

Derivation of Asymptotic $|\Delta\vec{I}| = \frac{1}{2}$ Rule

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It is argued that the origin of the observed approximate $|\Delta\vec{I}| = \frac{1}{2}$ rule is the presence of an asymptotic $|\Delta\vec{I}| = \frac{1}{2}$ rule which exists among certain two-body hadronic weak matrix elements, involving especially the ground-state hadrons.

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The understanding of the origin and also of the small violation of the $|\Delta\vec{I}| = \frac{1}{2}$ rule in nonleptonic weak processes as well as those of the Okubo-Zweig-Iizuka (OZI) rule¹ is one of the persistent problems of particle physics. The QCD short-distance correction to the W -exchange diagram was found unable^{2,3} to explain simultaneously the $|\Delta\vec{I}| = \frac{1}{2}$ rule and the Cabibbo-unsuppressed charmed-meson decays. The entirely opposite claim that nonleptonic physics is essentially determined by long-distance dynamics also exists.⁴

Although it does not work for the K mesons, it has long been recognized⁵ that one can impose $|\Delta\vec{I}| = \frac{1}{2}$ constraints on the *two-body hyperon* weak nonleptonic vertices, by incorporating essentially the color singlet nature of hadrons into the quark model wave functions. This observation is indicative of a close link between the possible origin of the $|\Delta\vec{I}| = \frac{1}{2}$ rule and the qqq structure of hyperons.

The purpose of this paper is to add a new point of view regarding the origin of the $|\Delta\vec{I}| = \frac{1}{2}$ rule and its violation, and we argue that the presence and violation of the OZI rule in hadron physics share exactly the same origin. We assert that the observed approximate $|\Delta\vec{I}| = \frac{1}{2}$ rule is a reflection of the existence of an *exact* $|\Delta\vec{I}| = \frac{1}{2}$ rule which holds for certain *asymptotic two-body* hadronic weak matrix elements involving ground-state hadrons. By applying a soft-pion extrapolation [actually only the $\vec{q}_\pi \rightarrow 0$ limit in the infinite-momentum frame of the parent particle instead of the usual $(\vec{q}_\pi)_\mu \rightarrow 0$ limit], one can relate the physical amplitudes to the above asymptotic two-body weak amplitudes satisfying the exact $|\Delta\vec{I}| = \frac{1}{2}$ rule. This causes an inevitable but slight violation of the rule for physical processes, providing an attractive explanation of why the rule is also slightly violated. Since this conjecture is already confirmed⁶ for the K -meson nonleptonic processes (as well as for their charmed counterparts⁷), in this paper we complete our assertion by deriving the rule for the more complicated cases of bary-

ons.

In our theoretical framework, the Hilbert space with which we deal consists essentially of observable hadrons. The underlying quarks and gluons however, control subtly the world of hadrons: Hadrons have to obey the (mainly $q\bar{q}$ and qqq) constituent quark level scheme and they are constrained severely by the presence of quark chiral algebras involving observable weak quark currents and their charges. The successful calculation of $g_A(0)$ by Adler and Weisberger lends support for this type of approach to hadron physics with confined quarks and gluons.

The central ingredient of our pattern recognition of the dynamical nonperturbative constraints is the *Ansatz of level* realization of *asymptotic* flavor symmetry⁸ in the chiral algebras involving the axial charges. This has produced⁸ a correct value of $g_A(0)$ and a good nucleon anomalous magnetic-moment relation $k_p = -k_n$, etc. In particular, it predicts the presence and the violation of the OZI rule for the *two-particle* asymptotic hadronic matrix elements of the vector and axial-vector currents and their charges.

