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Indices, Triality, and Ultraviolet Divergences for Supersymmetric Theories

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Transverse rotation-group representation indices are shown to appear in one-loop radiative corrections. Some conjectures are made on how such index structure generalizes to higher orders. An unusual correlation between the absence of UV divergences in supersymmetric theories and the equality of such indices for fermions and bosons is described. This correlation is argued to be a fundamental indicator of higher-order UV behavior for $D=10$ ($N=4$) supersymmetric Yang-Mills theory and $D=11$ ($N=8$) supergravity. Within this context, the significance of O(8) triality is discussed.

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In this Letter I discuss some striking group-theoretical features which appear in radiative corrections as computed perturbatively for relativistic quantum field theories. These features seem to be very general in character, and may lead to further simplifications in the rules used to determine the UV structure of field theories. Although I expect wider applications, I will primarily discuss supersymmetric theories in this Letter, since their UV structure is of paramount theoretical interest.

I shall show how radiative corrections involve "transverse rotation-group representation indices" by giving a simple one-loop example. On the basis of such low-order perturbation theory results, I conjecture that higher-loop processes involve generalized higher-order representation indices. I also suggest a factorized form for the dependence of higher-loop corrections on boson and fermion indices in supersymmetric theories such that generalized indices are a direct indicator of certain higher-order effects. I observe that these conjectures are consistent with an otherwise unusual correlation between the equality of group representation indices for the fermions and bosons found in certain supersymmetric theories and the known cancellation of UV divergences in those theories. Assuming the factorization conjecture is exact, we anticipate the results of future multiloop calculations of UV divergences in supersymmetric Yang-Mills theory and supergravity.

Even if these conjectures are incorrect in higher orders, they serve to emphasize some interesting mathematical properties of extended supersymmetry multiplets. Particularly notable among these properties are the effects of O(8) triangular symmetry (i.e., "triality") transformations acting on physical degrees of freedom. For $D=10$ ($N=4$) supersymmetric Yang-Mills theory, triality implies the equality of fermion and boson representation indices of all orders. For $D=11$ ($N=8$) supergravity, triality only leads to equality for the lowest few indices, which in fact are unequal beyond sixth order.

Indices[1] for a group representation are defined as sums of powers of the lengths of weights for that representation. Weights are essentially just arrays of eigenvalues for a maximal simultaneously diagonizable set of group generators (i.e., "good quantum numbers"). For a representation R, the pth index is

$$I^{(p)}[R]=\sum_{\vec{w}\text{ in }R}(\vec{w},\vec{w})^{p/2}. \tag{1}$$

The norm $(\vec{w},\vec{w})$ of a weight is defined with use of the inverted Cartan matrix to form a metric. An irreducible representation of a simple Lie group consists of a "spindle-shaped" array of weights which is completely determined once one is given the "highest" weight in the spindle.[2]

To demonstrate the physical significance of indices, consider a gauge-invariant current correlation function evaluated to one loop. This correlation function provides a very simple example of a physically interesting UV divergence. Upon reduction from D to d dimensions, the result is proportional to

$$\Pi_{mn}(q)=(-)^s f(q^2)[P_{mn}{}^{(d)}I^{(0)}/(d-1) - 4P_{mn}{}^{(D)}I^{(2)}/r], \tag{2}$$

where $f(q^2)$ is the basic one-loop integral (dimensionally regulated in d dimensions)

$$f(q^2)=\int \mathfrak{D}^d k[k^2(q+k)^2]^{-1},$$

and r is the rank of $O(D-2)$. The P tensors are essentially just projections given by

$$P_{mn}{}^{(d)}=q_m q_n-q^2 g_{mn}{}^{(d)},\quad P_{mn}{}^{(D)}=q_m q_n-q^2 g_{mn}{}^{(D)},$$

where $g^{(d)}$ and $g^{(D)}$ are the space-time metrics in d and D dimensions. Note that q_m is by assumption reduced to d dimensions, i.e., only the first d components of any momenta are possibly nonzero. One is allowed to take $d=D$ to obtain a less general result. The statistics phase factor $(-)^s$ in (2) is $+1$ for bosons and -1 for fermions. In general, of course, one must sum (2) over the various possible virtual particles that can contribute.

