.11\pm0.03)d/d_0.$$

This value is only weakly dependent on the relative energy in the range from 0.2 to 10 eV. The same applies for the electric field strength $|E(t)|$ as a function of time. The uncertainties arise mainly from statistical errors of the Monte Carlo procedure.

(4) Stable molecular ions $(\mathrm{He}\text{-}M\text{-}\mathrm{He})^+$ are formed in liquid helium with a considerable rate which varies quadratically with density because ternary collisions are required for the formation of a bound system:

$$\Gamma_{\mathrm{MIF}} = (2\pm1)\times10^{12}(d/d_0)^2\ \mathrm{sec}^{-1},$$

where MIF stands for molecular ion formation. The electron density experienced by the ion after MIF is increased by a factor of 5 with respect to a free motion in liquid helium. The MIF is insignificant for $n>5$ but increases the effective Auger rate by an average factor of about 2–3 for low-n levels. The formation of this type of molecular ion was at first presumed by Hughes[13] to explain depolarization effects for positive muons in gases. Russell[14] suspected this effect to be responsible for the K_β/K_α inversion in pionic helium.

(5) The electric field surpasses $|E|=10^8$ V/cm for about 2×10^{-14} sec during a collision, changing its direction twice and reaching a peak value of about 10^9 V/cm.

The external Auger rates were calculated by multiplying the internal Auger rates given by Burbidge and deBorde[15] for exotic atoms inside the closed electron K shell with the mean electron density in units of ρ_0:

$$\Gamma^{\mathrm{Aug}}_{\mathrm{ext}}(n,l) = (0.11\pm0.03)(d/d_0)\Gamma^{\mathrm{Aug}}_{\mathrm{int}}(n,l).$$

The increase of the electron density due to MIF after the average formation time was additionally taken into account.

Stark mixing of level $|n_i;l_i;m_i\rangle$ with $|n_i;l_f=l_i+1;\ m_f=m_i$ or $m_i\pm1\rangle$ is due to the time-dependent dipole matrix element

$$M=\langle n_f l_f m_f|\, e\vec{\mathbf{E}}(t)\cdot\vec{\mathbf{r}}\,|n_i l_i m_i\rangle.$$

The electric field $E(t)$ experienced in a typical collision changes nonadiabatically with respect to the quantum beat frequency between two neighboring fine-structure levels causing transitions between them. Several transitions may occur successively during one single collision thus allowing $\Delta l=\pm2,\pm3,\ldots$. The orientation of the electric dipole moment relative to the external electric field (i.e., the quantum number m_i) is conserved during a collision because then the electric field vector changes adiabatically with respect to the precession frequency of the dipole moment in this electric field, leading to a $\Delta m=0$ selection rule. The z axis is chosen parallel to the molecular symmetry axis. In contrast, the orientation is not conserved in between two collisions because the direction of the residual electric field from neighboring helium atoms changes faster than the electric dipole moment can follow. Therefore the quantum numbers m_i are conserved during collisions but have to be redefined for each new collision. The Stark transition probabilities $\Gamma^{St}(n_i l_i m_i \to n_i l_f m_i)$ for a transition from a given level $|n_i l_i m_i\rangle$ to a level $|n_i l_f m_i\rangle$ during one collision were calculated by solving numerically the time-dependent Schrödinger equations for all n_i^2 sublevels $|n_i l_i m_i\rangle$ in steps of 10^{-15} sec for the collision time. The transition probability increases with increasing dipole matrix element M and decreasing energy separation $\Delta E_{e_f,e_i}$. The energy separations originate from relativistic and vacuum polarization effects, for s states additionally from the finite nuclear size, and for pionic helium from strong-interaction effects.

The *cascade calculation* started with a given initial distribution which was either statistical $[\propto(2l+1)e^{al};\ a=0,\ldots,0.2]$ or according to a distribution derived by Haff and Tombrello.[16] As a result of the Stark mixing in highly excited levels no significant influence of the initial distribution or of the capture level (when varied within 15%) on the x-ray intensities could be found.

The depopulation of all l sublevels of the capture level n was calculated in between and during collisions. In between two collisions only radiative transitions, nuclear capture, and particle decay are allowed. During collisions external Auger transition and Stark mixing may additionally occur. The time between collisions depends on the target density and the kinetic energy of the exotic atom and influences the relative weight of radiative transitions compared to Auger or Stark mixing transitions.