The weak effective Hamiltonian (in the limit $m_w \rightarrow \infty$) H_w in the standard model contains a sizable 27-plet. For SU(3) we use the concept of asymptotic SU(3) symmetry,⁸ which states that the linearity of the SU(3) transformation is still maintained in *broken* SU(3) symmetry but only for the SU(3) multiplets with *infinite momenta*. It has been shown that the assumption can be made in the presence of Gell-Mann-Okubo mass splittings with SU(3) particle mixings. We consider the asymptotic ground-state baryon matrix elements, $\langle B'(\vec{p}, \lambda) | H_w | B(\vec{p}, \lambda) \rangle$, with $\vec{p} \rightarrow \infty$ and helicity $\lambda = \frac{1}{2}$. Hereafter we suppress \vec{p} and λ unless necessary. The SU(3) parametrizations of $\langle B' | H_w | B \rangle$ with $\vec{p} \rightarrow \infty$ can be carried out by using the quark constraints, i.e., the equal-time commutators involving H_w and the SU(3) charges [such as $V_{K^0} = -i \int d^3x V_{K^0}{}^4(x)$, where $V_{K^0}{}^4(x) \equiv i\bar{q}(x)\gamma^4$

$\times \frac{1}{2}(\lambda_6 + i\lambda_7)q(x)$, etc.]. The most useful ones are⁹ [$H_W \equiv H(\Delta S=1)$]

$$[[H_W, V_{\bar{K}^0}], V_{\bar{K}^0}] = -2H_W^\dagger, \quad (1)$$

$$[H_W, V_{K^0}] = 0, \quad (2)$$

$$[[H, V_{K^-}], V_{K^-}] = [[H, V_{\pi^-}], V_{\pi^-}], \quad H \equiv H_W + H_W^\dagger. \quad (3)$$

For various $\langle B'|H|B\rangle$, we introduce the following abbreviations:

$$\begin{aligned} x_1 &= \langle p|H_W|\Sigma^+\rangle, \quad x_2 = \langle n|H_W|\Sigma^0\rangle, \quad x_3 = \langle n|H_W|\Lambda\rangle, \quad x_4 = \langle \Sigma^-|H_W|\Xi^-\rangle, \quad x_5 = \langle \Sigma^0|H_W|\Xi^0\rangle, \quad x_6 = \langle \Lambda|H_W|\Xi^0\rangle, \\ y_1 &= \langle p|H_W|Y^+\rangle, \quad y_2 = \langle n|H_W|Y^0\rangle, \quad y_3 = \langle \Sigma^0|H_W|\Xi^{*0}\rangle, \quad y_4 = \langle \Sigma^-|H_W|\Xi^{*-}\rangle, \quad y_5 = \langle \Lambda|H_W|\Xi^{*0}\rangle, \quad y_6 = \langle \Xi^-|H_W|\Omega^-\rangle, \\ z_1 &= \langle \Delta^+|H_W|\Sigma^+\rangle, \quad z_2 = \langle \Delta^0|H_W|\Sigma^0\rangle, \quad z_3 = \langle \Delta^0|H_W|\Lambda\rangle, \quad z_4 = \langle \Delta^-|H_W|\Sigma^-\rangle, \quad z_5 = \langle Y^-|H_W|\Xi^-\rangle, \quad z_6 = \langle Y^0|H_W|\Xi^0\rangle, \\ w_1 &= \langle \Delta^+|H_W|Y^+\rangle, \quad w_2 = \langle \Delta^0|H_W|Y^0\rangle, \quad w_3 = \langle \Delta^-|H_W|Y^-\rangle, \quad w_4 = \langle Y^0|H_W|\Xi^{*0}\rangle, \quad w_5 = \langle Y^-|H_W|\Xi^{*-}\rangle, \quad w_6 = \langle \Xi^{*-}|H_W|\Omega^-\rangle. \end{aligned}$$