The indices appearing in Eq. (2) are those appropriate to the $O(D-2)$ representation for the massless virtual particle in the closed loop, which in general might be a reducible representation. This is somewhat surprising since the one-loop momentum integration has been rather indelicately reduced to d dimensions, and one might *a priori* expect only $O(d-2)$ indices to appear. Nonetheless, the net result is expressible in terms of $O(D-2)$ indices. One immediately sees that the divergence (in fact, the entire one-loop contribution) cancels if there are massless bosons and fermions contributing equal zeroth and second indices. Thus there is a correlation between the equality of the transverse rotation-group indices for fermions and bosons in higher dimensions and the cancellation of UV divergences.

Equalities among transverse rotation-group representation indices for fermions and bosons are easily found in supersymmetric theories when those theories are formulated in their "natural" space-time dimensions. For example, $D=10$ and $D=11$ are the natural space-times for $N=4$ supersymmetric Yang-Mills theory[3] and $N=8$ supergravity,[4] respectively, for which the transverse rotation groups are O(8) and O(9). It will be sufficient here to illustrate such index equalities with use of these maximal globally and locally supersymmetric models. First, however, let us observe that generalized index equalities for fermions and bosons are a logical extension to higher space-time dimensions of the spin-moment sum rules satisfied by linear supermultiplets in four dimensions,[5,6] with four-dimensional results straightforwardly obtainable by dimensional reduction. These spin-moment sum rules were previously associated with the absence of UV divergences in lowest-order perturbation theory[5] (e.g., the vanishing of the Gell-Mann–Low charge renormalization function to one loop). An excellent review of this and related effects is given by Duff.[7]

Now consider the $D=10$ supersymmetric Yang-Mills theory.[3] This model involves Majorana-

Weyl spinors, and vector gauge fields, both in the adjoint representation of some external gauge group (which we may ignore for this discussion). The physical states of these two fields form irreducible representations of the transverse rotation group O(8), whose highest weights are (1000) and (0001). One could also use (0010) for the spinor. If one computes the indices for either of these representations, one finds $I^{(0)} = I^{(2)} = 8$. In fact, all indices for these representations are identical. The obvious way to see this is to note that each of these representations consists of eight weights of unit norm.

It is more informative, however, to note that all indices agree for the boson and fermion fields of this model as a result of a symmetry. The symmetry is well-known mathematically,[8] but to my knowledge it has not been previously applied in the present physical context.[9] For every O(8) representation, R_1, there are two other O(8) representations, R_2 and R_3 (perhaps identical to R_1), which are related to R_1 by the principle of triality. That is, the representation R_2 (or R_3) may be obtained through a cyclic permutation (or the inverse permutation) of the first, third, and fourth weight components of R_1. Such a permutation may be understood in terms of automorphisms acting on the group algebra. It is easily seen that this permutation symmetry leaves the weight metric invariant, and hence in general it leads to a set of three representations whose indices are all identical. Obviously, the two O(8) representations for the $D=10$ supersymmetric Yang-Mills theory are related by such a triality transformation acting on their weights.

The contributions to the self-energy in (2) from bosons and fermions identically cancel for the $D=10$ supersymmetric Yang-Mills theory. An easily established corollary is that this leads to a vanishing one-loop charge renormalization for that theory. I believe that this vanishing charge renormalization may be viewed as a consequence of the triality properties of the model, since triality guarantees the equality of $I^{(0)}$, and $I^{(2)}$, for the fermions and bosons in the theory.

I have not yet extended the result in (2) beyond one loop for an arbitrary set of representations. However, we can make an obvious conjecture anticipating this extension. Let us conjecture that the k-loop result will involve the $2k$th (and lower order) indices of all the transverse rotation-group representations appearing in the k-loop Feynman diagrams. The order of the highest index to appear should equal the maximum number of vertices at which the current "probes" the space-time structure of the representation.

For supersymmetric theories, where the couplings among the various fermion and boson representations are strongly constrained, it is also quite conceivable that the net contribution to the particular gauge-invariant UV divergence of (2) is factorizable in all higher orders, permitting at least one factor of $I_{\text{boson}}^{(p)} - I_{\text{fermion}}^{(p)}$ to be extracted, generalizing the case for one loop following from (2). I suggest that this well-defined property can be proved in higher orders using supergraph techniques.