The calculated transition probabilities for all effects give the branching ratios for transitions into lower levels, levels of the same n but different l, or staying in the initial level after one collision. The distribution procedure was repeated for as many collisions as were necessary to decrease the population of the $|n_c,l\rangle$ sublevels to $\frac{1}{1000}$ of its original value. The calculation then proceeded to n_c-1, $n_c-2,\ldots$. The percentage of all radiative transitions from n_i to n_f was stored in a separate array. The rate for nuclear absorption from the 1s level in pionic helium was taken from the experimentally observed line broadening. It was left open as a free parameter for the 2p level. The ns and np widths were scaled according to the overlap of atomic and nuclear wave functions.

In *muonic helium* (see Table I) the strong interaction is not involved in the deexcitation process. Therefore the calculation did not use any free

TABLE I. Comparison of experimental data and calculations for muonic and pionic helium.

Density (d_o)		Experiment	Theory
System: μ^-He — No free parameters			χ^2_9 = 10.3
0.01[2] —	K_α / K_{tot}	0.62 ± 0.08	0.57
	K_{tot} / Stop	0.99 ± 0.10	0.94
	L_{tot} / K_{tot}	0.64 ± 0.05	0.66
	2s population	0.034 ± 0.007	0.043
1.0[3]	K_α / K_{tot}	0.60 ± 0.027	0.60
	K_{tot} / Stop	0.76 ± 0.19	0.88
	K_β / K_α	0.54 ± 0.03	0.54
	K_γ / K_α	0.108 ± 0.013	0.075
	K_∂ / K_α	0.026 ± 0.015	0.011
System: $\pi^-{}^4$He — Experimental input: 1s shift = −75.7 eV[3]; 1s width = 45 ± 3 eV[3]; Free parameter: 2p width = 2.2 ± 0.2 meV			χ^2_6 = 6.6
0.085[4]	K_β / K_α	0.38 ± 0.04	0.32
1.0[3] —	K_α / K_{tot}	0.36 ± 0.03	0.35
	K_α / Stop	0.071 ± 0.018	0.055
	K_β / K_α	1.24 ± 0.06	1.31
	K_γ / K_α	0.42 ± 0.02	0.43
	K_∂ / K_α	0.096 ± 0.006	0.088
	K_ε / K_α	0.030 ± 0.007	0.028

parameters at all. Accordingly muonic helium provides a good test for the validity of our calculations. Experiments have been performed at $10^{-2}d_0$ (Ref. 2) and d_0.[1,3,5] The comparison between the experimental and theoretical results g give a χ^2 of 10.3 for 9 degrees of freedom indicating good agreement.

In *pionic helium* (see Table I) the strong interaction leads to energy shifts and broadening of the s and p levels. The energy shift and width of the $1s$ level is well known[3]; the $2p$ width was left open as a free parameter. Experiment and theory agreed best for a $2p$ level width of 2.2 ± 0.2 meV with $\chi^2 = 6.6$ for 6 degrees of freedom indicating again very good agreement. The most important result is certainly the explanation of the high K_β/K_α ratio is liquid helium.

The time between the ejection of the second electron and the last observable transition was also calculated. For pionic helium in gas (17.5 atm $= 0.025 d_0$) the time is 0.7×10^{-10} sec; in liquid, 2.5×10^{-12} sec. These values have to be compared with the experimental results of $(1.4 \pm 0.7)\times10^{-10}$ sec in gas at 17.5 atm,[17] and $(3.19 \pm 0.23)\times10^{-10}$ sec in liquid.[18] The obvious disagreement for the value in liquid can be explained qualitatively by the "trapping hypothesis,"[19-22] suggesting that a small fraction of stopped particles is trapped in highly excited, nearly circular orbits forming an $\alpha\mu e$ or $\alpha\pi e$ system. The dominant $\Delta l = 1$ or $\Delta l = 0$ Auger transitions are then energetically forbidden and each additional multipole reduces the Auger transition rate by about 2 orders of magnitude. The neutral $\alpha\mu e$ or $\alpha\pi e$ atoms are not subject to the strong Stark mixing during collisions like $(\alpha\mu)^+$ or $(\alpha\pi)^+$ ions, and have lifetimes greater than 10^{-9} sec.[22] As the time needed to reach the ground state is less than 10^{-10} sec at densities where experiments have been carried out so far, the measured cascade times are always determined by the fraction of trapped particles. The longer "cascade time" of pionic helium in liquid indicates that this fraction seems to increase with density. Experiments measuring the cascade time of π-He in gas at 1 atm or below would provide a good test for this hypothesis because the calculated cascade time at 1 atm is 5.6×10^{-10} sec and therefore of the same order of magnitude as the lifetime of the almost-circular states.

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