We sandwich Eq. (1) between the following *asymptotic* hyperon states: (i) $\langle p|$ and $|\Sigma^+\rangle$, (ii) $\langle n|$ and $|\Sigma^0\rangle$, (iii) $\langle n|$ and $|\Lambda\rangle$, (iv) $\langle \Sigma^-|$ and $|\Xi^-\rangle$, (v) $\langle \Sigma^0|$ and $|\Xi^0\rangle$, and (vi) $\langle \Lambda^0|$ and $|\Xi^0\rangle$, and use asymptotic SU(3). Then the following four sum rules among the x_i ($i=1, \dots, 6$) are obtained in addition to the two relations automatically satisfied by T invariance: $x_2 + \sqrt{3}x_3 - 2x_5 = 0$, $-\sqrt{3}x_2 + x_3 + 2x_6 = 0$, $-2x_2 + x_5 + \sqrt{3}x_6 = 0$, and $-2x_3 + \sqrt{3}x_5 - x_6 = 0$. Equation (2) adds nothing new. However, between $\langle \Xi^-|$ and $|\Sigma^+\rangle$ and $\langle \Sigma^-|$ and $|p\rangle$, Eq. (3) produces $\sqrt{2}x_1 - \sqrt{2}x_4 - x_5 + \sqrt{3}x_6 = 0$ and $\sqrt{2}x_1 + x_2 - \sqrt{3}x_3 - \sqrt{2}x_4 = 0$, respectively. An inspection of these six asymptotic SU(3) constraints reveals that, among the $\langle B'_8|H_W|B_8\rangle$ with $\vec{p} \rightarrow \infty$, only *three* are independent and they can be identified with d_W , f_W , and ξ , where d_W and f_W represent the two independent octet [$H(8)$] couplings and ξ the 27-plet [$H(27)$] coupling. If we again insert Eqs. (1) and (2) between the asymptotic decuplet states, (vii) $\langle \Delta^+|$ and $|Y^+\rangle$, (viii) $\langle \Delta^0|$ and $|Y^0\rangle$, (ix) $\langle \Delta^-|$ and $|Y^-\rangle$, (x) $\langle Y^-|$ and $|\Xi^{*-}\rangle$, (xi) $\langle Y^0|$ and $|\Xi^{*0}\rangle$, and (xii) $\langle \Xi^{*-}|$ and $|\Omega^-\rangle$, the nontrivial sum rules are $-w_2 + w_4 = 0$, $2w_3 + \sqrt{3}w_5 = 0$, $-\sqrt{3}w_3 - 3w_5 + \sqrt{3}w_6 = 0$, and $\sqrt{3}w_5 - 2w_6 = 0$. Inserting Eq. (3) between $\langle Y^-|$ and $|\Delta^+\rangle$ and $\langle \Xi^{*-}|$ and $|Y^+\rangle$, we also obtain $w_1 + \sqrt{2}w_2 - \sqrt{3}w_3 - w_5 = 0$ and $w_1 + \sqrt{2}w_4 - w_5 + \sqrt{3}w_6 = 0$, respectively. These sum rules imply that $\langle B'_{10}|H_W|B_{10}\rangle$ with $\vec{p} \rightarrow \infty$ can be described in terms of two independent parameters corresponding to $H(8)$ and $H(27)$. In the same way, both $\langle B'_{10}|H_W \times |B\rangle$ and $\langle B'_8|H_W|B_{10}\rangle$ with $\vec{p} \rightarrow \infty$ can be parametrized in terms of two independent couplings corresponding to $H(8)$ and $H(27)$. These observations are not trivial. In the usual treatment of SU(3) these statements can make sense only in the exact SU(3) symmetry limit, whereas the above *asymptotic* SU(3) sum rules are valid in broken SU(3) symmetry.

We now demonstrate that the *Ansatz* of level

realization of asymptotic SU(3) symmetry implies that the 27 parts of $\langle B'_8|H_W|B_8\rangle$ and $\langle B'_{10}|H_W|B_{10}\rangle$ vanish, $\langle B'_8|H_W|B_{10}\rangle = \langle B'_{10}|H_W|B_8\rangle = 0$, and there exist SU(6)-like relations among $\langle B'_8|H_W|B_8\rangle$ and $\langle B'_{10}|H_W|B_{10}\rangle$, but all *only* in the *asymptotic* limit $\vec{p} \rightarrow \infty$. We now consider another valuable quark constraint:

$$[[H_W, A_{\pi^-}], A_{\pi^+}] = [[H_W, V_{\pi^-}], V_{\pi^+}], \quad (4)$$

where V_{π^\pm} are the isospin operators and A_{π^\pm} the axial charges. We insert Eq. (4) between the hyperon states $\langle B'_8(\vec{p})|$ and $|B_8(\vec{p})\rangle$ with $\vec{p} \rightarrow \infty$ for the combinations previously considered, (i)–(vi). On the left-hand side of Eq. (4), among the set of complete single-hadron intermediate states inserted between the factors H_W , A_{π^-} , and A_{π^+} , we distinguish the contributions coming from various *levels* of hadrons. We demand that the fractional contribution of each level to Eq. (4) should be (asymptotically) invariant under the SU(3) rotation produced by the variations (i)–(vi), i.e., the flavor symmetry in Eq. (4) should be asymptotically secured *levelwise*. We now look at the realization at the ground-state ($\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet) level. We define $d \equiv \langle \Sigma^-|A_{\pi^-}| \Lambda(\vec{p})\rangle$, $f \equiv \langle \Sigma^0|A_{\pi^-}| \Sigma^+(\vec{p})\rangle$, $g \equiv \langle Y^0|A_{\pi^-}| Y^+(\vec{p})\rangle$, and $h \equiv \langle \Sigma^0| \times A_{\pi^-}| Y^+(\vec{p})\rangle$ where $\vec{p} \rightarrow \infty$. The same level-realization *Ansatz* applied for similar algebras, $[A_{\pi^+}, A_{\pi^-}] = 2V_3$, $[[j_3^\mu(x), A_{\pi^+}], A_{\pi^-}] = 2j_3^\mu(x)$, etc., already fixed⁸ the ratios of d , f , g , and h consistently:

$$\begin{aligned} d &= (\frac{2}{3}k)^{1/2}, \quad f = -\frac{2}{3}(2k)^{1/2}, \quad g = -\frac{1}{3}(2k)^{1/2}, \\ h &= \pm \frac{2}{3}\sqrt{k}. \end{aligned} \quad (5)$$

Here, k denotes the fractional contribution of the ground state to the algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_3$ and $k \approx 0.6$. Denoting the ground-state contribution to each case (i)–(vi) as $G(j)$ ($j=i, ii, \dots, vi$) and

the rest (i.e., higher-level contributions) as $H(j)$ we obtain

$$G(i) + H(i) = x_1 + \sqrt{2}x_2, \quad G(ii) + H(ii) = \sqrt{2}(x_1 + \sqrt{2}x_2), \quad G(iii) + H(iii) = 0, \quad G(iv) + H(iv) = x_4 + \sqrt{2}x_5, \\ G(v) + H(v) = \sqrt{2}(x_4 + \sqrt{2}x_5), \quad G(vi) + H(vi) = 0.$$

G involves the weak matrix elements x_i , y_i , z_i , and w_i as well as the strong couplings, d , f , g , and h . For example,

$$G(i) = (\frac{1}{2}f^2 - \sqrt{3}df + \frac{3}{2}d^2 + 2h^2)x_1 + [(\frac{1}{2})^{1/2}f - (\frac{3}{2})^{1/2}d]fx_2 + [(\frac{1}{2})^{1/2}f - (\frac{3}{2})^{1/2}d]dx_3 \\ + [(\frac{1}{2})^{1/2}f - (\frac{3}{2})^{1/2}d]hy_2 + (\frac{1}{2}f - \frac{1}{2}\sqrt{3}d - 2g)hz_1 - \sqrt{2}fhz_2 - \sqrt{2}dhz_3 - \sqrt{2}h^2w_2.$$

The cases $j = iii$ and $j = vi$ show that the level-realization pattern should be $0 + \dots = 0$, which in turn implies $x_1 = -\sqrt{2}x_2$ and $x_4 = -\sqrt{2}x_5$. These are the $|\Delta \bar{1}\rangle = \frac{1}{2}$ rules for the asymptotic $\langle B_8' | H_W | B_8 \rangle$ vertices and they imply that $\langle B_8' | H_W(27) | B_8 \rangle$'s vanish, i.e., $\xi \rightarrow 0$ as $\vec{p} \rightarrow \infty$. Thus the x_i 's can now be expressed in terms of two independent parameters (corresponding to d_W and f_W couplings), for example, x_2 and x_3 , as $x_1 = -\sqrt{2}x_2$, $x_4 = -(x_2 + \sqrt{3}x_3)/\sqrt{2}$, $x_5 = (x_2 + \sqrt{3}x_3)/2$, $x_6 = (\sqrt{3}x_2 - x_3)/2$. This is the asymptotic octet rule for the $\langle B_8' | H_W \times | B_8 \rangle$. The realization patterns require that all G 's vanish. The complete octet dominance over $\langle B_{10}' | H_W | B_{10} \rangle$ in the limit $\vec{p} \rightarrow \infty$ can also be derived, by inserting Eq. (4) between $\langle B_{10}' |$ and