These conjectures on the index structure of higher-loop radiative corrections are consistent with known properties of the $D=10$ ($N=4$) supersymmetric Yang-Mills theory. This model is UV-finite through three-loop order[10] in four dimensions [e.g., $d=4$ in (2)] where the current correlation is the only independent physical UV divergence, and it is widely believed to be finite in all higher orders of perturbation theory. Our factorization conjecture links this UV behavior to the previously noted equality of fermion and boson representation indices of all orders. Given this link, we would understand the UV finiteness of the theory as a consequence of the triality symmetry. Even if the connection is not this direct, it is nevertheless fascinating to see such a correlation between all orders of perturbation theory (cancellation of UV divergences) and all orders of representation indices [vanishing $I_{\text{boson}}^{(p)} - I_{\text{fermion}}^{(p)}$].

I now discuss whether there is a similar correlation between the physical (on shell) UV divergence structure of $N=8$ supergravity and the generalized transverse rotation-group representation indices for the boson and fermion physical degrees of freedom in that theory.

For the $N=8$ supergravity theory,[4] the natural dimension of space-time is $D=11$. Physical states for the model form irreducible representations of the transverse rotation group O(9). The highest weights labeling those representations are

(2000), 44 "gravitons",

(0010), 84 "formitons",

(1001), 128 "gravitinos",

corresponding to symmetric rank-2 and antisymmetric rank-3 tensors and a Majorana spinor-vector, respectively.

One can now compare the higher indices for these representations to check equality for bosons and fermions. Since the group is no longer O(8), one might expect that triality is not available to facilitate this check. However, there is a theorem[1] that states that

$$I^{(p)}(\mathrm{O}(2n+1)) = \sum I^{(p)}(\mathrm{O}(2n)),$$

where the sum extends over all representations of O(2n) contained in the O(2n+1) representation in question. Thus we may embed O(8) in O(9) and use triality for O(8) as an aid in comparing higher boson and fermion indices for O(9).

The supergravity O(9) representations branch into O(8) representations as

$$(2000) = (2000) + (0000) + (1000),$$

$$(0010) = (0011) + (0100),$$

$$(1001) = (1001) + (1010) + (0001) + (0010).$$

Among the O(8) representations, we recognize several boson/fermion pairings for which all indices are guaranteed to be equal by triality. We may choose these to be $\{(1000), (0001)\}$ and $\{(0011), (1010)\}$. However, not all the representations allow pairing in this fashion. Hence the overall matching of the O(9) boson and fermion indices will occur if and only if the indices agree for the following sets of O(8) representations:

$$(2000) + (0100) + (0000) = \text{bosons}, \qquad (1001) + (0010) = \text{fermions}. \tag{3}$$

One immediately recognizes these sets as the Kronecker products

$$(1000) \times (1000) = (2000) + (0100) + (0000),$$

$$(1000) \times (0001) = (1001) + (0010).$$

Since the second terms on the left-hand sides form a boson/fermion pair whose indices match by triality, one might expect some, if not all, of the indices for the two products to agree.

Indeed, using several simple theorems,[1] one can see without computing any of the weights that the zeroth, second, fourth, and even the sixth indices for fermions and bosons are equal. Unfortunately, the agreement ends there. All O(9) indices higher than the sixth are unequal for the fermions and bosons of supergravity in 11 dimensions.

For example, by explicitly computing the lengths of the 64 weights for both bosons and fermions in (3), one finds that

$$I^{(8)}[(1000)\times(1000)] = 2816$$

while

$$I^{(8)}[(1000)\times(0001)] = 2624.$$

One may easily check these, and all other indices, by noting that the boson representations in (3) consist of 48 norm-2, 8 norm-4, and 8 norm-0 weights, while the fermion representations consist of 32 norm-3 and 32 norm-1 weights.

A priori, we expect that the eighth indices will arise when computing the UV divergences in on-shell invariant amplitudes for two-two scattering at the three-loop level. For example, as one can easily see, there are eight vertices in several diagrams contributing to such a scattering process. In supergravity, it is well-known[11] that this is the lowest-order process for which on-shell infinities are possible.

If the conjecture regarding the index structure of higher-order radiative corrections is exact, we would expect the $N = 8$ theory to be infinite at the three-loop level, since the eighth indices for fermions and bosons disagree. It is important to see if these simple group-theoretic considerations are actually supported by explicit three-loop calculations for the model. Such explicit calculations are being seriously considered.

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Hydrogen-Antihydrogen Oscillations, Double Proton Decay, and Grand Unified Theories

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The $\Delta B = \Delta L = 2$ processes hydrogen-antihydrogen oscillation and $pp \to e^+e^+$ or $\mu^+\mu^+$ (double proton decay) in grand unified theories are examined. Although these reactions are very suppressed in the minimal SU(5) model, their rates may be significantly increased by appending a 50-plet of Higgs scalars. In that case relatively light color sextet and doubly charged colorless scalars might mediate $pp \to e^+e^+$ or $\mu^+\mu^+$ at a rate observable in ongoing proton decay experiments. Consequences of this scenario are discussed.

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Several years ago Feinberg, Goldhaber, and Steigman[1] (FGS) suggested the possibility of hydrogen-antihydrogen oscillations $\mathrm{H} \equiv p + e \to \bar{p} + \bar{e} \equiv \bar{\mathrm{H}}$, via $\Delta B = \Delta L = 2$ interactions (B = baryon number, L = lepton number). In their scenario (neglecting CP violation) the linear combinations[2,3]

$$H_1 \equiv \frac{1}{\sqrt{2}}(\mathrm{H} + \bar{\mathrm{H}}), \tag{1a}$$

$$H_2 \equiv \frac{1}{\sqrt{2}}(\mathrm{H} - \bar{\mathrm{H}}) \tag{1b}$$

are mass eigenstates with mass difference $\delta \equiv |m_{H_1} - m_{H_2}|$. (Note the similarity with the K_L-K_S system.[2]) An initial H state, *free from external interactions*, will oscillate into $\bar{\mathrm{H}}$ and back to H with a period[1]

$$T_{\mathrm{H}\bar{\mathrm{H}}} = 2\pi/\delta \tag{2}$$

Of course, δ must be *very* small. Indeed, from astrophysical data on γ-ray flux (which would be perturbed by $\mathrm{H} \to \bar{\mathrm{H}}$ followed by annihilation) FGS set the rather stringent bound[1]

$$\delta < 2 \times 10^{-33} \text{ eV}, \tag{3a}$$

$$T_{\mathrm{H}\bar{\mathrm{H}}} > 7 \times 10^{10} \text{ yr}. \tag{3b}$$

The same $\Delta B = \Delta L = 2$ interaction that would allow H-$\bar{\mathrm{H}}$ oscillations could also give rise to the reaction $pp \to e^+e^+$ in nuclei, which we shall refer to as double proton decay. FGS estimated that the lifetime for this nuclear decay mechanism was approximately related to $T_{\mathrm{H}\bar{\mathrm{H}}}$ by[1]

$$\tau(pp \to e^+e^+) \simeq (3 \times 10^4 \text{ yr}^{-1})\, T_{\mathrm{H}\bar{\mathrm{H}}}^2. \tag{4}$$

So, experimental searches for proton decay (employing nuclei) can be used to bound $T_{\mathrm{H}\bar{\mathrm{H}}}$ via Eq. (4). They give[1] $T_{\mathrm{H}\bar{\mathrm{H}}} \gtrsim 6 \times 10^{12}$ yr, which is about two orders of magnitude better than the astrophysical bound in Eq. (3b).

The $\Delta B = \Delta L = 2$ interactions required for H-$\bar{\mathrm{H}}$ oscillation and double proton decay arise quite naturally in grand unified theories, which are better known for their $\Delta B = \Delta L = 1$ interactions and prediction that the proton decays (e.g., $p \to \pi^0 + e^+$). In the minimal Georgi-Glashow SU(5) model,[4] $\Delta B = \Delta L = 1$ superweak interactions are mediated by superheavy gauge bosons with masses $m_S \simeq 2 \times 10^{14}$ GeV. They give rise to a predicted proton lifetime of about[5] $\tau_p \simeq 10^{30}$–10^{31} yr (it scales like m_S^4). In such a theory $\Delta B = \Delta L = 2$ reactions can obviously occur as second-order superweak effects; but they are then highly suppressed. In the minimal SU(5) model $T_{\mathrm{H}\bar{\mathrm{H}}} \propto m_S^4$