$|B_{10}\rangle$ for the combination (vii)-(xii). Corresponding to $G(i)$ and $H(i)$ we introduce $I(i)$ and $J(i)$, respectively where $i = vii, \dots, xii$. We then obtain

$$I(vii) + J(vii) = 2\sqrt{2}(\sqrt{2}w_1 + w_2),$$

$$I(viii) + J(viii) = 2\sqrt{2}w_1 + 5w_2 - \sqrt{6}w_3,$$

$$I(ix) + J(ix) = \sqrt{2}(-\sqrt{3}w_2 + \sqrt{2}w_3),$$

$$I(x) + J(x) = \sqrt{2}w_4 + w_5,$$

$$I(xi) + J(xi) = \sqrt{2}(\sqrt{2}w_4 + w_5),$$

$$0 + J(xii) = 0 \quad [\text{i.e., } I(xii) = 0].$$

$I(vii)$ is, for example, given by

$$I(vii) = \sqrt{2}h^2x_2 + \sqrt{6}h^2x_3 + (f - \sqrt{3}d - 2g)hy_1 + \sqrt{2}ghy_2 + \sqrt{2}ghz_2 + \sqrt{6}ghz_3 + 2(h^2 + g^2)w_1 + \sqrt{2}g^2w_2.$$

We see that the realization pattern is again $0 + \dots = 0$. Then all the right-hand sides of the sum rules, as well as $I(j)$'s ($j = vii, \dots, xi$), must all vanish. Combined with the asymptotic SU(3) sum rules involving w 's, we now find that $\langle B_{10}' | H_W \times | B_{10} \rangle$ can now be parametrized in the $\vec{p} \rightarrow \infty$ limit by one parameter, i.e., $w_2 = -\sqrt{2}w_1$, $w_3 = -\sqrt{3}w_1$, $w_4 = -\sqrt{2}w_1$, $w_5 = 2w_1$, and $w_6 = \sqrt{3}w_1$. This is the asymptotic octet rule.

We now show that $\langle B_8' | H_W | B_{10} \rangle = \langle B_{10}' | H_W | B_8 \rangle = 0$. If we sandwich Eq. (1) between $\langle B_8' |$ and $|B_{10}\rangle$, then the z_i 's are expressed linearly in y_i . Conversely, the same procedure between $\langle B_{10}' |$ and $|B_8\rangle$ expresses the y_i 's linearly in z_i . We now insert Eq. (5) between $\langle B_8' |$ and $|B_{10}\rangle$ and apply the level realization. By just studying the realization pattern suggested by the right-hand side of Eq. (5), we find the following: We first see $\langle B_8' | H_W(27) | B_{10} \rangle = 0$ as $\vec{p} \rightarrow \infty$, i.e., y_i and, therefore, z_i also satisfy the octet rule and, in particular, $z_3 = \langle \Delta^0 | H_W | \Lambda \rangle = 0$. Then, to accommodate $z_3 = 0$ in the realization scheme the $\langle B_8' | H_W(8) \times | B_{10} \rangle$'s are required to vanish, namely $y_i = z_i = 0$, $i = 1, \dots, 6$. The result obtained so far is independent of the structure of other realization constraints such as $G(i) = I(j) = 0$ and will be satisfied

also by higher-lying baryons. If we now substitute the information obtained for x_i , w_i , y_i , and z_i into the two realization constraints $I(vii) = 0$ and $I(viii) = 0$, the following SU(6)-like relations are obtained:

$$x_2 = w_1/\sqrt{2}, \quad \text{i.e.,} \quad (6)$$

$$\langle n | H_W | \Sigma^0 \rangle = (1/\sqrt{2})\langle \Delta^+ | H_W | Y^+ \rangle,$$

$$x_3 = -(\sqrt{6}/3)w_1, \quad \text{i.e.,} \quad (7)$$

$$\langle n | H_W | \Lambda^0 \rangle = -(\sqrt{6}/2)\langle \Delta^+ | H_W | Y^+ \rangle.$$

These solutions satisfy not only all the remaining level-realization constraints, i.e., $G(j) = I(k) = 0$ ($j = i, \dots, vi$ and $k = ix, \dots, xii$), but also all the level-realization requirements of another independent quark constraint,

$$[[H_W, A_{\pi^+}] = [[H_W, V_{\pi^+}], V_{\pi^-}]. \quad (8)$$

In the latter case, the fraction of the ground-state level contribution l is found to be equal to k in Eq. (5). Had Eq. (5) not been input, Eq. (5) could have been derived, producing the correct value of $g_A(0)$, $g_{\Delta\rho\pi}$, etc. In exactly the same way, we see that the $\langle B'(\lambda = \frac{3}{2}) | H_W | B(\lambda = \frac{3}{2}) \rangle$'s also satisfy

the asymptotic octet rule. We stress that the assumption of saturation by a particular level is never used. Had we assumed the saturation ($l=1$) of Eq. (8) by the ground-state baryons, $k=l=1$ implies $g_A(0)=\frac{5}{3}$ from Eq. (5). This is the unsatisfactory result of SU(6).

We summarize the difference between the results based explicitly on the quark wave function⁵ (QW) and ours. (i) Our $|\Delta\vec{I}|=\frac{1}{2}$ rule and also the octet rule for the *two-body hadronic* weak vertices hold *only* in the $\vec{p}\rightarrow\infty$ limit, but they are valid for baryons *as well as* for mesons. In QW no asymptotic statement has been made and the $|\Delta\vec{I}|=\frac{1}{2}$ rule was not obtained for mesons. (ii) In QW, $\langle B_8'|H_W|B_{10}\rangle=\langle B_{10}'|H_W|B_8\rangle=\langle B_{10}'|H_W|B_{10}\rangle=0$, whereas we find only $\langle B_8'|H_W|B_{10}\rangle=\langle B_{10}'|H_W|B_8\rangle=0$ for $\vec{p}\rightarrow\infty$. Furthermore, the $\langle B_{10}'|H_W|B_{10}\rangle$'s are related to the $\langle B_8'|H_W|B_8\rangle$'s by SU(6)-like relations at $\vec{p}\rightarrow\infty$, as seen from Eqs. (6) and (7). (iii) The same equations imply that the $\langle B_8'|H_W\times|B_8\rangle$'s are of *pure f* type at $\vec{p}\rightarrow\infty$. This is, at least, compatible with the known fact that the observed hyperon *s*-wave amplitudes prefer a predominantly *f*-type coupling. On the other hand, in QW $d_w/f_w=-1$ is predicted which makes the *s* waves difficult to fit (see Pakvasa's remark in Ref. 5).

As for the octet hyperon decays, a new calculation may be in order using asymptotic SU(3) symmetry and the new soft-pion technique mentioned, to accommodate the asymptotic octet sum rules obtained in this paper.

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¹S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, CERN Report No. CERN-TH-401, 1964 (unpublished); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. 35, 1061 (1965).

²G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974); M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. 22, 123 (1975) [JETP Lett. 22, 55 (1975)], and Nucl. Phys. B120, 316 (1977), and Zh. Eksp. Teor. Fiz. 72, 1275 (1977) [Sov. Phys. JETP 45, 670 (1977)].

³N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418 (1978).

⁴G. Nardulli and G. Preparata, CERN Report No. CERN-TH-3055, 1981 (unpublished).

⁵For example, J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920 (1971). For exhaustive references and also references on the $|\Delta\vec{I}|=\frac{1}{2}$ rule, see S. Pakvasa, in *High Energy Physics—1980*, edited by L. Durand and L. G. Pondrom, AIP Conference Proceedings No. 68 (American Institute of Physics, New York, 1981), p. 1164.

⁶S. Oneda, Phys. Lett. 102B, 433 (1981); S. Oneda and T. Tanuma, University of Maryland Physics Publication No. 81-910, 1981 (unpublished).

⁷T. Tanuma, S. Oneda, and K. Terasaki, Phys. Lett. 110B, 260 (1982).

⁸S. Oneda and Seisaku Matsuda, Phys. Rev. D 5, 2287 (1972); S. Oneda, T. Tanuma, and Milton D. Slaughter, Phys. Lett. 88E, 343 (1979).

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